

Harnessing the Power of Modeling Tasks through the Lens of a Math Progression

Graham Fletcher

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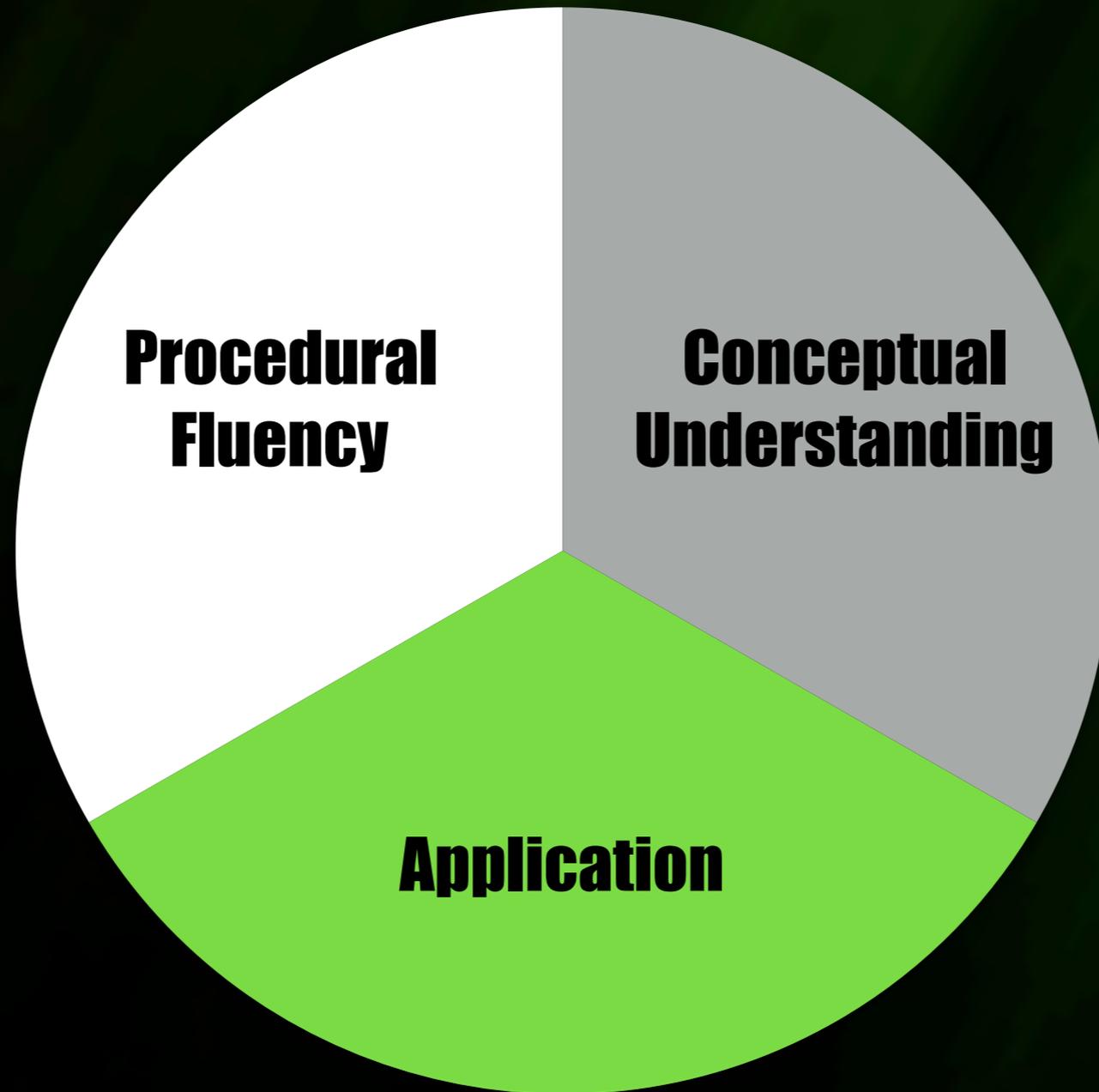


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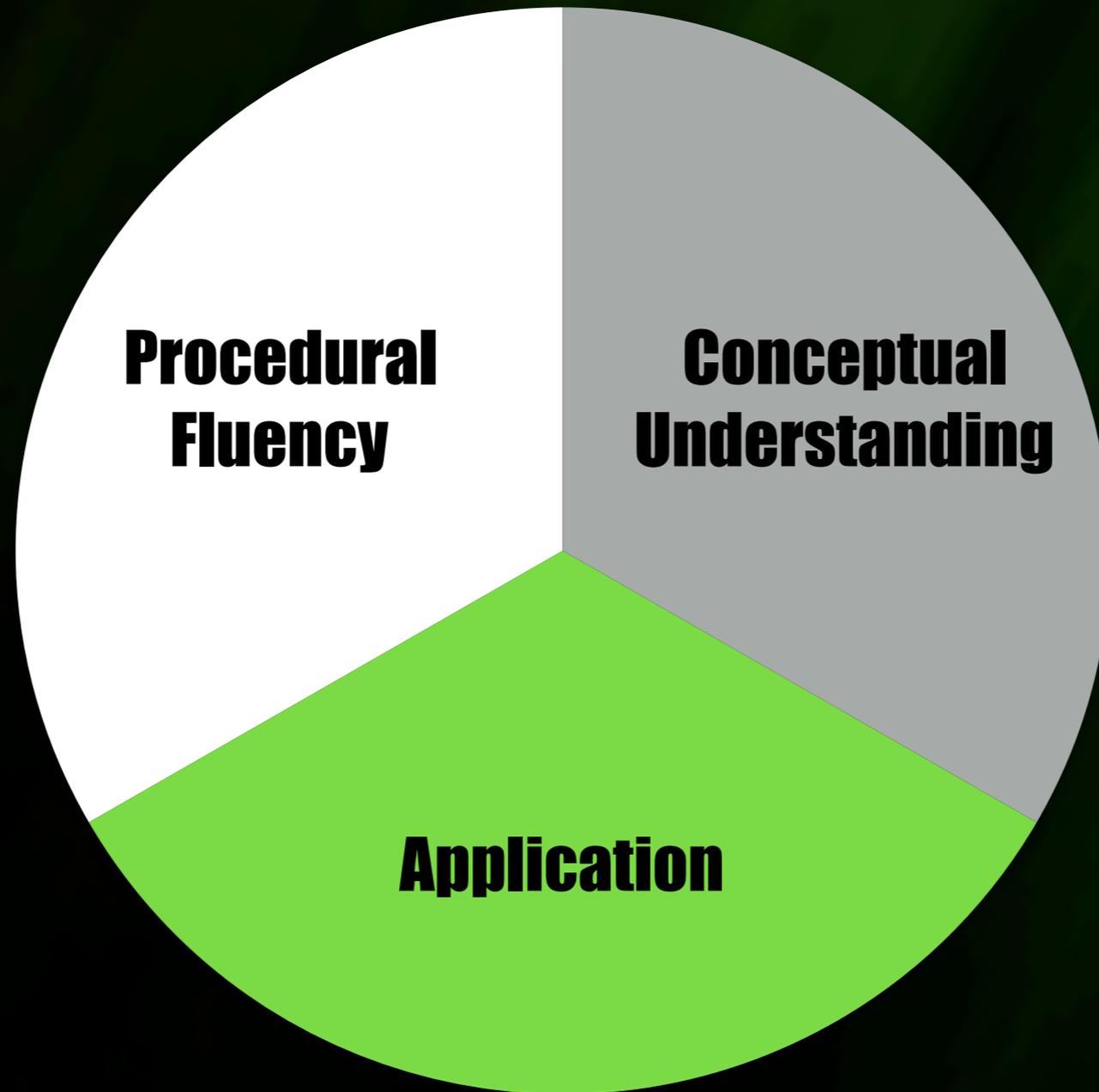


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**Procedural
Fluency**



**NEXT TIME YOU'RE AFRAID
TO SHARE IDEAS
REMEMBER SOMEONE
ONCE SAID IN A MEETING
LET'S MAKE A FILM WITH A
TORNADO FULL OF SHARKS**

Joe had some playing cards in his bag. Ashley gave him 13 more cards. Joe now has 21 cards. How many cards did Joe have in his bag?

13

21

You little plucker!

13

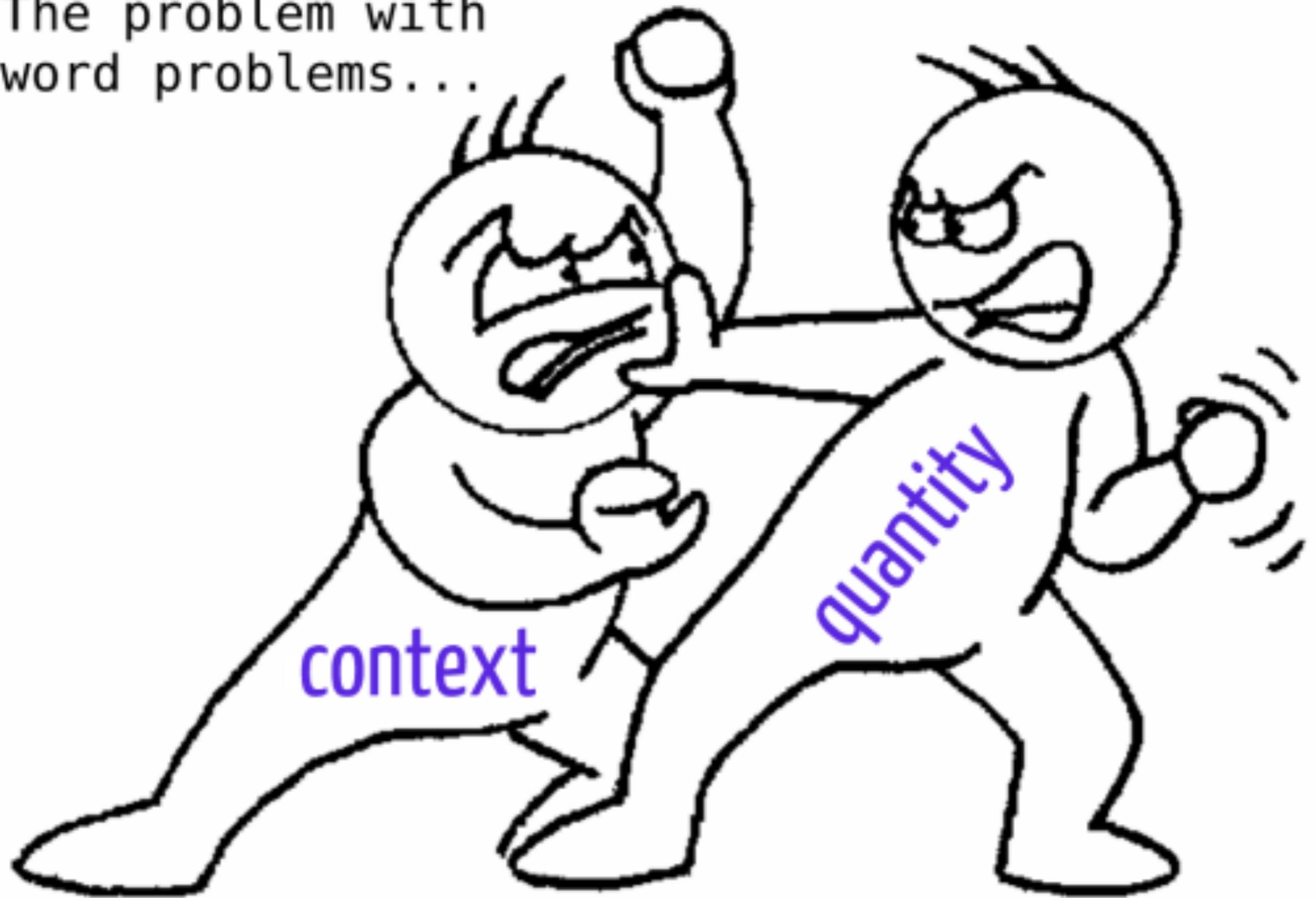
21

number
^

You little plucker!

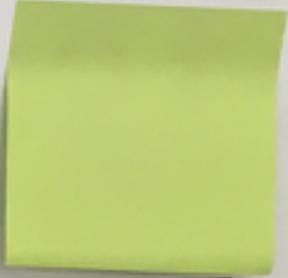
13
21

The problem with
word problems...



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Joe had some playing cards in his bag. Ashley gave him 13 more cards. Joe now has 21 cards. How many cards did Joe have in his bag?

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in his bag. Ashley gave him 
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How many cards did Joe
have in his bag?

$$6 \times \frac{5}{8}$$

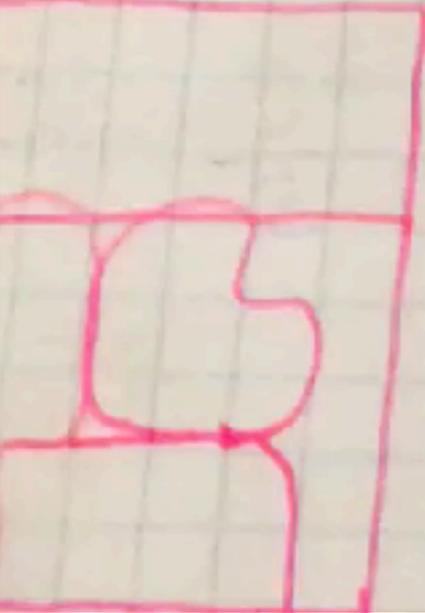
$$6 \times \frac{5}{8}$$

$$6 \text{ @ } \frac{10 + 3 + 5/8}{600 | 180}$$

$$6 \times 5/8 = 3 \frac{6}{8}$$

$$\begin{array}{r} 10 + 3 + 5/8 \\ \hline 600 \mid 180 \end{array}$$

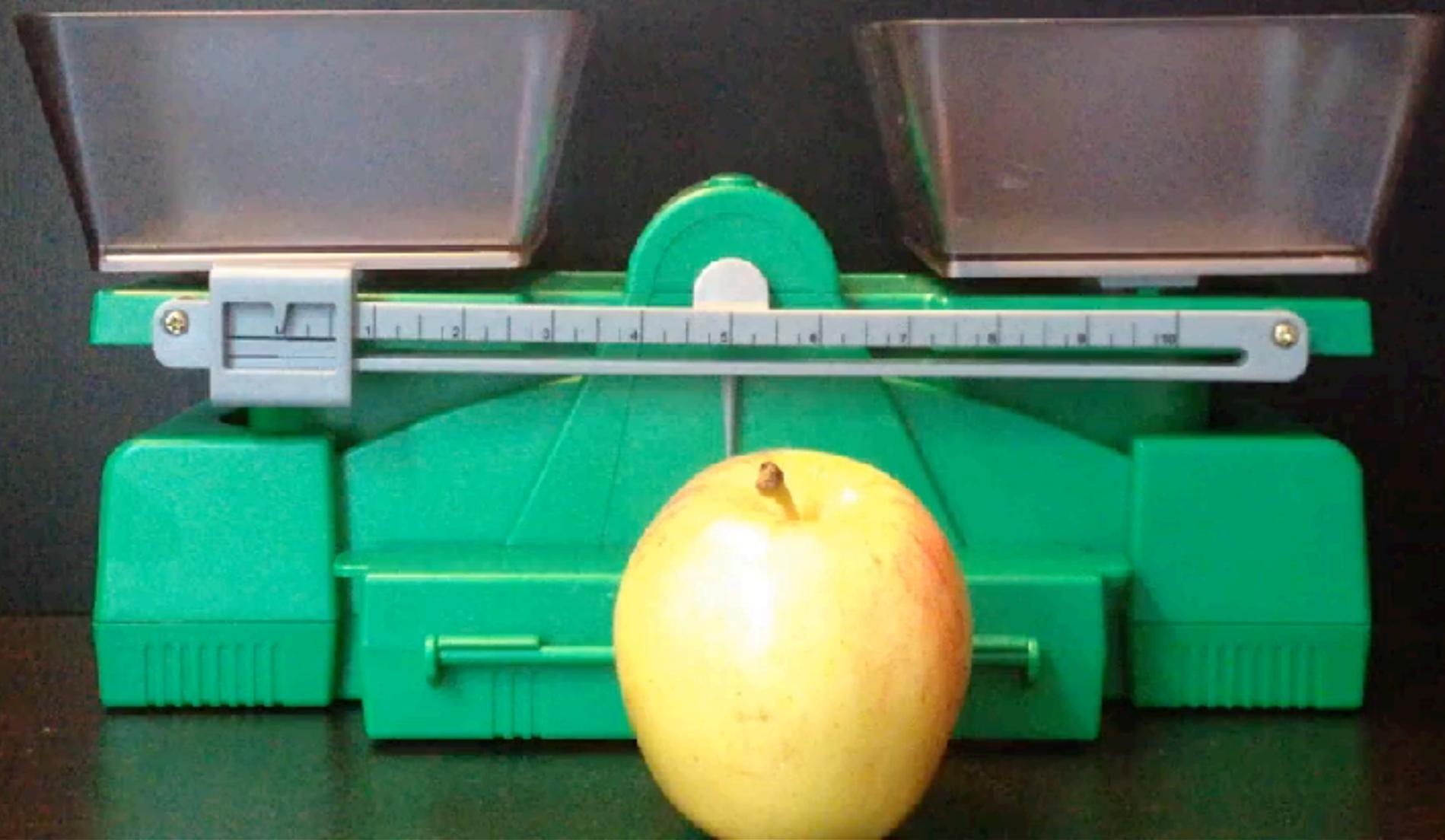
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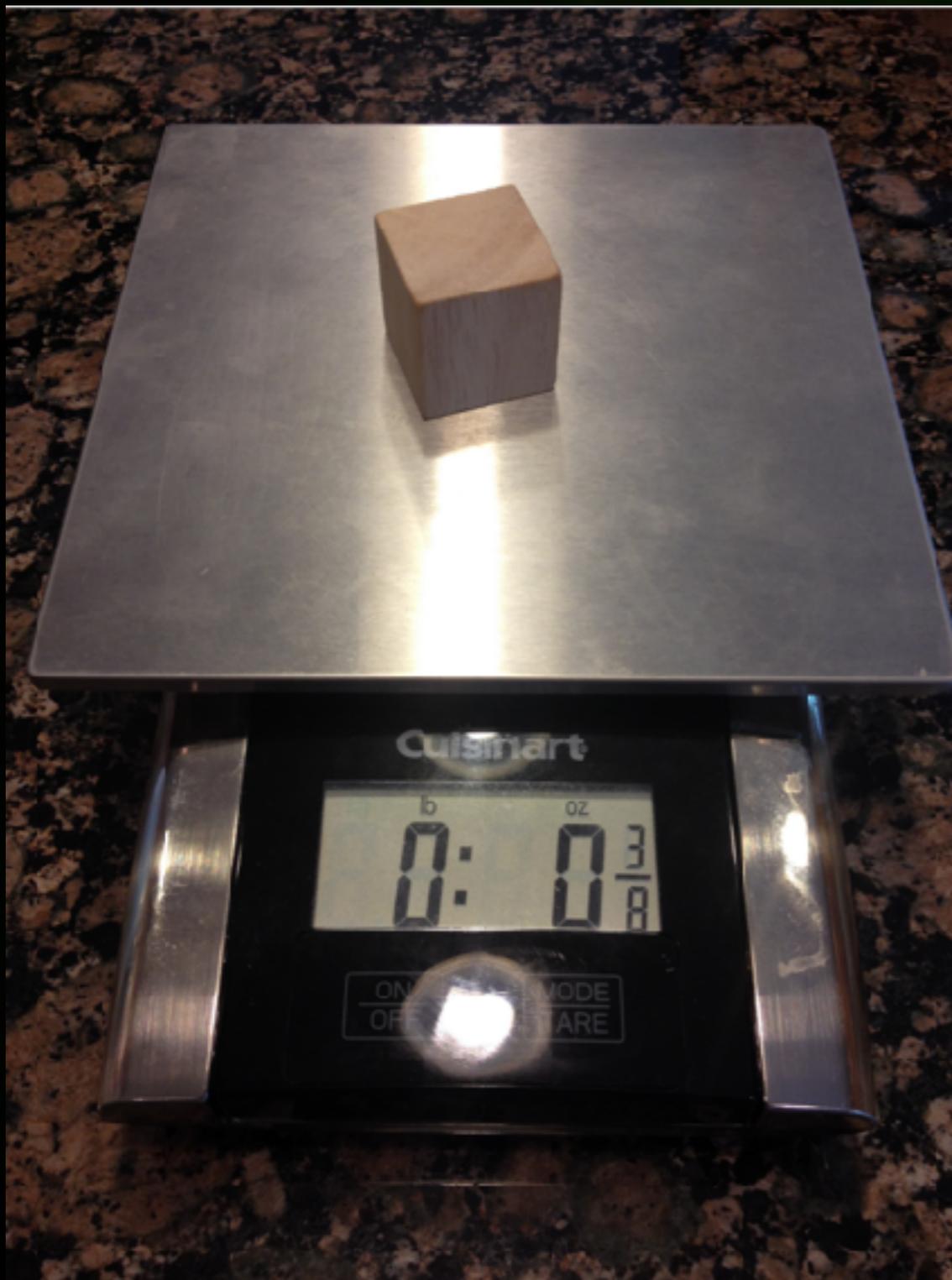

$$6 \times \frac{5}{8}$$

8

Today's Goals

- Understand the structure of 3-act task and see how they fit into the scope and sequence of a unit.
- Explore the importance of progressional understanding and how a good task can be used as formative assessment.
 - Numbers and Operations in Fractions
- Understand the importance of an effective closing and the role it plays in deciding our next move.







0:00 / 0:14



3-Act Tasks

Act 1:

- Real world problem or scenario presented
- What do you notice? What do you wonder?
- Make estimates

Act 2:

- Identify missing variables and missing variables to solve
- Define solution path using variables

Act 3:

- Solve and interpret results of the solution
- Validate answer

Most asked questions:

- How often should we use 3-Act Tasks?
How do they fit into the scope of a unit?
- How long does one task usually take?
- What if we don't have the time?

Orchestrating Discussions

Five practices constitute a model for effectively using student responses in whole-class discussions that can potentially make teaching with high-level tasks more manageable for teachers.

Margaret S. Smith, Elizabeth K. Hughes, Randi A. Engle, and Mary Kay Stein



Margaret S. Smith, pegso@pitt.edu, is an associate professor of mathematics education at the University of Pittsburgh. Over the past decade, she has been developing research-based materials for use in the professional development of mathematics teachers and studying what teachers learn from the professional development in which they engage. Elizabeth K. Hughes, elizabeth.hughes@pitt.edu, recently finished her doctorate in mathematics education at the University of Pittsburgh. Her areas of interest include preservice secondary mathematics teacher education and the use of practice-based materials in developing teachers' understanding of what it means to teach and learn mathematics. Randi A. Engle, raengle@berkeley.edu, is an assistant professor of mathematics education and the social context of learning at the University of California Berkeley. She is interested in developing practical theories for how mathematics teachers can create discussion-based learning environments that promote strong student engagement, learning, and transfer. Mary Kay Stein, mstein@pitt.edu, is a professor of learning solutions and policy and the director of the Learning Policy Center at the University of Pittsburgh. Her research focuses on instructional practice and the organizational and policy conditions that shape it.

Discussions that focus on cognitively challenging mathematical tasks, namely, those that promote thinking, reasoning, and problem solving, are a primary mechanism for promoting conceptual understanding of mathematics (Hartano and Inagaki 1991; Michaels, O'Connor, and Resnick forthcoming). Such discussions give students opportunities to share ideas and clarify understandings, develop convincing arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things from other perspectives (NCTM 2000).

Although discussions about high-level tasks provide important

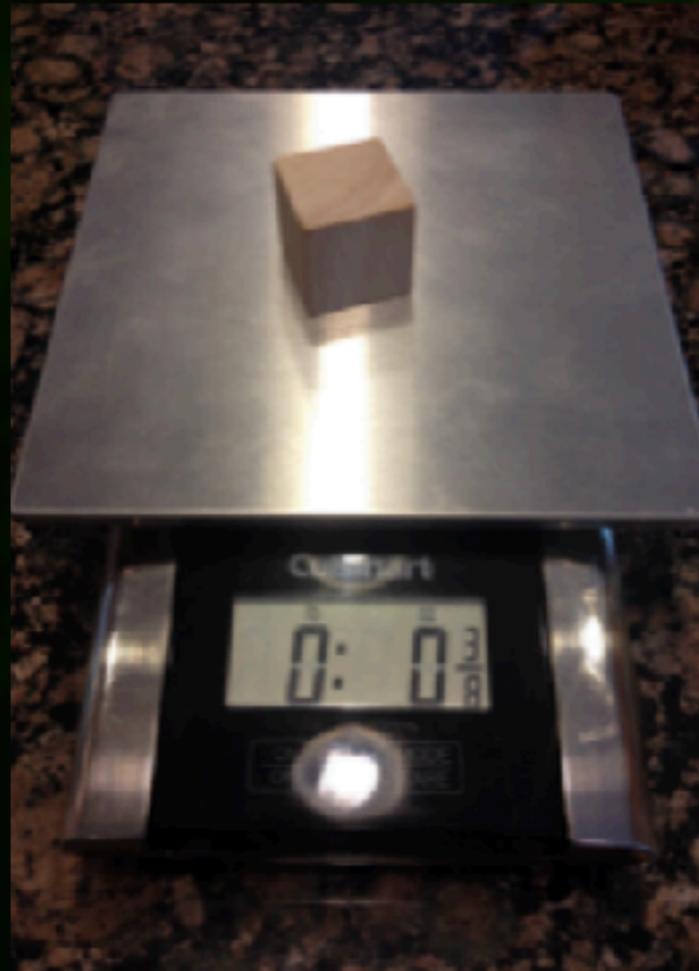
The **5** practices are:

1. Anticipating student responses to challenging mathematical tasks;
2. Monitoring students' work on and engagement with the tasks;
3. Selecting particular students to present their mathematical work;
4. Sequencing the student responses that will be displayed in a specific order and;
5. Connecting different students' responses and connecting the responses to key mathematical ideas.

Task Planning Page

Learning Targets:		
Questions and Look-Fors:		
Strategy	Who and What	Order
Notes:		

Anticipating → Monitoring → Selecting → Sequencing → Connecting



The **5** practices are:

1. **Anticipating** student responses to challenging mathematical tasks;
2. **Monitoring** students' work on and engagement with the tasks;
3. **Selecting** particular students to present their mathematical work;
4. **Sequencing** the student responses that will be displayed in a specific order and;
5. **Connecting** different students' responses and connecting the responses to key mathematical ideas.

$$\frac{3}{8} + \frac{3}{8} = \frac{6}{8} + \frac{6}{8} = \frac{12}{8} \quad | \quad \frac{1}{2} + \frac{1}{9} = 3\frac{2}{9}$$

$$3\frac{0}{8} + \frac{4}{8} = 4\frac{4}{8}$$

S1

$$\begin{array}{r} 2 \\ 4 \\ 4 \\ 4 \\ 4 \\ \hline 18 \end{array}$$

S2

$$5\frac{1}{4} \div \frac{3}{8} = 5\frac{2}{8} \div \frac{3}{8} = \frac{42}{8} \div \frac{3}{8} = 14$$

S3

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} = \frac{9}{8} = 1\frac{1}{8}$$

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} = \frac{9}{8} = 1\frac{1}{8}$$

$$\frac{12}{4} + \frac{3}{8} + \frac{3}{8} = \frac{15}{4} = 3\frac{3}{4}$$

$$\frac{6}{8} + \frac{3}{8} + \frac{3}{8} = \frac{12}{8} = 1\frac{4}{8} = 1\frac{1}{2}$$

S4

$$5\frac{1}{4} \div \frac{3}{8} = \frac{42}{8} \div \frac{3}{8}$$

$$\frac{42}{8} \times \frac{8}{3} = \frac{336}{24} = 14$$

S5

6. Construct a viable argument or share a reflection:

A = $5\frac{1}{4}$ B = $\frac{3}{8}$

$\frac{3}{8} \times 2 = \frac{6}{8} + \frac{3}{8} = \frac{9}{8}$ it takes 4 $\frac{3}{8}$ blocks to make $5\frac{1}{4}$ of an apple

Answer: 14

S6

6. Construct a viable argument or share a reflection:

Answer: 14

$$3\frac{6}{8} - \frac{6}{8} - 1\frac{1}{8} - 1\frac{4}{8} - 1\frac{7}{8} - 2\frac{2}{8} = 14$$

$$2\frac{5}{8} - 3 - 3\frac{3}{8} - 3\frac{6}{8} - 1\frac{1}{8} - 4\frac{4}{8} - 4\frac{7}{8} - 5\frac{2}{8} = 14$$

S7

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8}$$

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8}$$

$$1\frac{1}{8}$$

$$2\frac{2}{8}$$

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8}$$

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8}$$

$$3\frac{3}{8}$$

$$4\frac{4}{8}$$

$$1\frac{1}{4} + 3\frac{3}{8}$$

$$5$$

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8}$$

$$5\frac{1}{4}$$

$$5\frac{1}{4} \div \frac{3}{8} = \frac{42}{8} \div \frac{3}{8}$$

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$$5\frac{1}{4} \div \frac{3}{8} = 5\frac{2}{8} \div \frac{3}{8} = \frac{42}{8} \div \frac{3}{8} = 14$$

$$\frac{3}{8} + \frac{3}{8} = \frac{6}{8} + \frac{6}{8} = \frac{12}{8} \quad | \quad \frac{4}{8} + \frac{4}{8} = 3\frac{0}{8}$$

$$3\frac{0}{8} + 1\frac{4}{8} = 4\frac{4}{8} + \frac{12}{8}$$

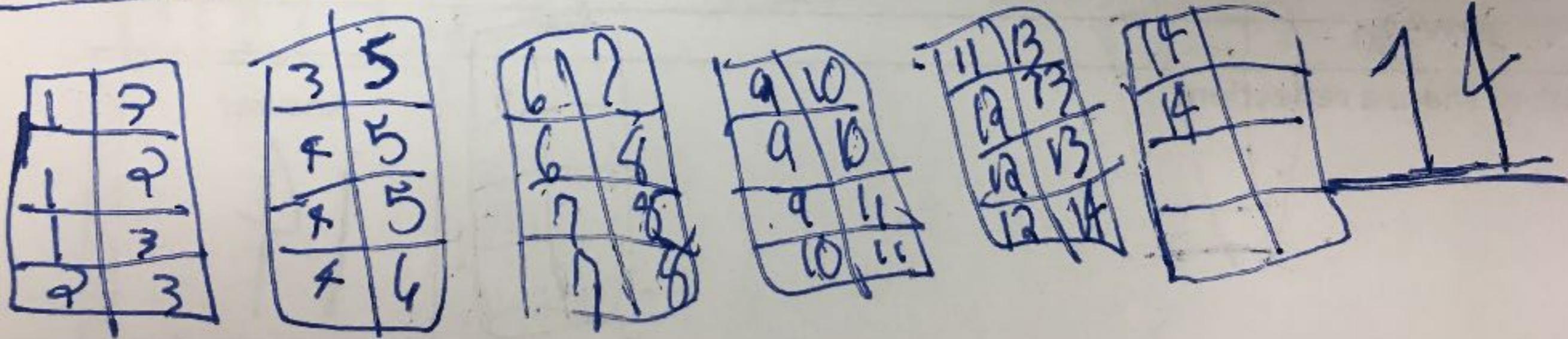
6. Construct a viable argument or share a reflection:

Answer

14

B $-\frac{3}{8} - \frac{6}{8} - 1\frac{1}{8} - 1\frac{4}{8} - 1\frac{7}{8} - 2\frac{2}{8} -$

$2\frac{5}{8} - 3 - 3\frac{3}{8} - 3\frac{6}{8} - 4\frac{1}{8} - 4\frac{4}{8} - 4\frac{7}{8} - 5\frac{2}{8}$



$$\frac{3}{8} + \frac{3}{8} = \frac{6}{8} + \frac{4}{8} = \frac{12}{8} \quad | \quad \frac{4}{8} + \frac{4}{8} = 3\frac{0}{8}$$

$$3\frac{0}{8} + 1\frac{4}{8} = 4\frac{4}{8} + \frac{12}{8}$$

S1

$$\begin{array}{r} 2 \\ 4 \\ + 4 \\ 4 \\ \hline 6 \\ 18 \end{array}$$

S2

S3

$$5\frac{1}{4} \div \frac{3}{8} = 5\frac{2}{8} \div \frac{3}{8} = \frac{42}{8} \div \frac{3}{8} = 14$$

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} = \frac{3}{8} + \frac{3}{8} + \frac{3}{8}$$

$$1\frac{1}{8} \quad 2\frac{2}{8}$$

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} = \frac{3}{8} + \frac{3}{8} + \frac{3}{8}$$

$$3\frac{3}{8} \quad 4\frac{4}{8}$$

$$1\frac{1}{4} + 3\frac{3}{8} = 5$$

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} = 5\frac{1}{4}$$

S4

$$5\frac{1}{4} \div \frac{3}{8} = \frac{42}{8} \div \frac{3}{8}$$

$$\frac{42}{8} \times \frac{8}{3} = \frac{336}{24} = 14$$

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Answer: 14

S6

6. Construct a viable argument or share a reflection:

Answer: 14

$$\frac{3}{8} + \frac{6}{8} + 1\frac{1}{8} + 1\frac{4}{8} + 1\frac{7}{8} + 2\frac{2}{8} = 14$$

$$2\frac{5}{8} - 3 - 3\frac{3}{8} - 3\frac{6}{8} - 4\frac{1}{8} - 4\frac{4}{8} - 4\frac{7}{8} - 5\frac{2}{8} = 14$$

S7

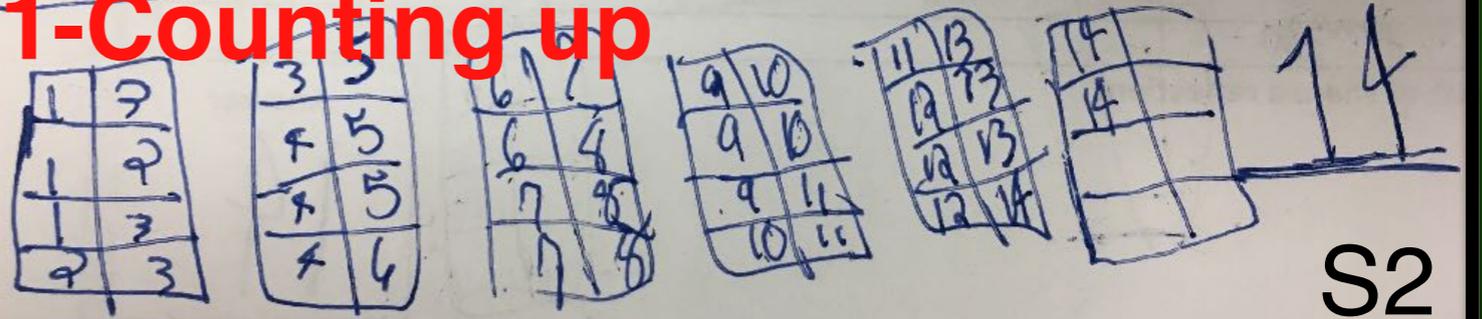
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$$3\frac{0}{8} + 1\frac{4}{8} = 4\frac{4}{8} + \frac{12}{8}$$

S1

$$\begin{array}{r} 2 \\ 4 \\ + 4 \\ 4 \\ \hline 6 \\ 18 \end{array}$$

1-Counting up



S2

$$5\frac{1}{4} \div \frac{3}{8} = 5\frac{2}{8} \div \frac{3}{8} = \frac{42}{8} \div \frac{3}{8} = 14$$

S3

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} = \frac{3}{8} + \frac{3}{8} + \frac{3}{8}$$

$$1\frac{1}{8} \quad 2\frac{2}{8}$$

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} = \frac{3}{8+8+8} = 4\frac{4}{8}$$

$$1\frac{1}{4} + 3\frac{3}{8} = 5$$

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S4

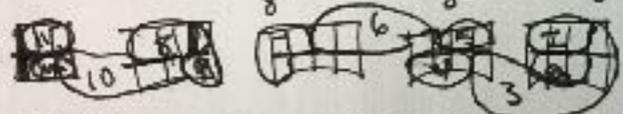
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S5

6. Construct a viable argument or share a reflection:

1-Counting up



it takes 4 $\frac{3}{8}$ blocks to make $5\frac{1}{4}$ of an apple

Answer

14

S6

6. Construct a viable argument or share a reflection:

$$3 - \frac{1}{8} - \frac{1}{8} - \frac{1}{8} - \frac{1}{8} - \frac{1}{8} - \frac{1}{8} - 2\frac{2}{8} = 14$$

$$2\frac{5}{8} - 3 - 3\frac{3}{8} - 3\frac{3}{8} - 4\frac{7}{8} - 4\frac{7}{8} - 4\frac{7}{8} - 5\frac{2}{8}$$

1-Counting up

Answer

14

S7

$$\frac{3}{8} + \frac{3}{8} = \frac{6}{8} + \frac{4}{8} = \frac{12}{8} \quad | \quad \frac{4}{8} + \frac{4}{8} = 3\frac{0}{8}$$

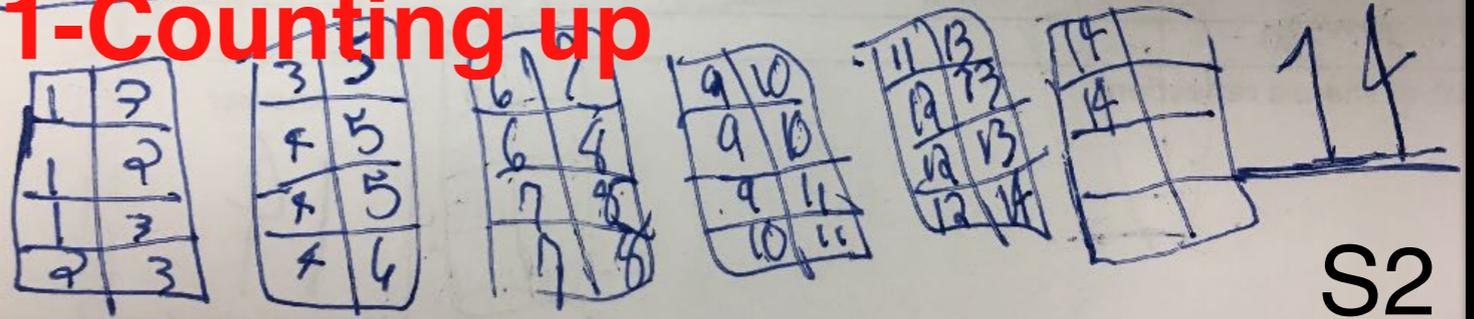
$$3\frac{0}{8} + 1\frac{4}{8} = 4\frac{4}{8} + \frac{12}{8}$$

2-Counting up then doubling

S1

$$\begin{array}{r} 4 \\ + 4 \\ + 4 \\ \hline 6 \\ \hline 18 \end{array}$$

1-Counting up



S2

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S3

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$$1\frac{1}{8} \quad 2\frac{2}{8}$$

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} = \frac{3}{8+8+8} = 4\frac{4}{8}$$

2-Counting up then doubling

S4

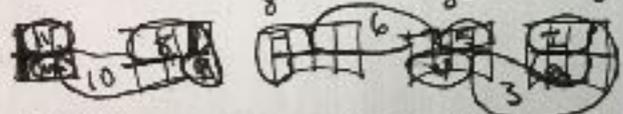
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S5

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1-Counting up



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1-Counting up

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S7

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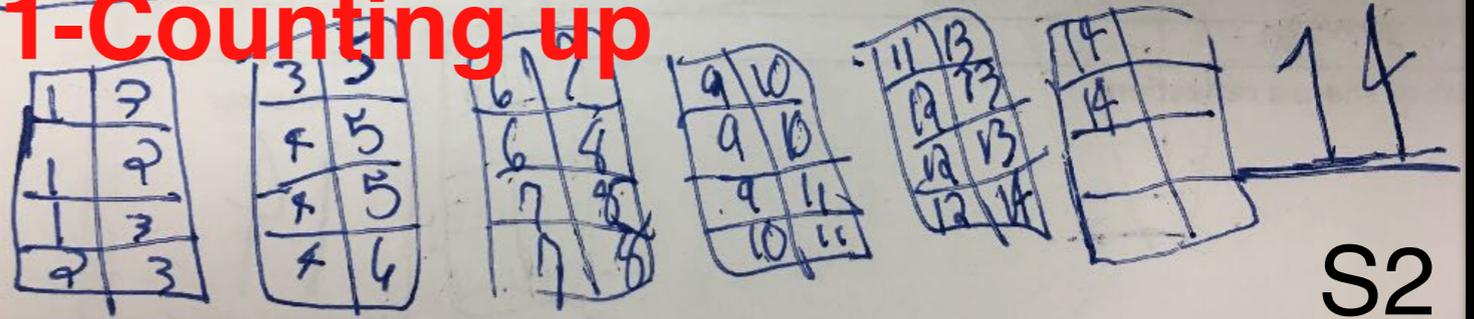
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2-Counting up then doubling

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1-Counting up



S2

$$5\frac{1}{4} \div \frac{3}{8} = 5\frac{2}{8} \div \frac{3}{8} = \frac{42}{8} \div \frac{3}{8} = 14$$

3-Common Denominator

S3

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} = \frac{3}{8} + \frac{3}{8} + \frac{3}{8}$$

$$1\frac{1}{8} \quad 2\frac{2}{8}$$

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} = \frac{3}{8+8+8} = 4\frac{4}{8}$$

2-Counting up then doubling

S4

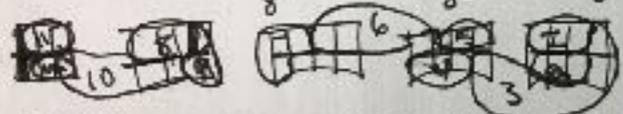
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S5

6. Construct a viable argument or share a reflection:

1-Counting up



it takes 4 $\frac{3}{8}$ blocks to make $5\frac{1}{4}$ of an apple

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1-Counting up

Answer

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S7

$$\frac{3}{8} + \frac{3}{8} = \frac{6}{8} + \frac{4}{8} = \frac{12}{8} \quad | \quad \frac{4}{8} + \frac{4}{8} = 3\frac{0}{8}$$

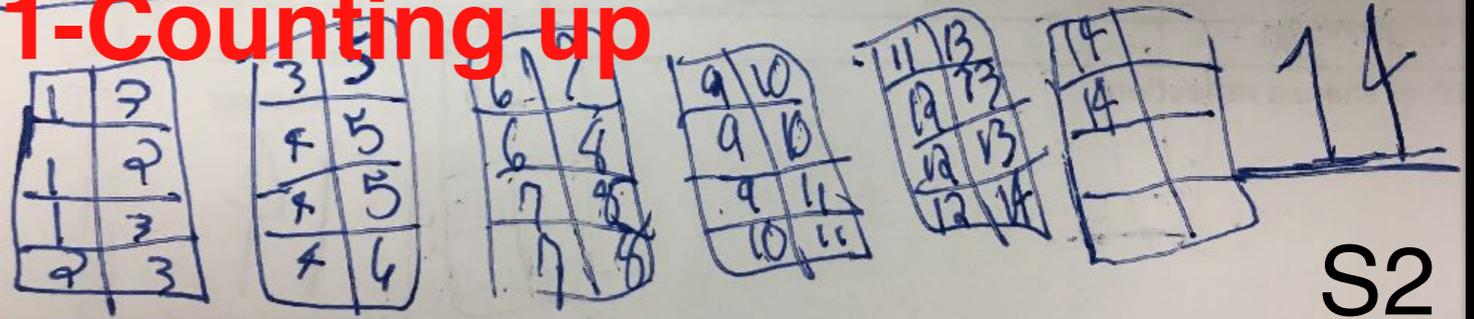
$$3\frac{0}{8} + 1\frac{4}{8} = 4\frac{4}{8} + \frac{4}{8}$$

2-Counting up then doubling

S1

$$\begin{array}{r} 4 \\ + 4 \\ + 4 \\ \hline 6 \\ \hline 18 \end{array}$$

1-Counting up



S2

$$5\frac{1}{4} \div \frac{3}{8} = 5\frac{2}{8} \div \frac{3}{8} = \frac{42}{8} \div \frac{3}{8} = 14$$

3-Common Denominator

S3

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} = \frac{3}{8} + \frac{3}{8} + \frac{3}{8}$$

$$1\frac{1}{8} \quad 2\frac{2}{8}$$

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} = \frac{3}{8+8+8} = \frac{9}{24} = 4\frac{4}{8}$$

2-Counting up then doubling

S4

$$5\frac{1}{4} \div \frac{3}{8} = \frac{42}{8} \div \frac{3}{8}$$

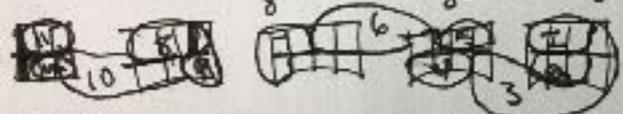
Procedural

$$\frac{42}{8} \times \frac{8}{3} = \frac{336}{24} = 14$$

S5

6. Construct a viable argument or share a reflection:

1-Counting up



it takes 4 $\frac{3}{8}$ blocks to make $5\frac{1}{4}$ of an apple

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6. Construct a viable argument or share a reflection:

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1-Counting up

Answer

14

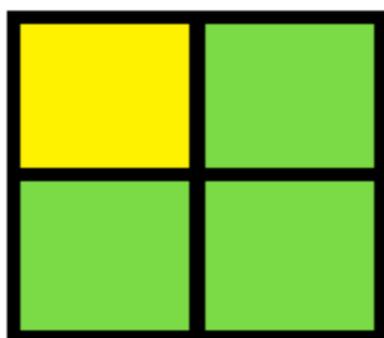
S7

Practice

Unit Fractions

Representation of a Fraction

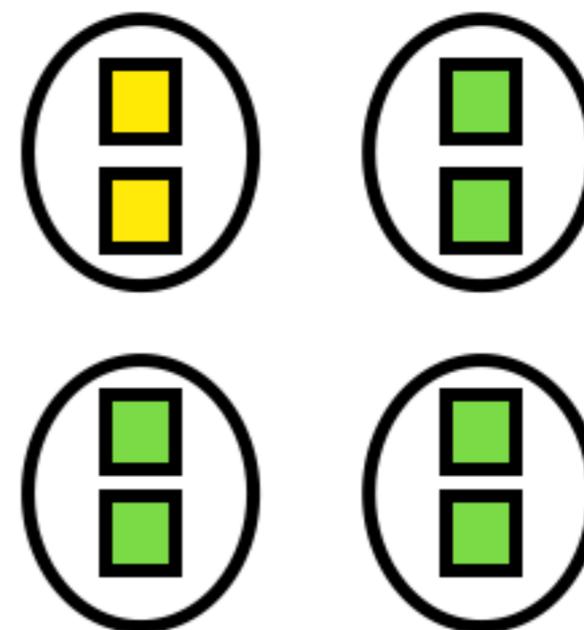
Area



Length



Set



unit fraction $\frac{1}{a}$

Say this fraction

$$\frac{3}{4}$$

Say this fraction

$$\frac{3}{4}$$

three one-fourths

$$3 = 1 + 1 + 1$$

$$3 = 1 + 1 + 1$$

$$\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$

Equivalent Fractions

Equal Fraction

$$\frac{2}{3} = \frac{\blacksquare}{\blacksquare}$$

$$\frac{3}{4} = \frac{\blacksquare}{\blacksquare}$$

$$\frac{2}{6} = \frac{\blacksquare}{\blacksquare}$$

Equal Fraction

$$\frac{2}{3} = \frac{5}{6} \quad \frac{3}{4} = \frac{7}{8}$$

$$\frac{2}{6} = \frac{5}{9}$$

Equal Fraction

WWTTFF??

$$\frac{2}{6} = \frac{5}{9}$$

Equal Fraction

What's **T**his **F**raction?

$$\frac{2}{6} = \frac{5}{9}$$

Equal Fraction

$$\frac{2}{3} = \frac{5}{6} \quad \frac{3}{4} = \frac{7}{8}$$

$$\frac{2}{6} = \frac{5}{9}$$

Name _____

Date _____

1. Draw a rectangular fraction model to find the sum. Simplify your answer, if possible.

a. $\frac{1}{4} + \frac{1}{3} =$

b. $\frac{1}{4} + \frac{1}{5} =$

Name _____

Date _____

1. Draw a rectangular fraction model to find the sum. Simplify your answer, if possible.

a. $\frac{1}{4} + \frac{1}{3} =$

b. $\frac{1}{4} + \frac{1}{5} =$

It is possible to over-emphasize the importance of simplifying fractions in this way. There is no mathematical reason why fractions must be written in simplified form, although it may be convenient to do so in some cases.

What about “the test”?

$\frac{3}{6} + \frac{1}{6}$ is equal to which of the following?

a. $\frac{4}{12}$

b. $\frac{8}{12}$

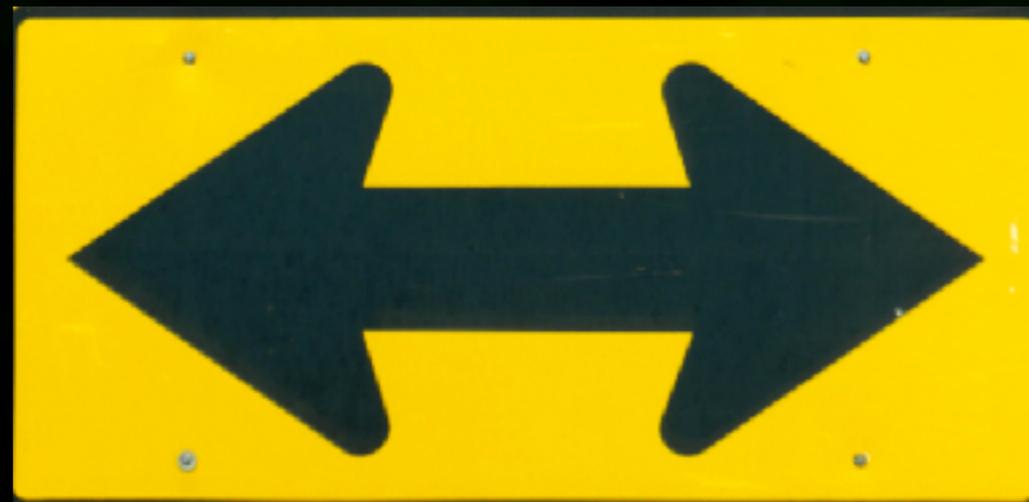
c. $\frac{3}{6}$

d. None of the above



Simplifying

Equivalence



Comparing Fractions



Which girl ate more apple?



Slices: 10

Slices: 7



twelfths

Pause

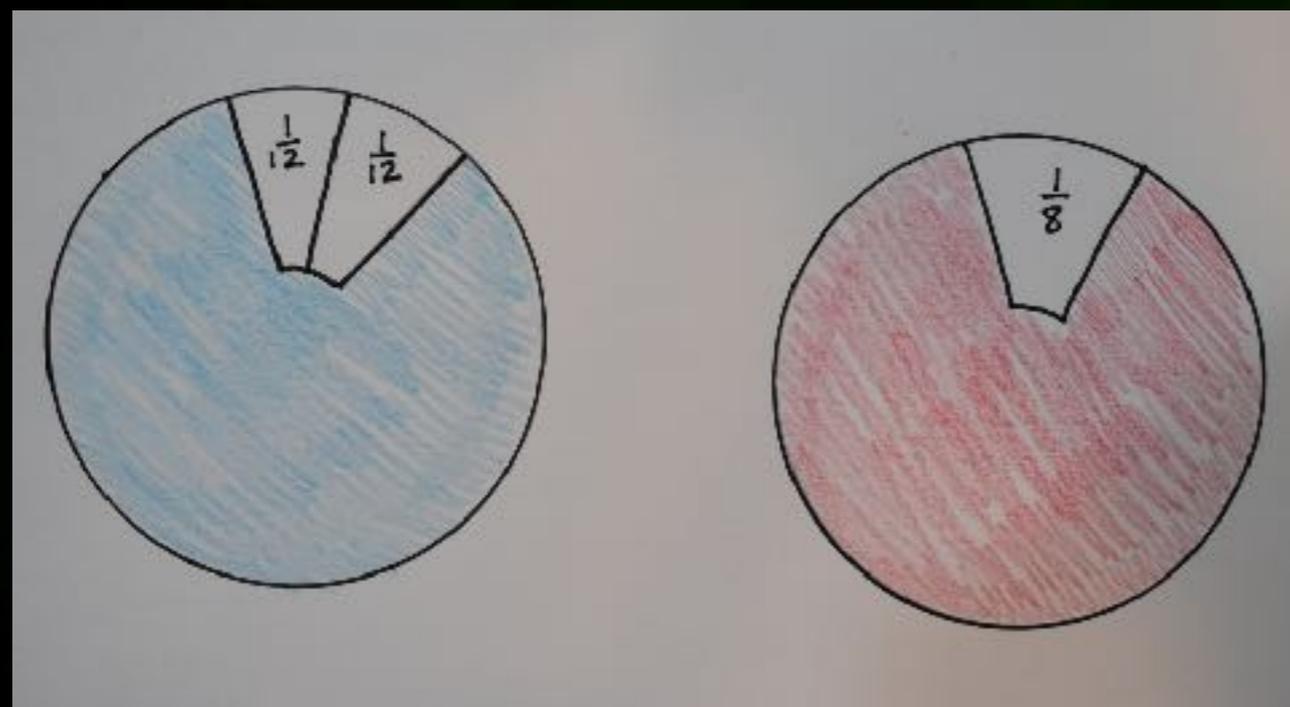
||



eighths

Apple Eat Off

Act-3



It Takes 3 to Prove it to Me

$$\frac{1}{4}$$

$$\frac{3}{4}$$

Common Denominator

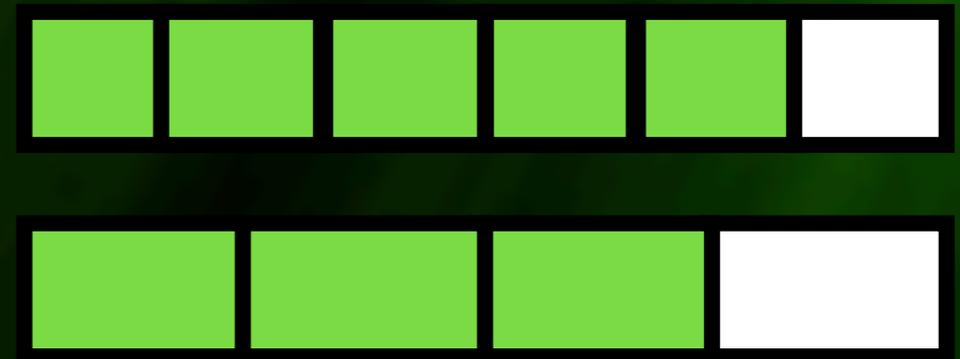


Unit Fraction Understanding

$$\frac{5}{6}$$

$$\frac{3}{4}$$

Missing Parts

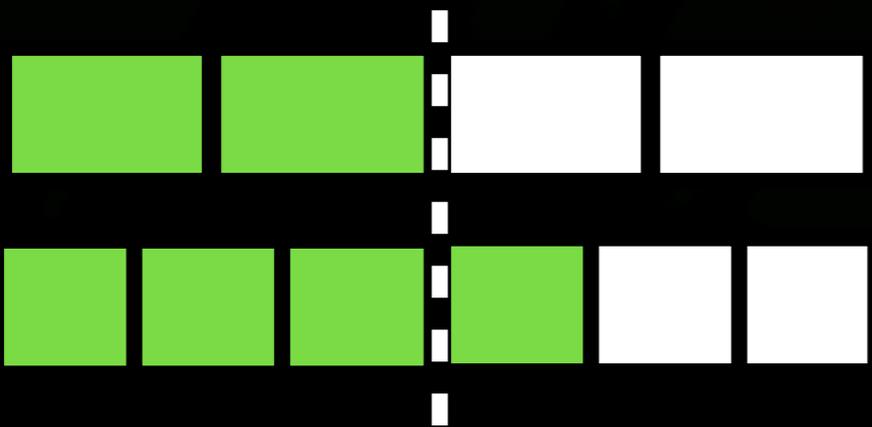


Unit Fraction Understanding

$$\frac{2}{4}$$

$$\frac{4}{6}$$

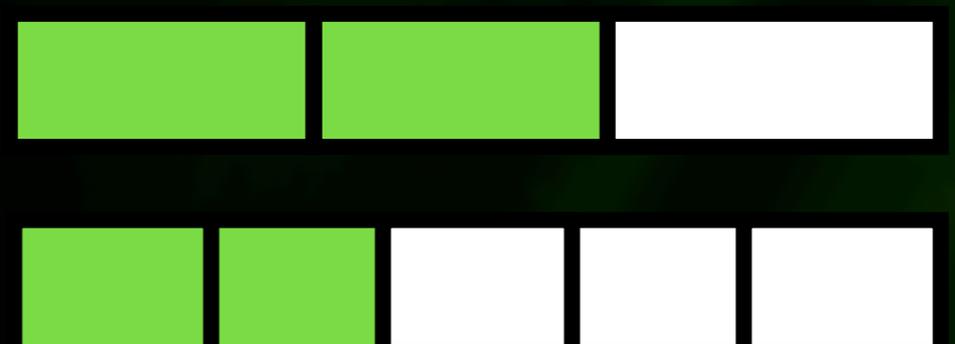
Benchmark



$$\frac{2}{3}$$

$$\frac{2}{5}$$

Common Numerator



Unit Fraction Understanding

Comparing Fractions

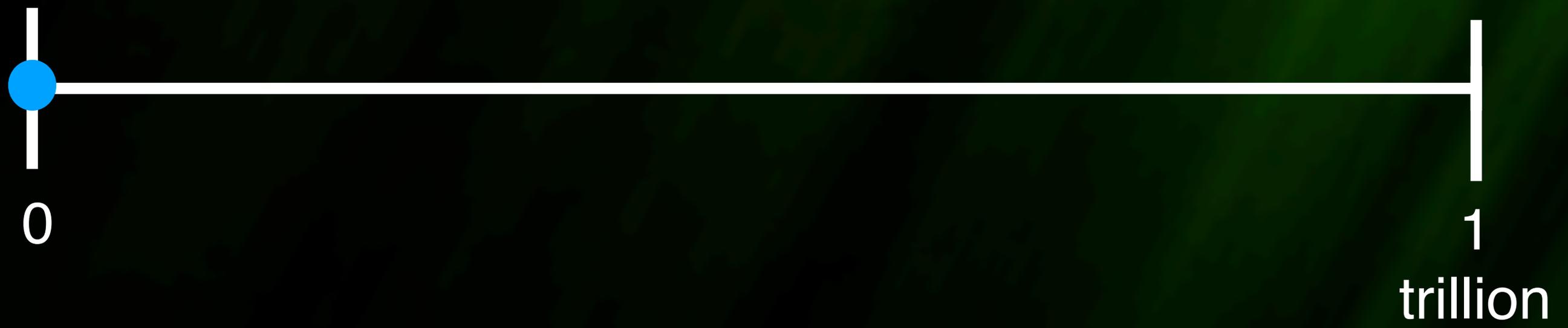
CCSS.MATH.CONTENT.3.NF.A.3.D

Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

CCSS.MATH.CONTENT.4.NF.A.2

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

Where does 1 billion go on the number line?



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