

The Power of Progressions: Untangling the Knotty Areas of Teaching and Learning Mathematics

Graham Fletcher

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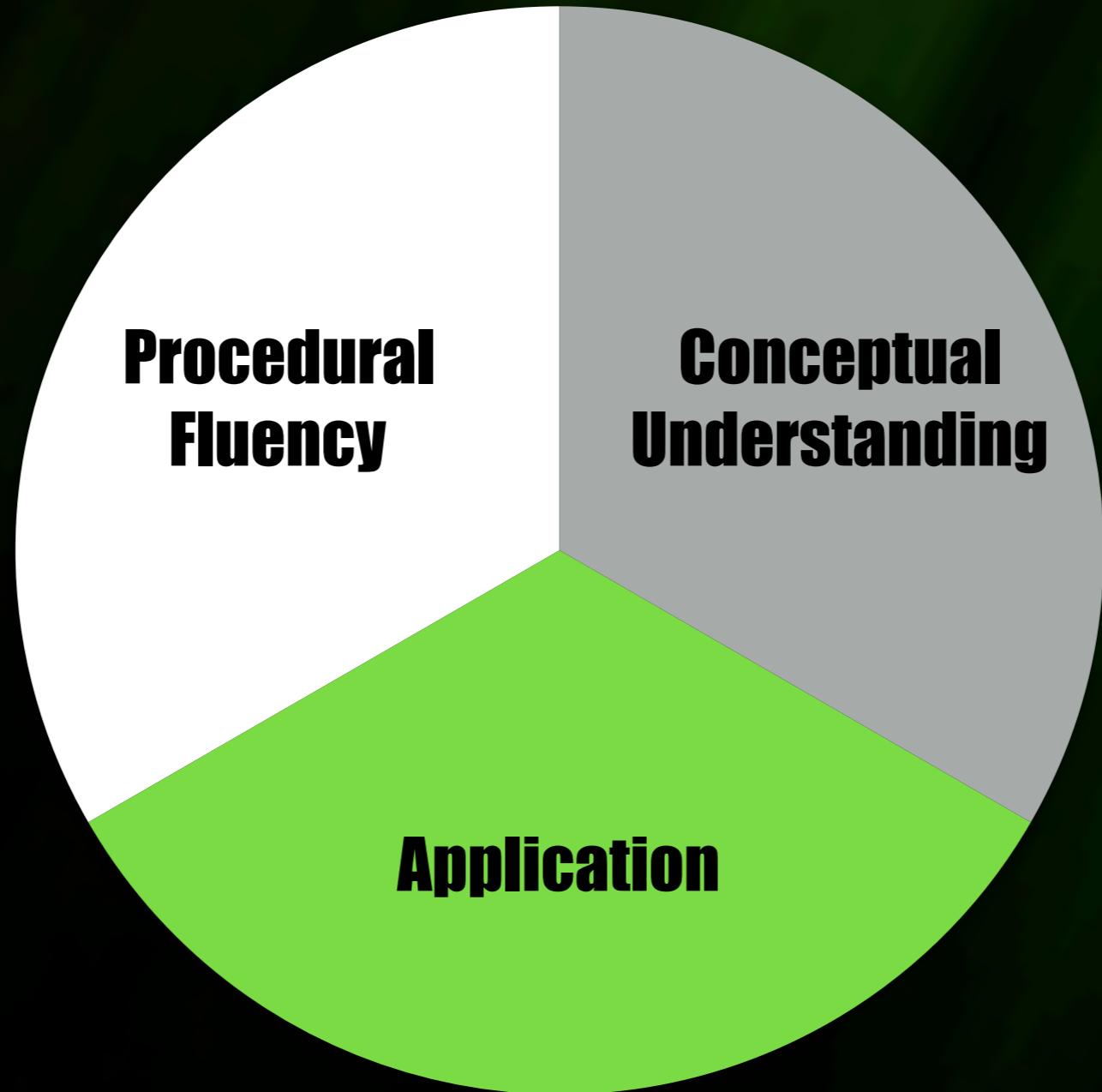
[@gfletchy](https://twitter.com/gfletchy)

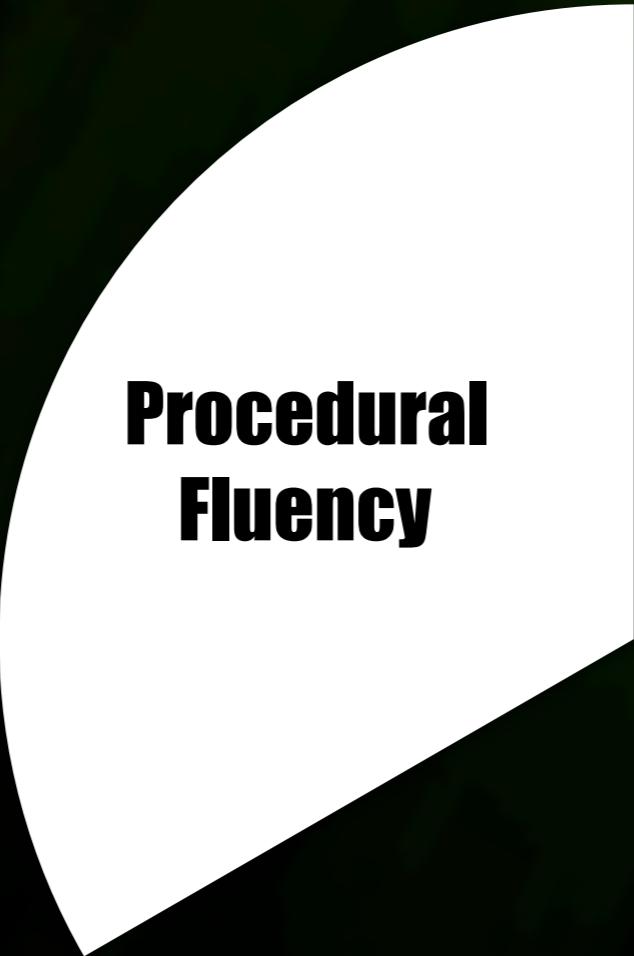


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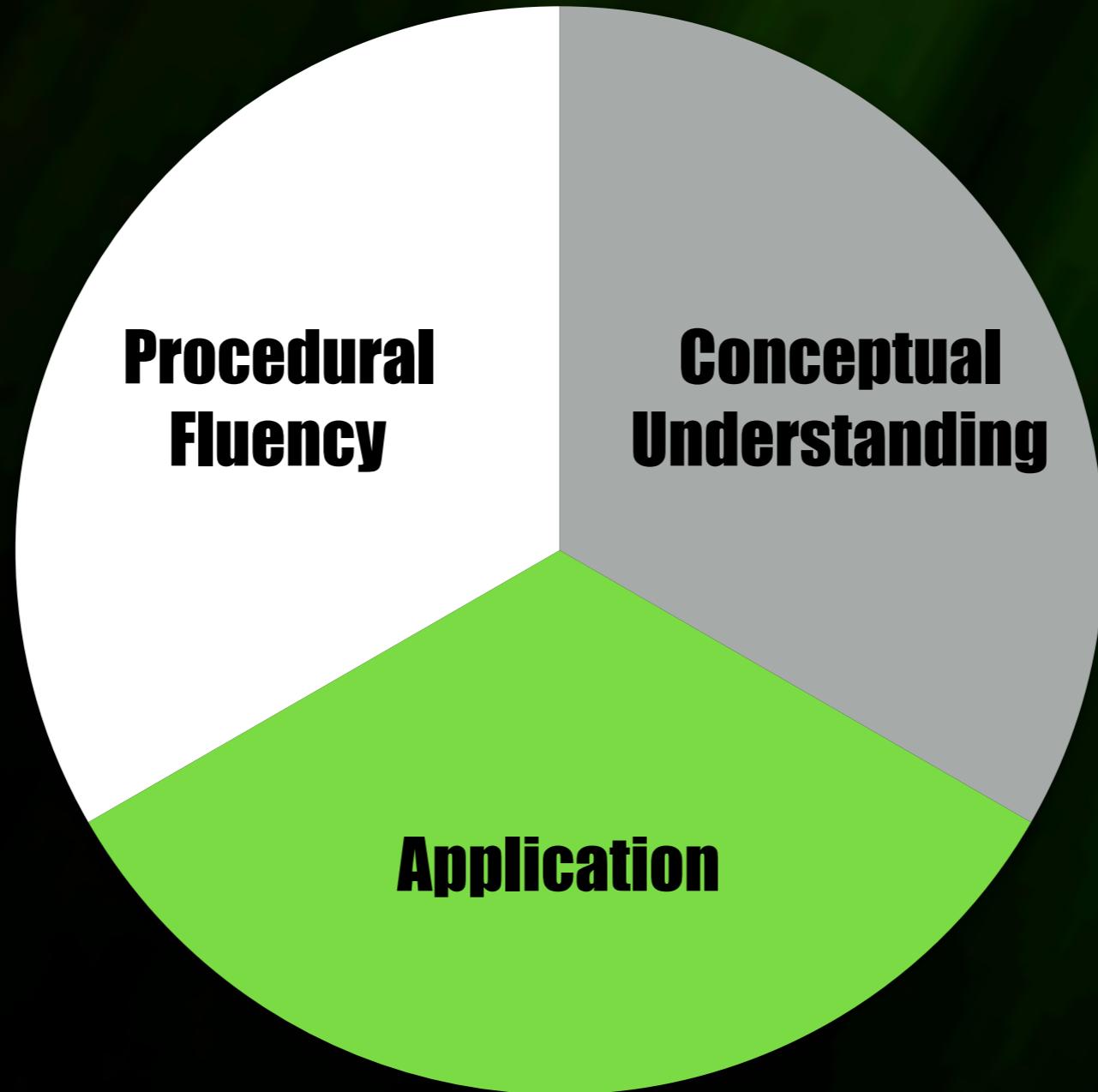


NEXT TIME YOU'RE AFRAID
TO SHARE IDEAS
REMEMBER SOMEONE
ONCE SAID IN A MEETING
LET'S MAKE A FILM WITH A
TORNADO FULL OF SHARKS





**Procedural
Fluency**

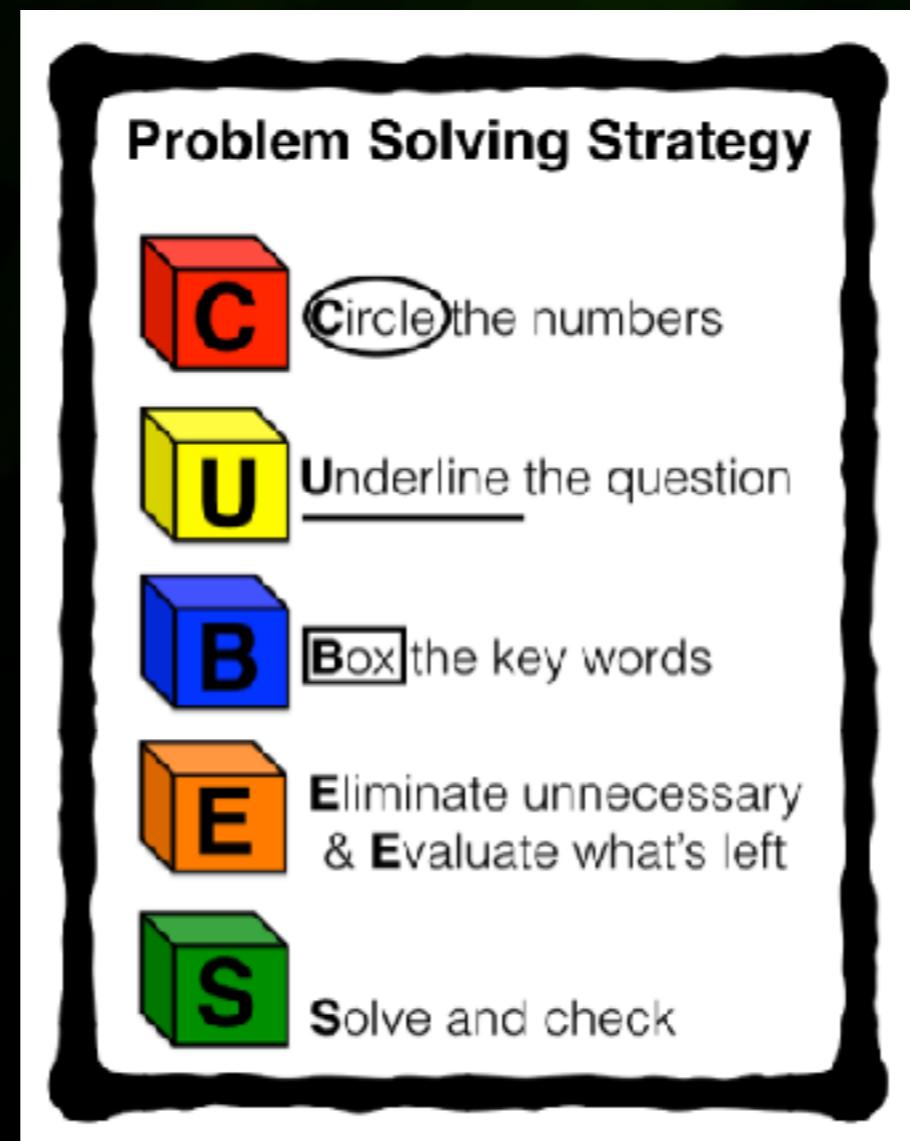


@RobertKaplinsky

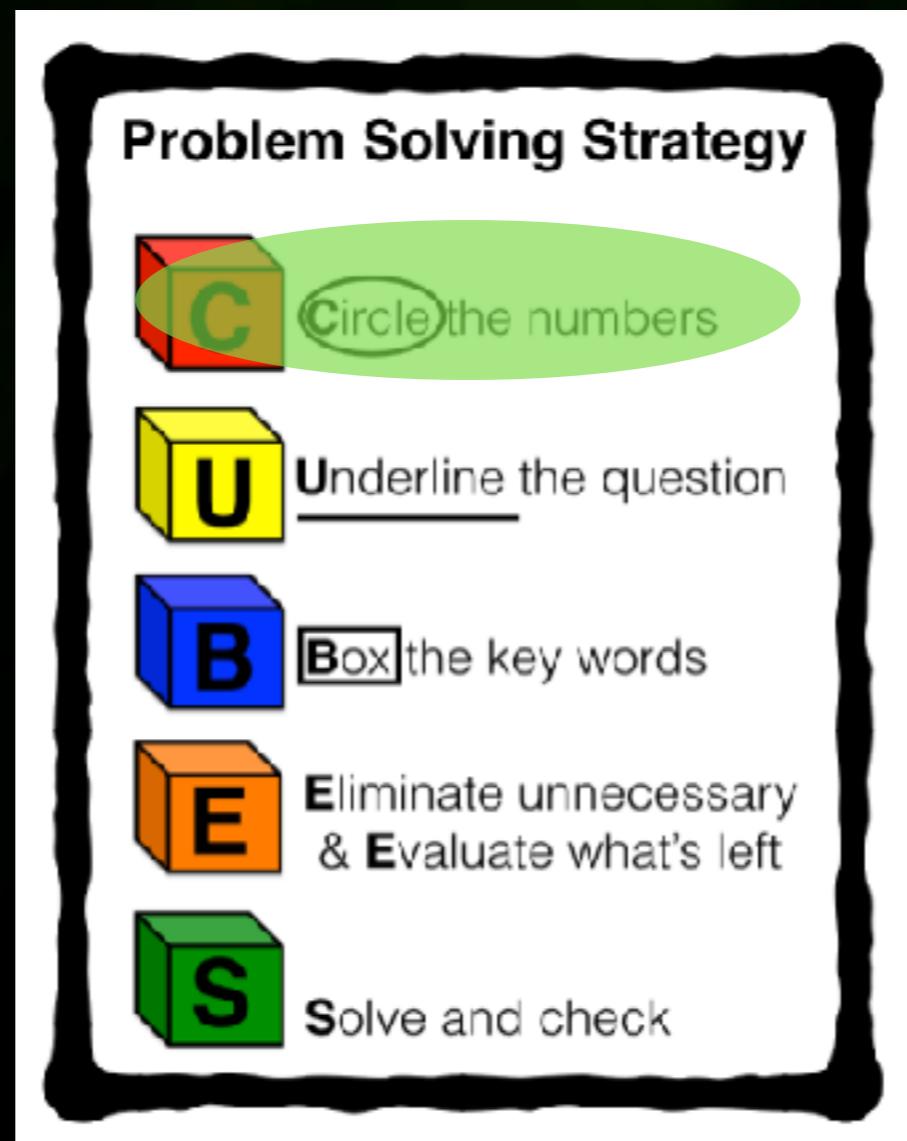
Demetrius has 17 Skittles which is 12 fewer than Alicia.

How many Skittles does Alicia have?

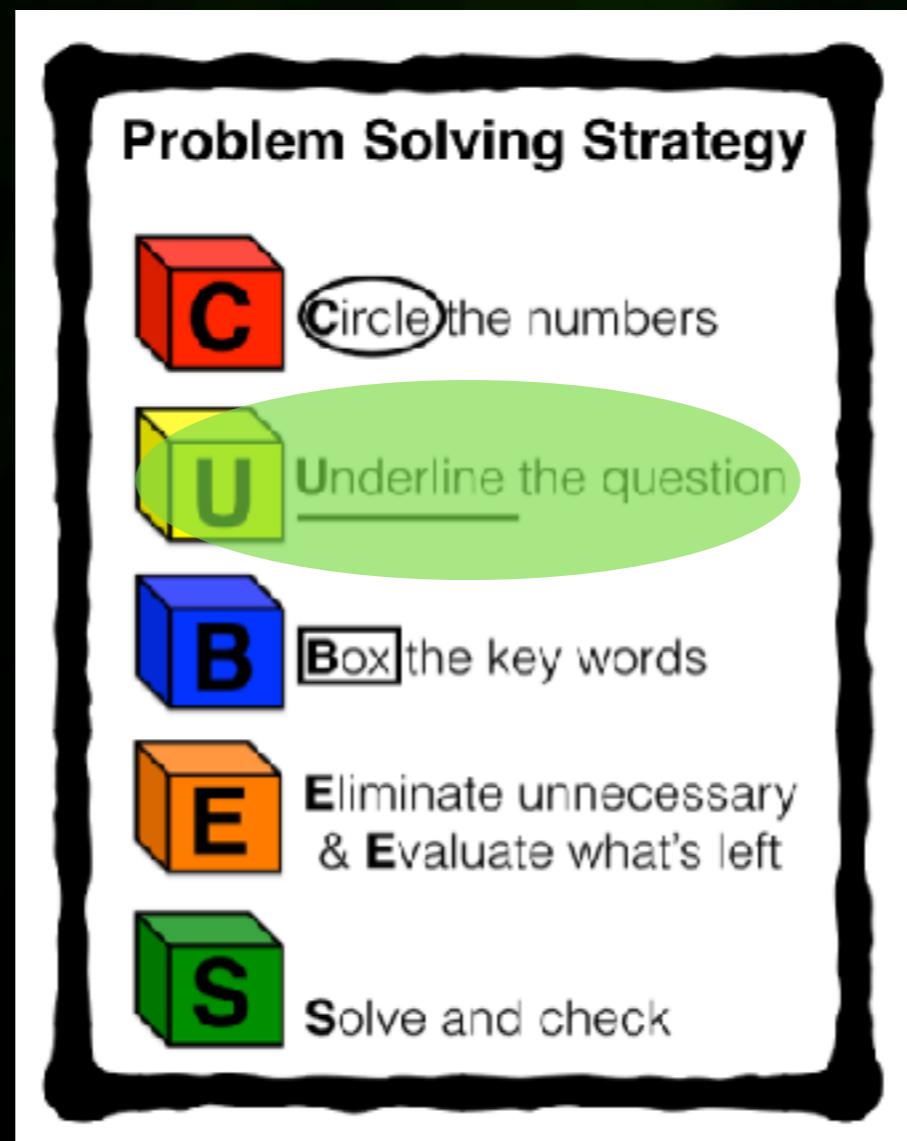
Demetrius has 17 Skittles which is 12 fewer than Alicia.
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Demetrius has 17 Skittles which is 12 fewer than Alicia.
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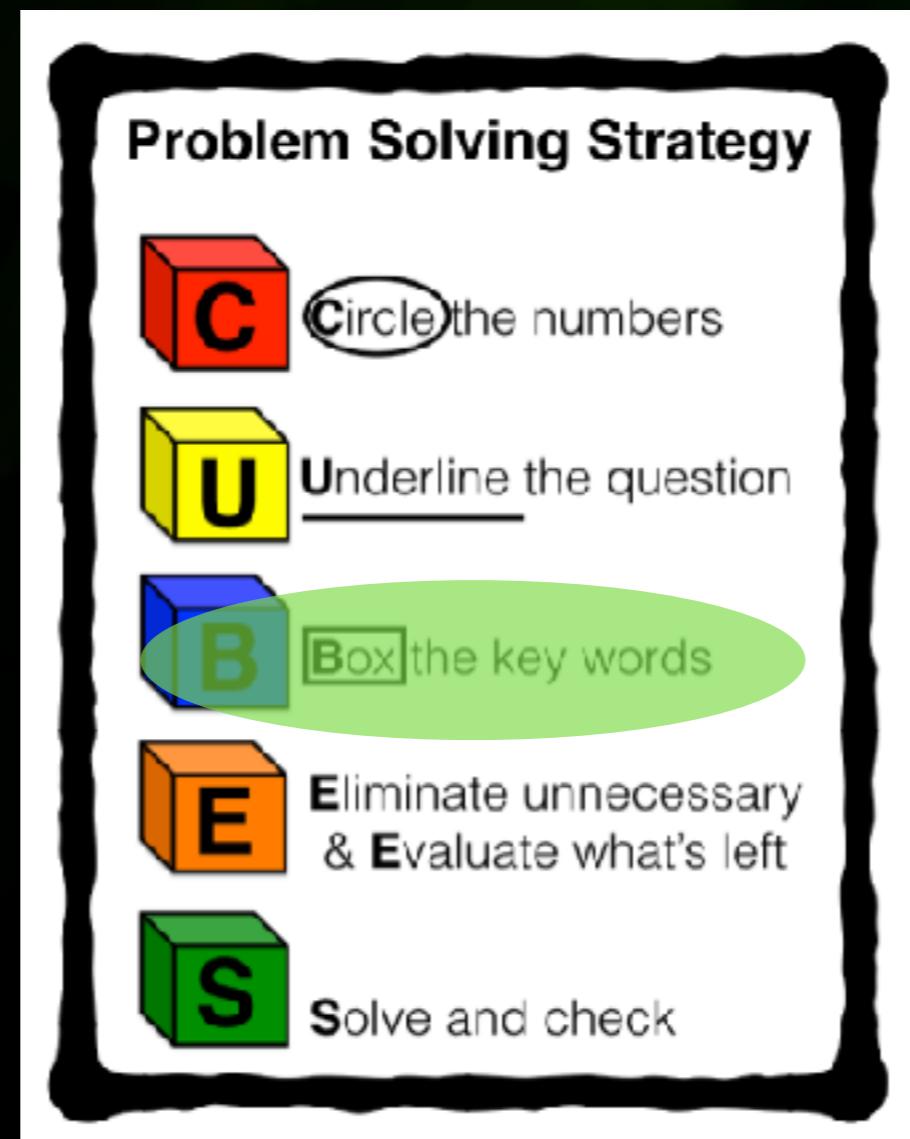


Demetrius has 17 Skittles which is 12 fewer than Alicia.
How many Skittles does Alicia have?



Demetrius has **17** Skittles which is **12**
fewer than Alicia.

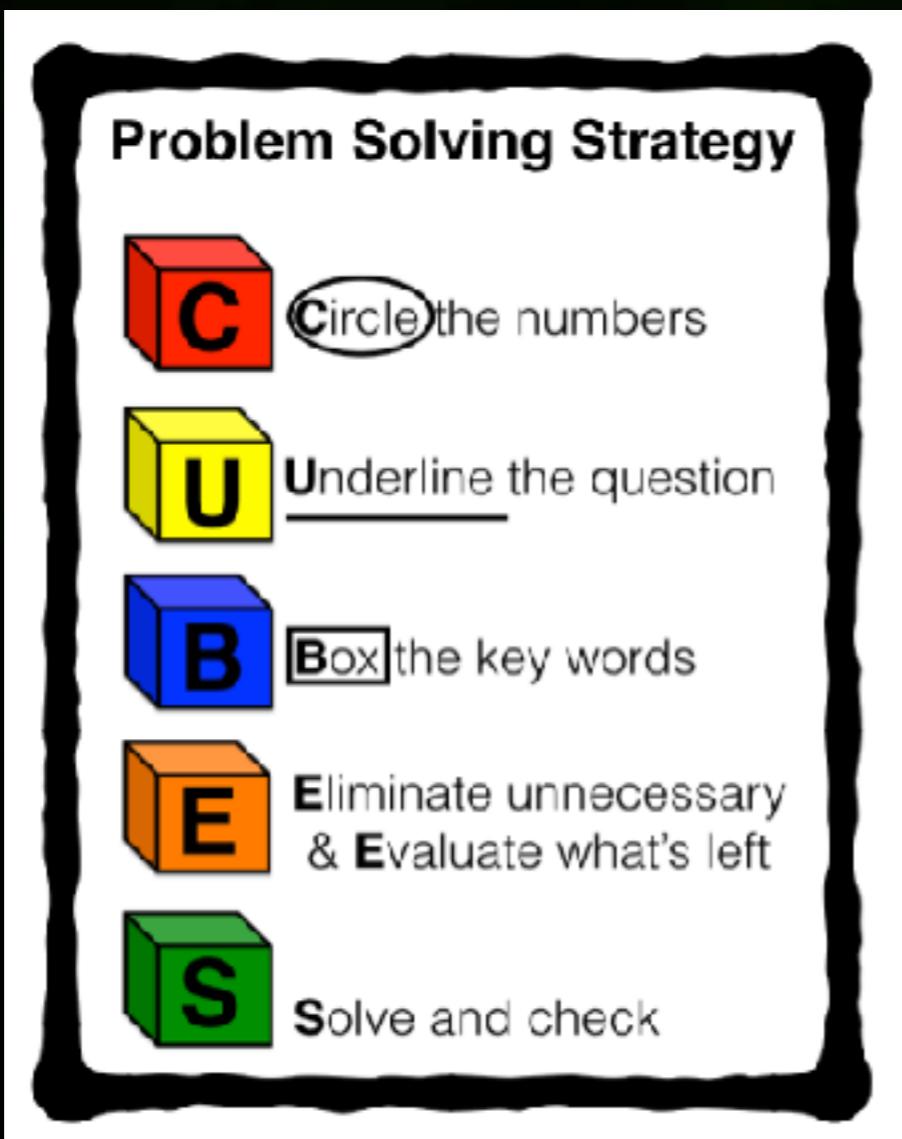
How many Skittles does Alicia have?

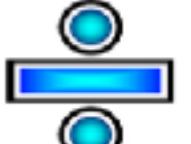


fewer

17

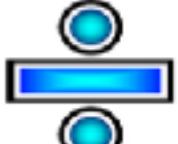
12



The Key Word in Word Problems	
 Add Sum Total All together Plus In all	 Multiply Product Times Twice Total Multiplied by
 Subtract Remain Difference Less than Fewer How many more Minus	 Divide Quotient Goes into Split Equally Each

$$17 - 12$$

Problem Solving Strategy	
	Circle the numbers
	Underline the question
	Box the key words
	Eliminate unnecessary & Evaluate what's left
	Solve and check

The Key Word in Word Problems	
	Multiply Product Times Twice Total Multiplied by
	Subtract Remain Difference Less than Fewer How many more Minus
	Divide Quotient Goes into Split Equally Each

17 – 12

WTFP

17 – 12

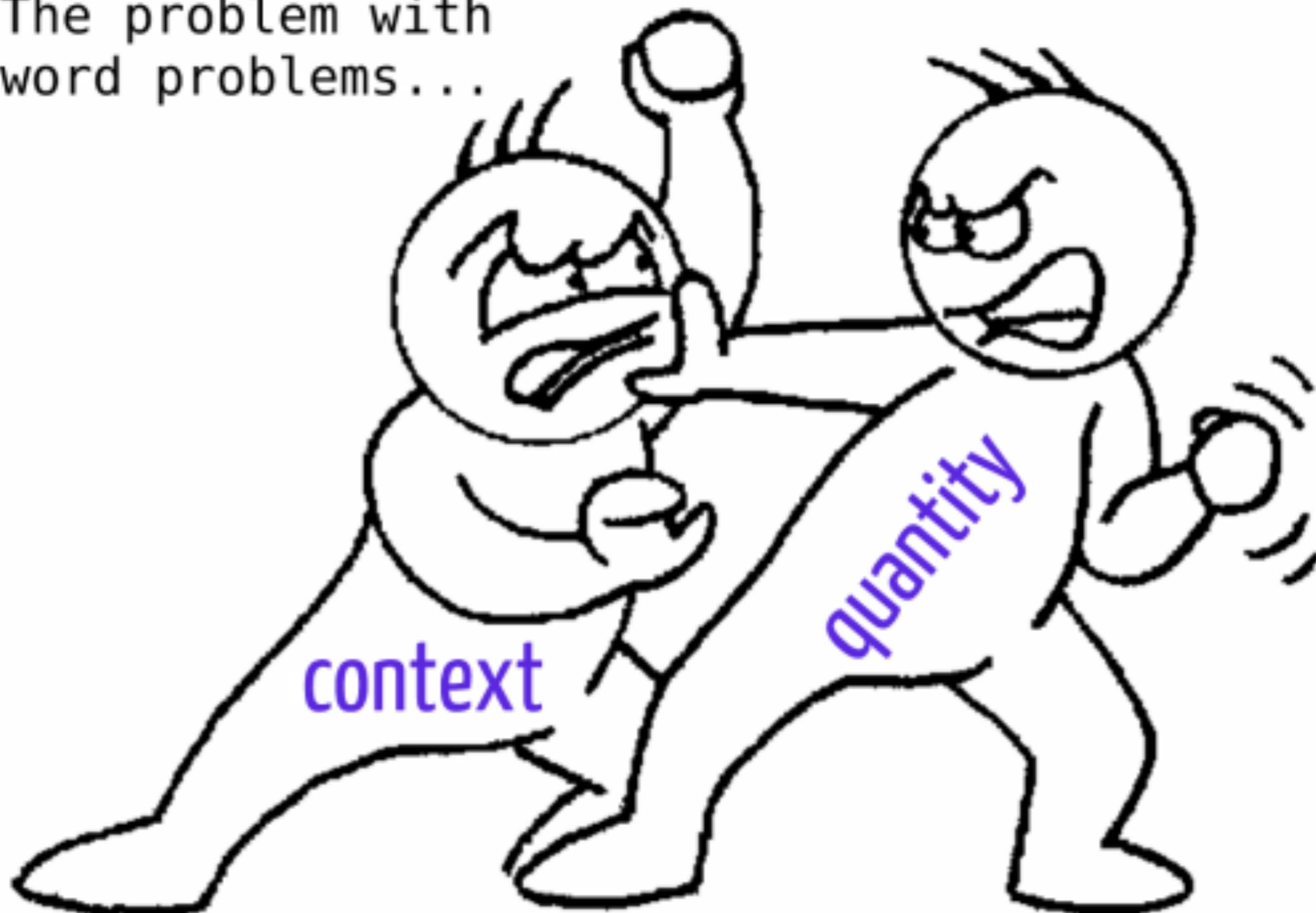
WTF?

hat's he five

Joe had some playing cards in his bag. Ashley gave him 13 more cards. Joe now has 21 cards. How many cards did Joe have in his bag?

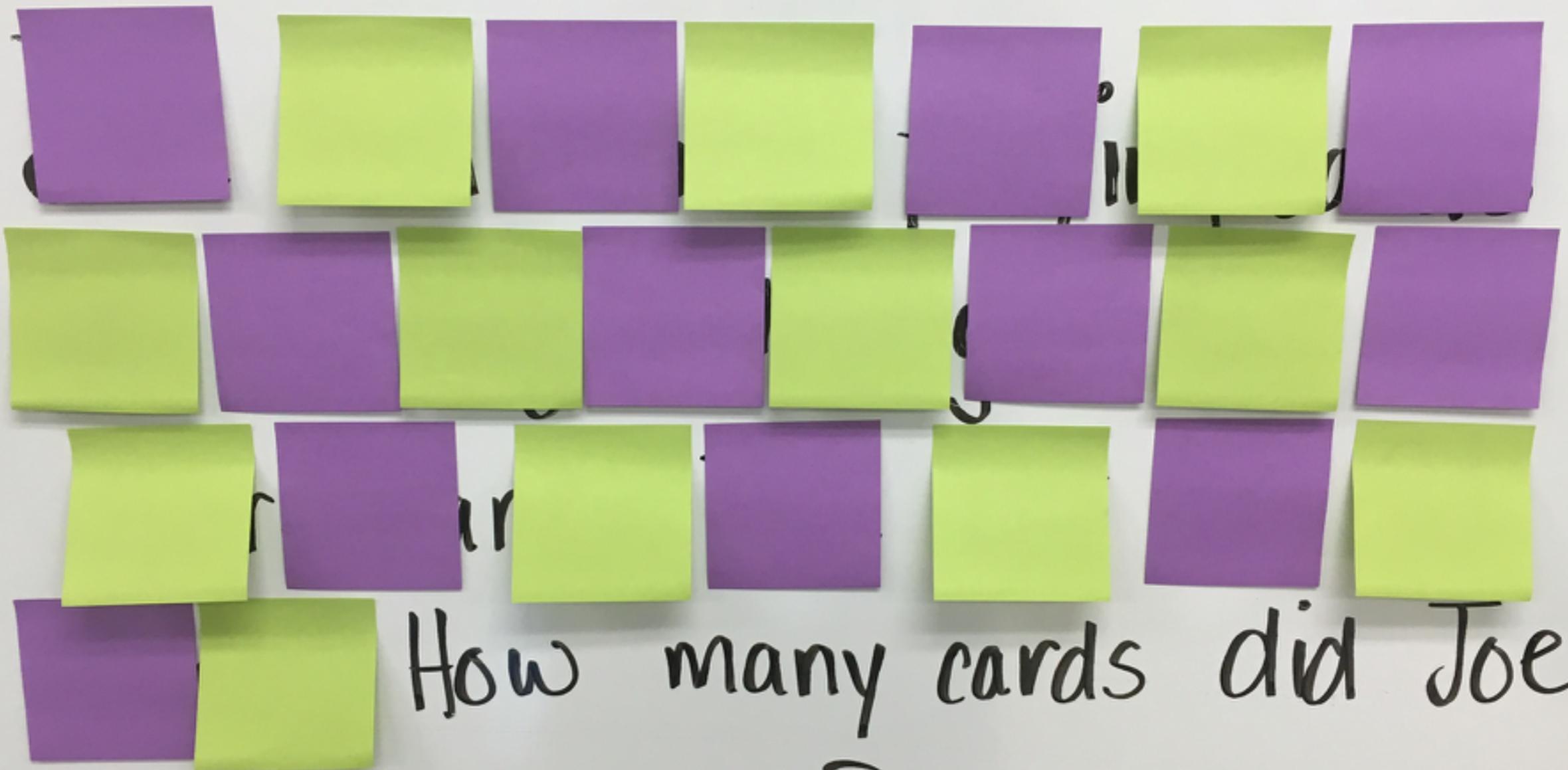
13
21

The problem with
word problems . . .



Joe had some playing cards in his bag. Ashley gave him 13 more cards. Joe now has 21 cards. How many cards did Joe have in his bag?

Joe had some playing cards
in his bag. Ashley gave him 
more cards. Joe now has 
cards. How many cards did Joe
have in his bag?



How many cards did Joe
have in his bag?

Name: _____

Date: _____

1. What did you notice?

2. What do you wonder?

3. Main Question:

4. Make an estimate.



Place an "X" to represent your estimate on the number line.

5. What information do you need?

6. Construct a viable argument or share a reflection:

Answer

Name: _____

Estimate

Draw a picture to show your thinking:

Use numbers to show your thinking:

Answer:





?



-19 yellow



-15 orange



-19 green



-17 purple



-21 red

The Big Reveal



Graham had some Skittles. He had 19 yellow, 15 orange, 19 green, 17 purple, and 21 red. How many Skittles did Graham have?

3-Act Tasks

Act 1:

- Real world problem or scenario presented
- What do you notice? What do you wonder?
- Make estimates

Act 2:

- Identify missing variables and missing variables to solve
- Define solution path using variables

Act 3:

- Solve and interpret results of the solution
- Validate answer

Most asked questions:

- How often should we use 3-Act Tasks?
- When should we use 3-Act tasks? How do they fit into the scope of a unit?
- How long does one task usually take?
- What if we don't have the time?



Orchestrating Discussions

Five practices constitute a model for effectively using student responses in whole-class discussions that can potentially make teaching with high-level tasks more manageable for teachers.

Margaret S. Smith, Elizabeth K. Hughes, Randi A. Engle, and Mary Kay Stein



past decade, she has been developing research-based materials for use in the professional development of mathematics teachers and studying what teachers learn from the professional development in which they engage. Elizabeth K. Hughes, elizabeth.hughes@uni.edu, recently finished her doctorate in mathematics education at the University of Pittsburgh. Her areas of interest include preservice secondary mathematics teacher education and the use of practice-based materials in developing teachers' understanding of what it means to teach and learn mathematics. Randi A. Engle, rengan@hsk.edu, is an assistant professor of mathematics education and the social context of learning at the University of California Berkeley. She is immersed in developing practical theories for how mathematics teachers can create discussion-based learning environments that promote strong student engagement, learning, and transfer. Mary Kay Stein, mstein@pitt.edu, is a professor of learning sciences and policy and the director of the Learning Policy Center at the University of Pittsburgh. Her research focuses on instructional practice and the organizational and policy conditions that shape it.

Discussions that focus on cognitively challenging mathematical tasks, namely, those that promote thinking, reasoning, and problem solving, are a primary mechanism for promoting conceptual understanding of mathematics (Herrano and Ingalski 1991; Michaels, O'Connor, and Resnick forthcoming). Such discussions give students opportunities to share ideas and clarify understandings, develop convincing arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things from other perspectives (NCTM 2000).

Although discussions about high-level tasks provide important

The **5** practices are:

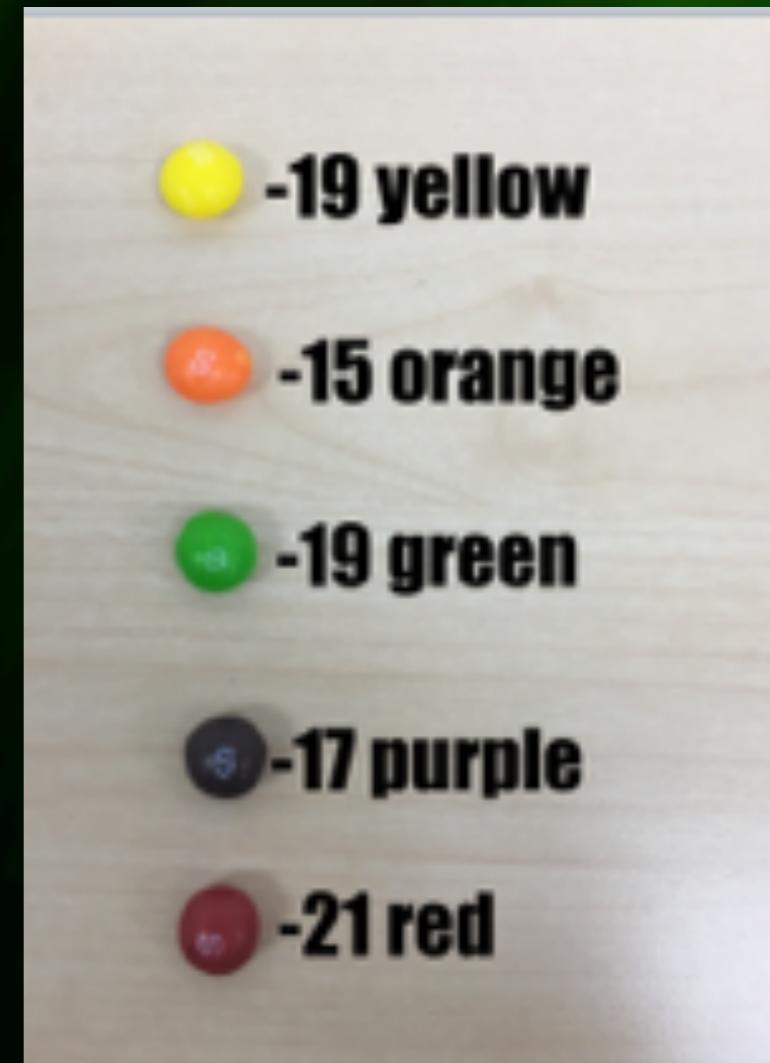
1. Anticipating student responses to challenging mathematical tasks;
2. Monitoring students' work on and engagement with the tasks;
3. Selecting particular students to present their mathematical work;
4. Sequencing the student responses that will be displayed in a specific order and;
5. Connecting different students' responses and connecting the responses to key mathematical ideas.

Task Planning Page

Learning Target:

Questions and Look-Fors:

Strategy	Who and What	Order
Notes:		



Task Planning Page

Learning Target:

Questions and Look-Fors:

Strategy	Who and What	Order
Notes:		

Anticipating →

Monitoring →

Selecting



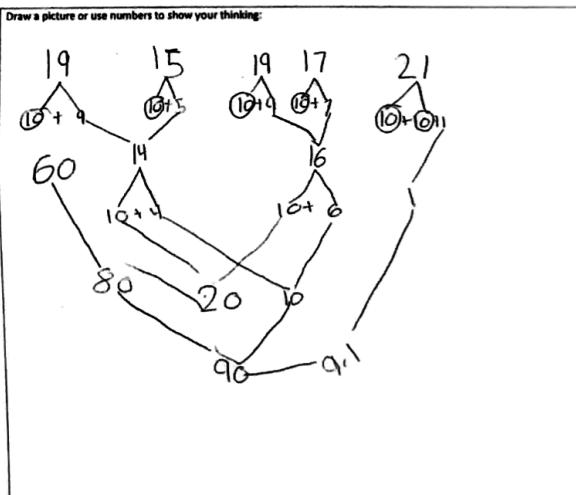
Sequencing →

Connecting

Name: 14

Student #1

Estimate
14

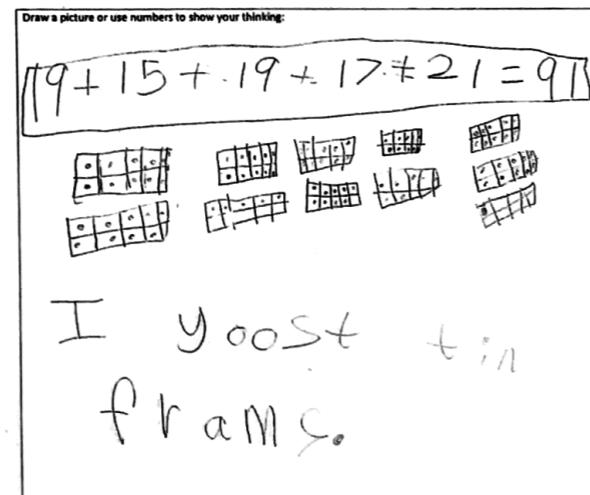


Answer:
91

Name: 11

Student #2

Estimate
13

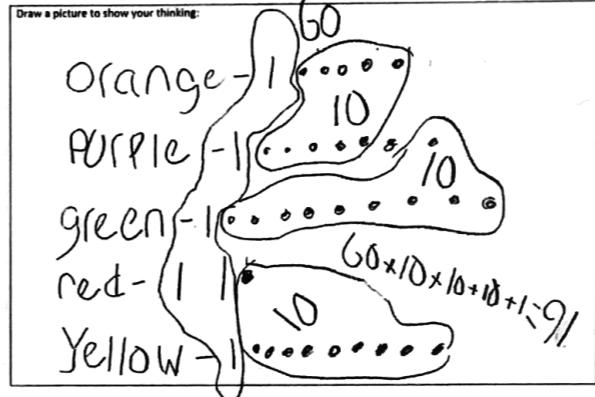


Answer:
91

Name: _____

Student #3

Estimate
42



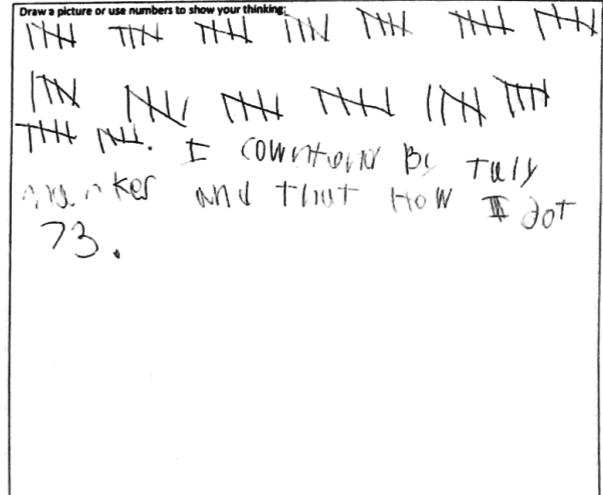
Use numbers to show your thinking:

Answer:
91

Name: 13

Student #4

Estimate
18



Answer:
73

Identify and name the strategy used, then place the student work in order in terms of efficiency (least to greatest)

Name: _____

Student #5

Estimate
50

Draw a picture to show your thinking:

$$\begin{array}{r} 19 \\ 15 \\ 19 \\ 17 \\ + 21 \\ \hline 91 \\ 60 \\ \hline 91 \end{array}$$

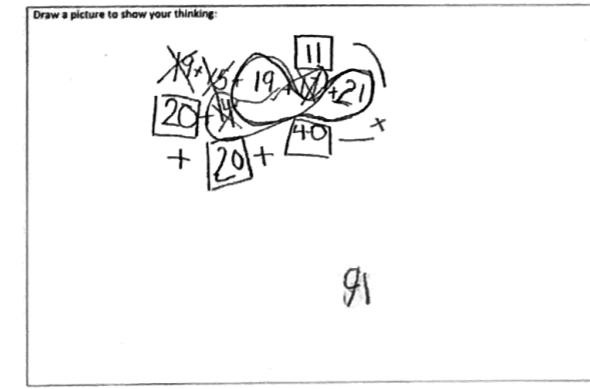
Use numbers to show your thinking:

Answer:
91

Name: _____

Student #6

Estimate
31



91

Use numbers to show your thinking:

I moved the numbers around.

Answer:
91

Name: 203

Student #7

Estimate
81



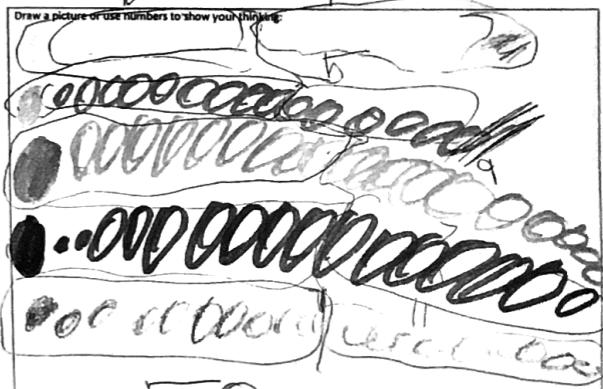
"I counted them all"

Answer:
91

Name: 4

Student #8

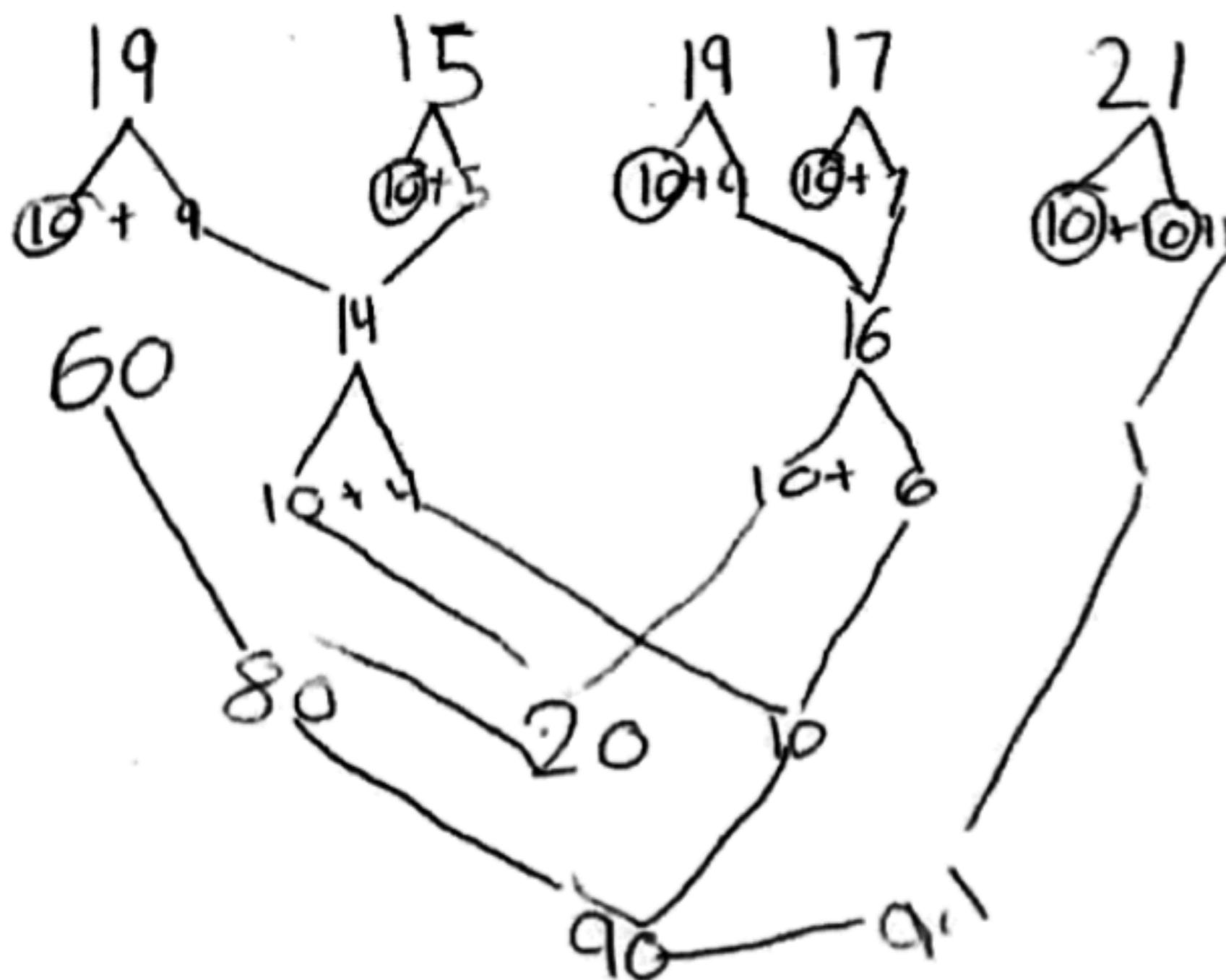
Estimate
810



50
61
30

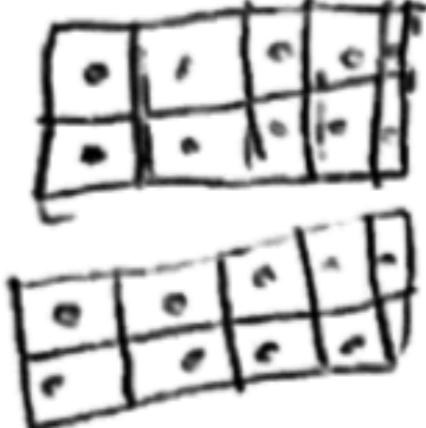
Answer:
91

Draw a picture or use numbers to show your thinking:



Draw a picture or use numbers to show your thinking:

$$19 + 15 + 19 + 17 + 21 = 91$$



I yooSt t in
framC.

Draw a picture to show your thinking:

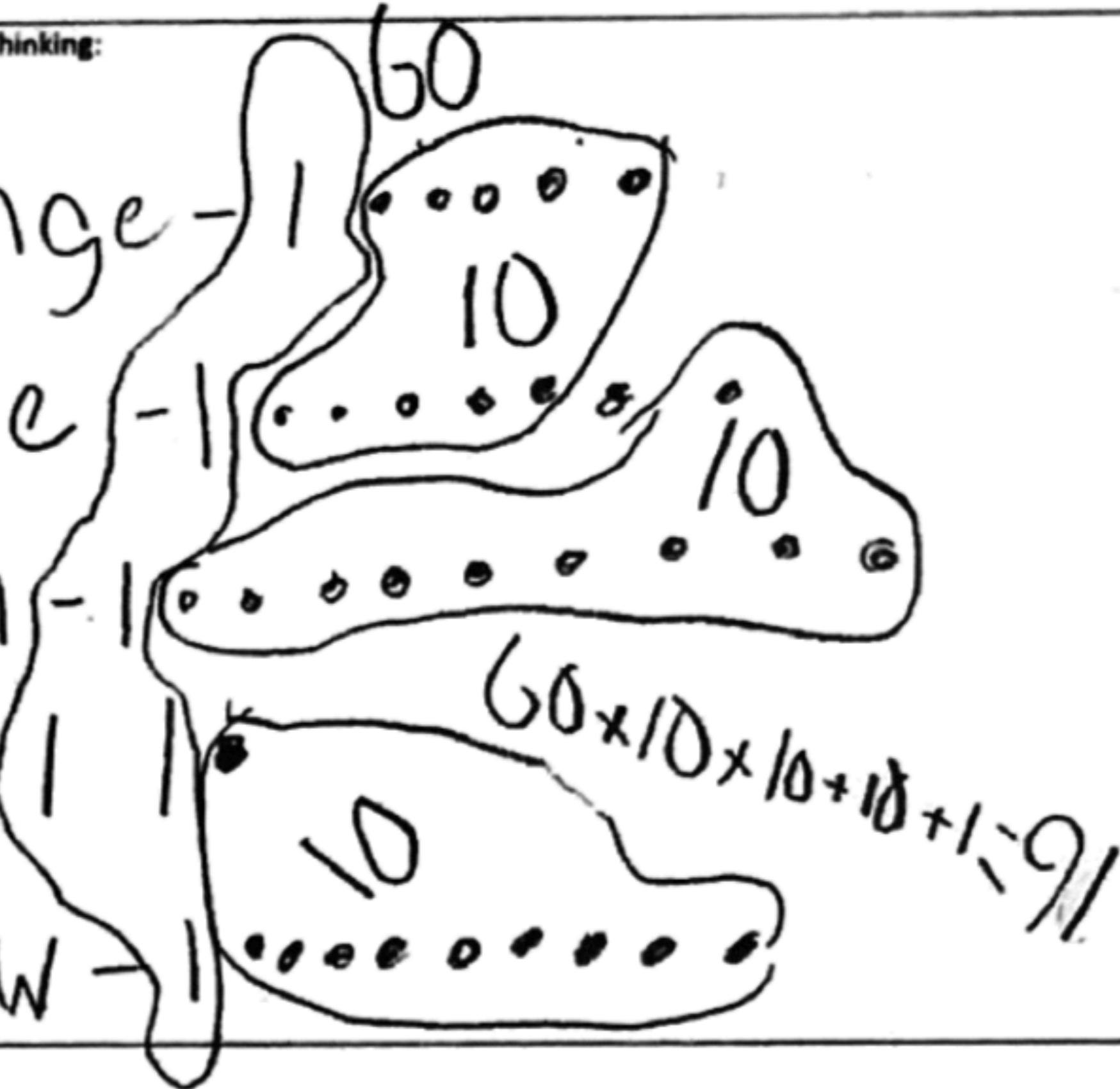
orange - 1

PURPLE - 1

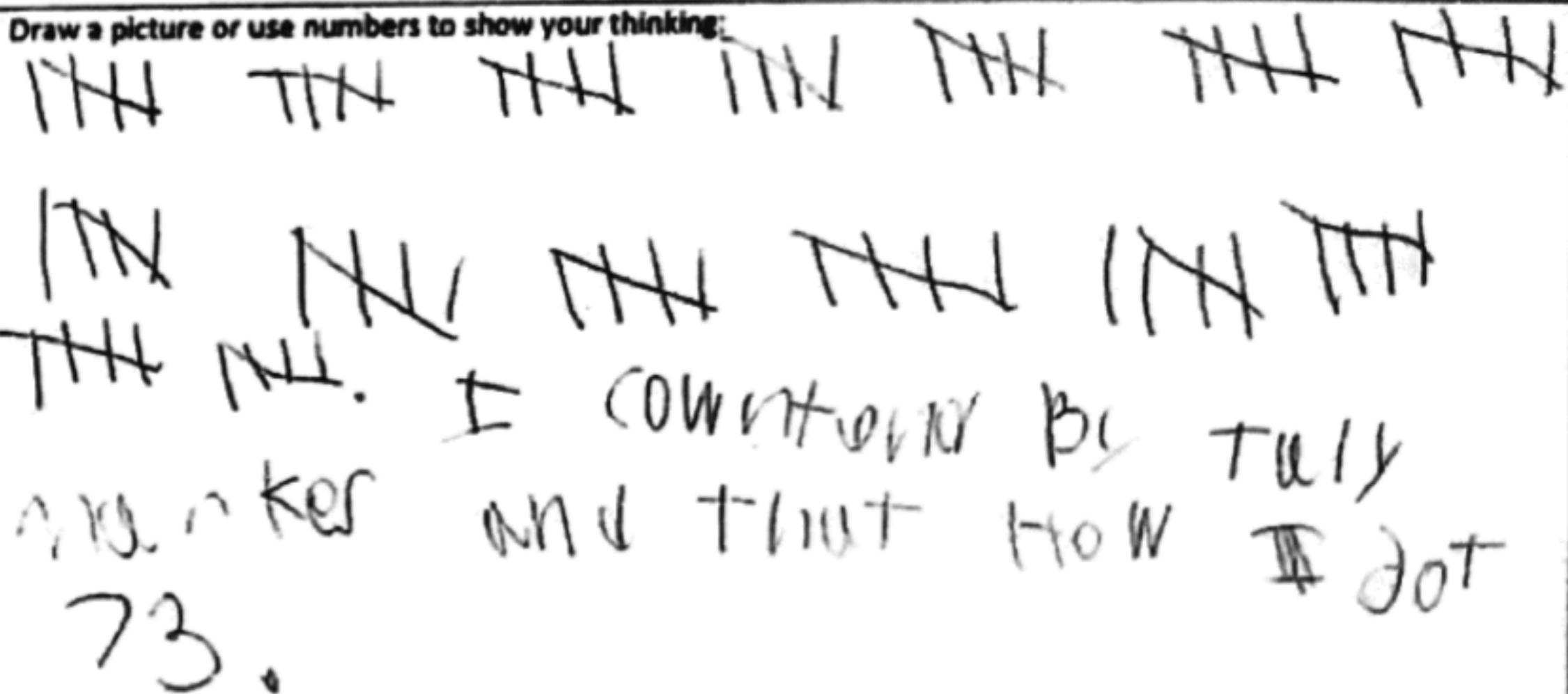
green - 1

red - 1

Yellow - 1



Draw a picture or use numbers to show your thinking:



Draw a picture to show your thinking:

$$\begin{array}{r} 19 \\ 15 \\ 19 \\ + 17 \\ \hline 91 \\ 60 \\ \hline 91 \end{array}$$

$$19+15+19+17+21$$

A hand-drawn addition problem on a white background. The numbers 19, 15, 19, 17, and 21 are written vertically in a column. A horizontal line is drawn across the top of the column, followed by a plus sign, another horizontal line, and then the number 40. To the right of the 40 is a minus sign and a blank space for a remainder. The entire problem is enclosed in a large circle. Above the circle, there is a small square containing three vertical lines.

$$\begin{array}{r} 19 \\ + 15 \\ + 19 \\ + 17 \\ + 21 \\ \hline 40 \end{array}$$

Draw a picture or use numbers to show your thinking:



"I counted them all"

10

9

Draw a picture or use numbers to show your thinking:



50
61

30

<p>Estimate</p> <p>14</p>	<p>Name: _____</p> <p>Student #1</p>
<p>Draw a picture or use numbers to show your thinking:</p>	
<p>Answer:</p> <p>91</p>	

Estimate	13	Name: _____	(11)
		student #2	
<p>Draw a picture or use numbers to show your thinking:</p> $19 + 15 + 19 + 17 + 21 = 91$ <p>I yooost + in frams.</p>			
		<p>Answer:</p> <div style="border: 1px solid black; padding: 5px; text-align: center;"> 91 </div>	

<p>Estimate</p> <p>42</p>	<p>Name: _____</p> <p>Student # 3</p>
<p>Draw a picture to show your thinking:</p> <p>Orange - 1 60 Purple - 1 10 green - 1 10 red - 1 10 Yellow - 1 10</p> $60 \times 10 \times 10 + 10 + 1 = 91$	
<p>Use numbers to show your thinking:</p>	
<p>Answer: 91</p>	

Estimate	<u>18</u>	Name: <u>13</u>
		Student #4
<p>Draw a picture or use numbers to show your thinking:</p> <p>I counted by tally marks and that how I got 73.</p>		
		Answer: <u>73</u>

Estimate	Student # 5
<p>Draw a picture to show your thinking:</p> $ \begin{array}{r} 19 \\ 15 \\ 19 \\ 17 \\ + 21 \\ \hline 31 \\ \hline 60 \\ \hline 91 \end{array} $	
<p>Use numbers to show your thinking:</p> <div style="height: 100px;"></div>	
Answer: 	

Estimate	1 810	4
Name: _____		
Student #8		
Draw a picture or use numbers to show your thinking:		
<p>Total 100 dots</p>		
50 30		

Answer:

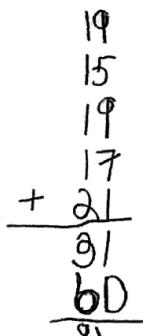
Estimate	Student #1
14	

Draw a picture or use numbers to show your thinking:

Estimate	Name: _____
13	student #2
Draw a picture or use numbers to show your thinking:	
$(9 + 15) + (19 + 17) = 21 = 91$	
<p>I yooost t in frames.</p> <h1>1-counting all</h1>	
Answer: 91	

<p>Estimate</p> <p>42</p>	<p>Name: _____</p> <p>Student # 3</p>
<p>Draw a picture to show your thinking:</p>	
<p>Use numbers to show your thinking:</p> <p>_____</p>	

Name:	13
Estimate	18
Student #4	
Draw a picture or use numbers to show your thinking:	
<p>13. I counted by 5's and that how I got 73.</p>	
Answer:	

<p>Estimate</p>	<p>Student # 5</p>
<p>Draw a picture to show your thinking:</p>  $ \begin{array}{r} 19 \\ 15 \\ \hline 34 \end{array} $	
<p>Use numbers to show your thinking:</p>	
<p>Answer:</p>	

<p>Estimate</p> <p><u>34</u></p>	<p>Name: _____</p> <p>student # b</p>
<p>Draw a picture to show your thinking:</p>	
<p>Use numbers to show your thinking:</p> <p>I moved the numbers around.</p>	
<p>Answer:</p>	

Estimate	Name: <u>203</u>
	Student # 7
<p>Draw a picture or use numbers to show your thinking:</p> <p><i>Yours</i></p> <p><i>I counted them all</i></p>	
<h1>1-counting all</h1>	

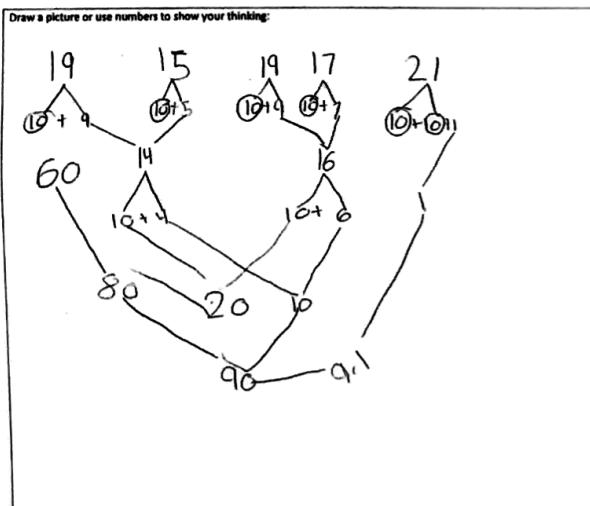
Estimate	1 810	Name: _____
		Student #8
Draw a picture or use numbers to show your thinking:		
	50 61	30

1-counting all

Name: 14

Student #1

Estimate
14

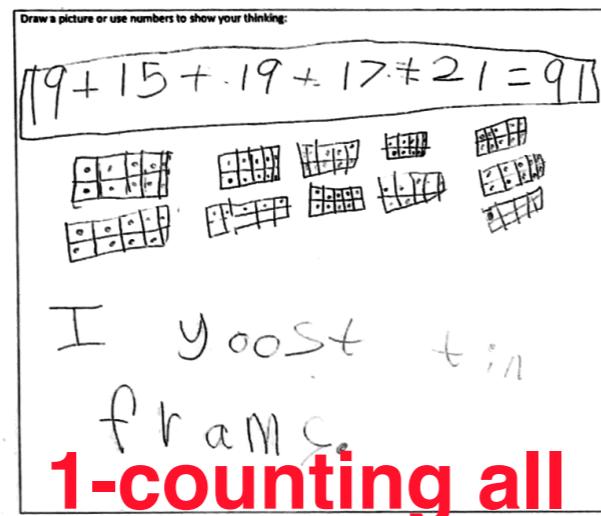


Answer:
91

Name: 11

Student #2

Estimate
13



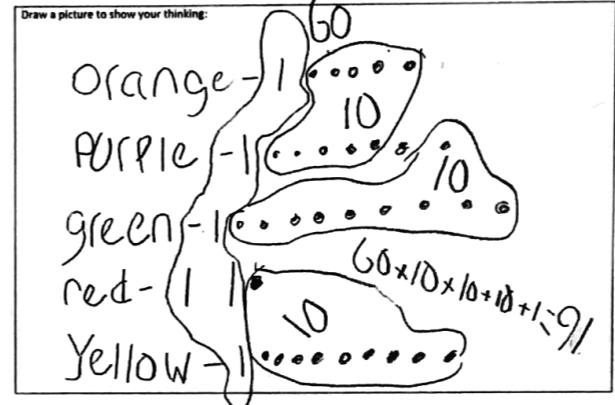
1-counting all

Answer:
91

Name: 12

Student #3

Estimate
12



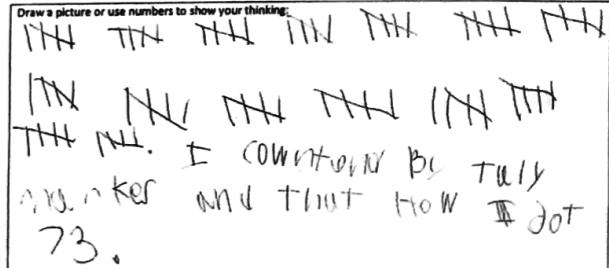
Use numbers to show your thinking:

Answer:
91

Name: 13

Student #4

Estimate
18



Answer:
73

2-counting all with skip counting

Name: _____

Student #5

Estimate
50

Draw a picture to show your thinking:

$$\begin{array}{r} 19 \\ 15 \\ 19 \\ 17 \\ + 21 \\ \hline 91 \\ 60 \\ \hline 91 \end{array}$$

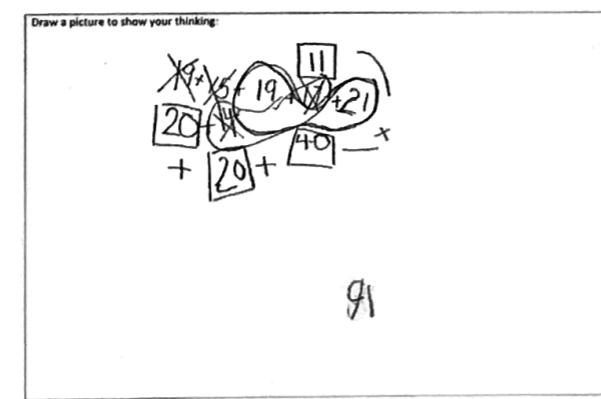
Use numbers to show your thinking:

Answer:

Name: _____

Student #6

Estimate
34



Use numbers to show your thinking:

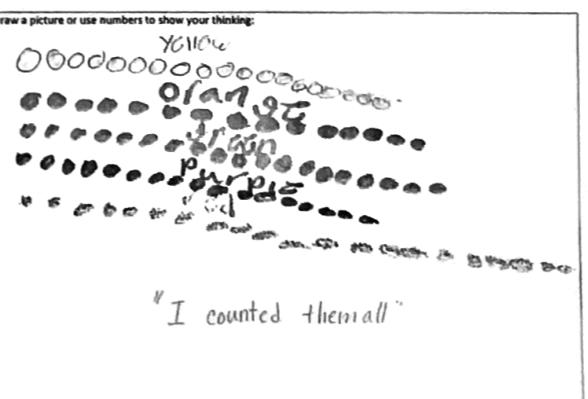
I mixed the numbers around.

Answer:

Name: 203

Student #7

Estimate
81



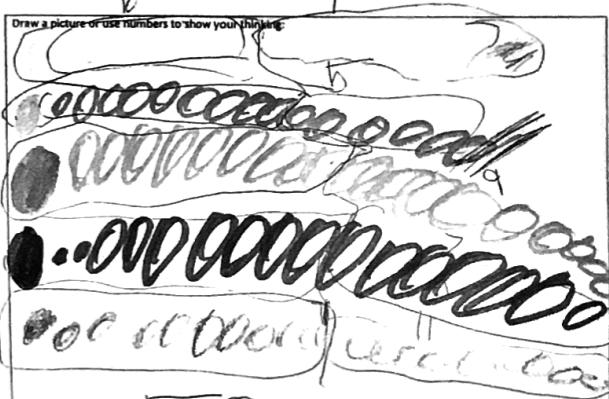
1-counting all

Answer:
91

Name: 4

Student #8

Estimate
81

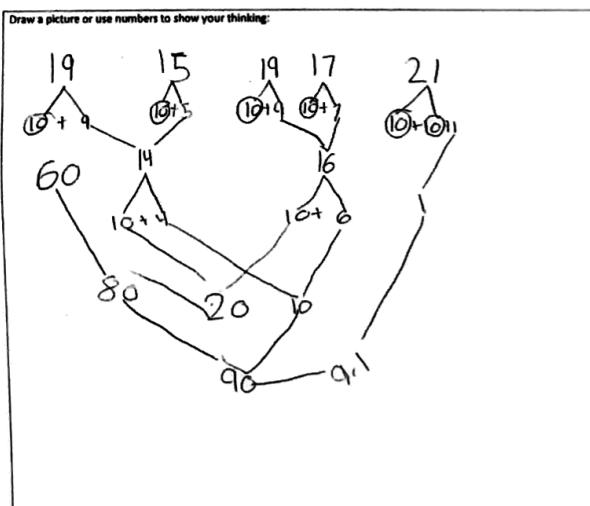


Answer:
91

Name: 14

Student #1

Estimate
14

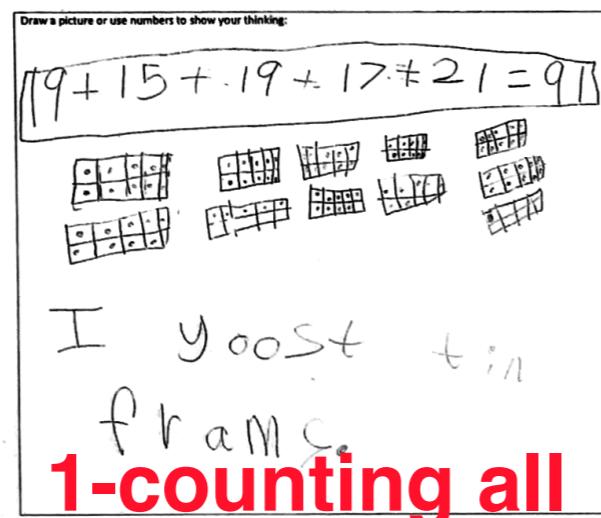


Answer:
91

Name: 11

Student #2

Estimate
13

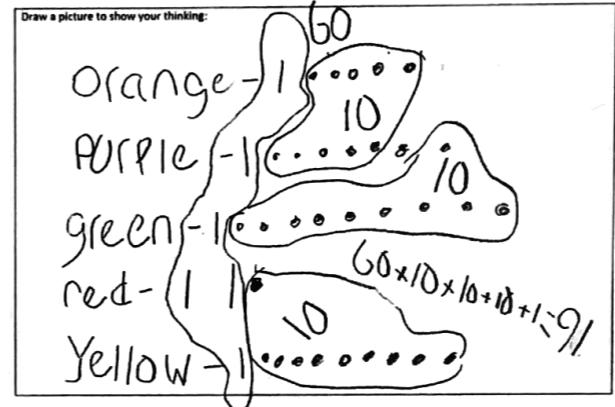


Answer:
91

Name: 12

Student #3

Estimate
12



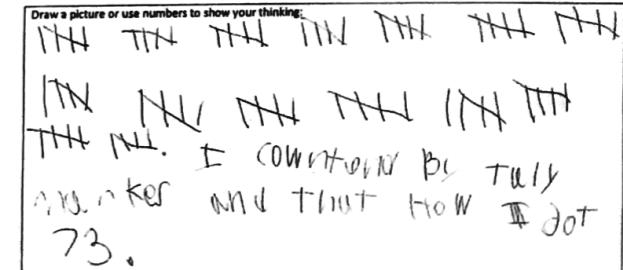
Use numbers to show your thinking:

Answer:
91

Name: 13

Student #4

Estimate
18

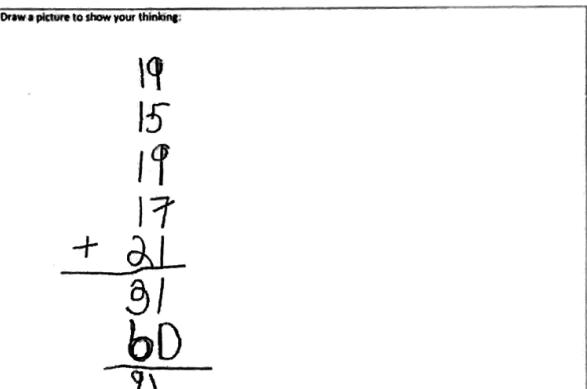


Answer:
73

2-counting all with skip counting

Name: 50

Student #5

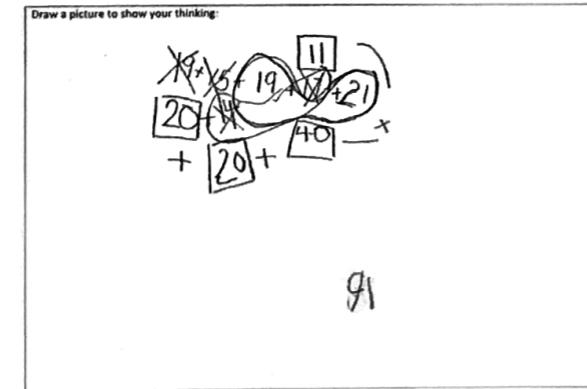


Use numbers to show your thinking:

Answer:
91

Name: 31

Student #6



Use numbers to show your thinking:

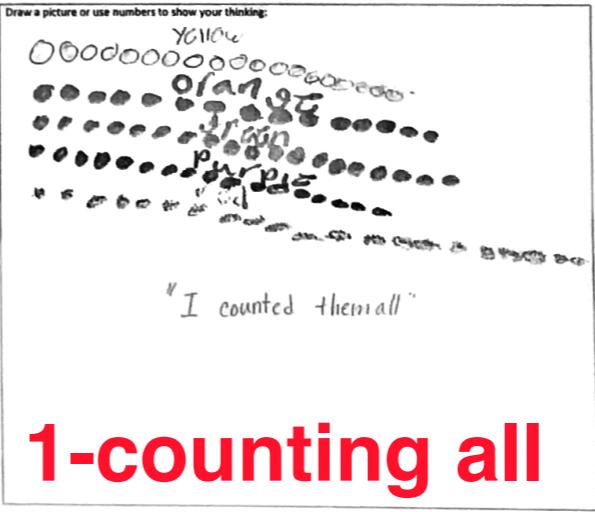
I mixed the numbers around.

Answer:
91

Name: 203

Student #7

Estimate
81

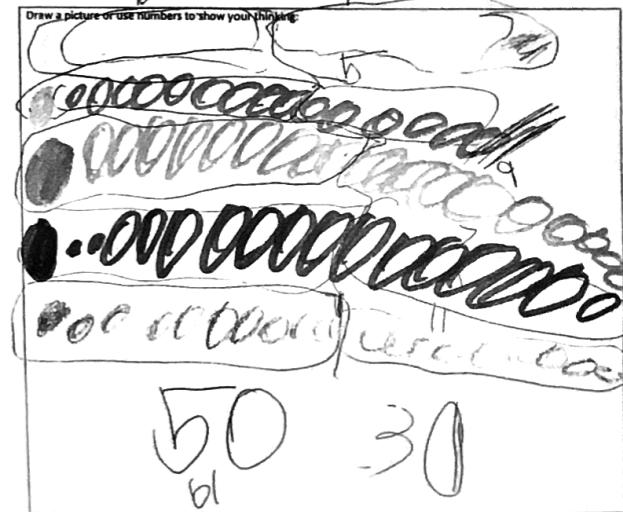


Answer:
91

Name: 4

Student #8

Estimate
81

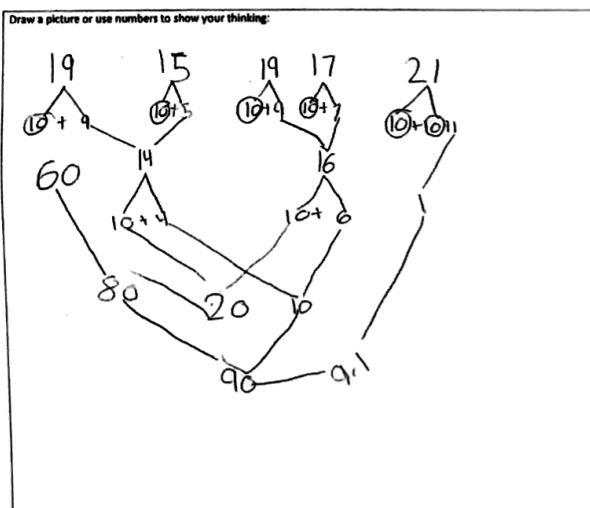


3-making tens from ones

Name: 14

Student #1

Estimate
14

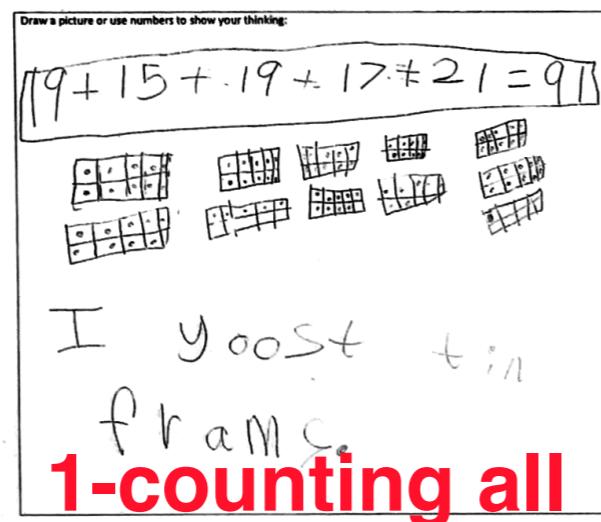


Answer:
91

Name: 11

Student #2

Estimate
13



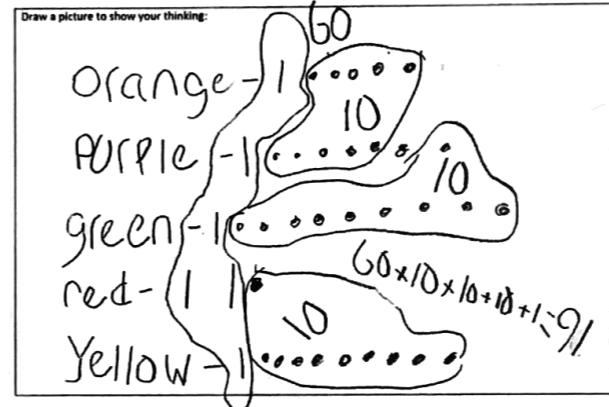
1-counting all

Answer:
91

Name: 12

Student #3

Estimate
12



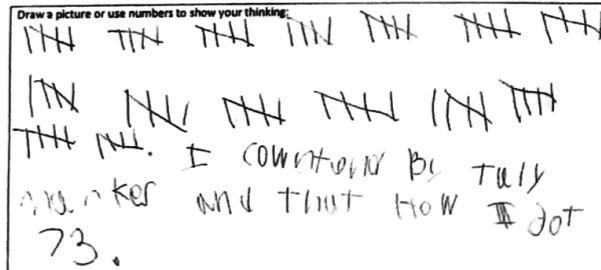
Use numbers to show your thinking:
4-making tens from ones

Answer:
91

Name: 13

Student #4

Estimate
18



Answer:
73

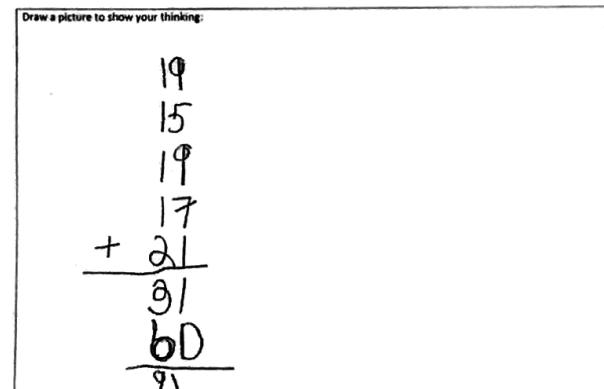
2-counting all with skip counting

Name: _____

Student #5

Estimate
50

$$\begin{array}{r} 19 \\ 15 \\ 19 \\ 17 \\ + 21 \\ \hline 91 \\ 60 \\ \hline 91 \end{array}$$

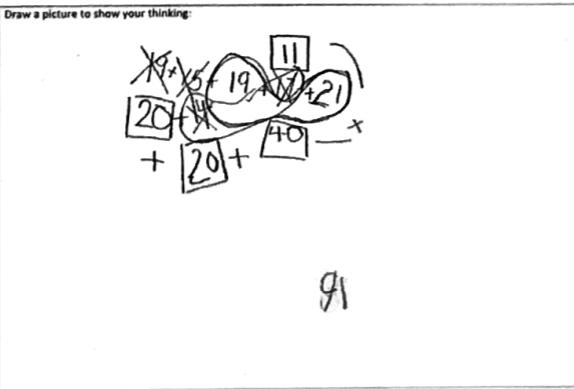


Use numbers to show your thinking:
Answer:

Name: _____

Student #6

Estimate
34



Use numbers to show your thinking:
I mixed the numbers around.
Answer:

Name: 203

Student #7

Estimate
81

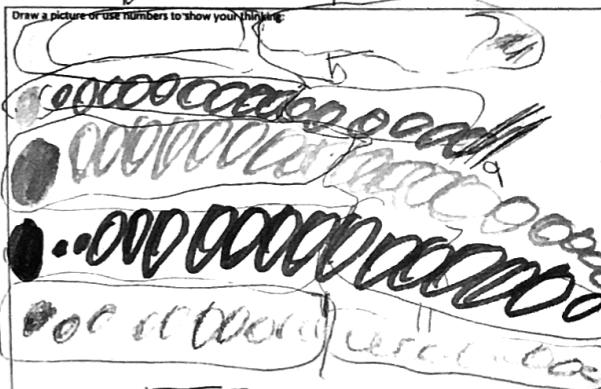


Answer:
91

Name: 4

Student #8

Estimate
810



Answer:
41

1-counting all

3-making tens from ones

Estimate	Student #1
14	

Draw a picture or use numbers to show your thinking:

5-Place value partitioning

Answer:	91
---------	----

5-Place value partitioning

Estimate	(11)
13	Student #2
<p>Draw a picture or use numbers to show your thinking:</p> $(9+15+19+17+21=91)$	
<p>I yooST + in fr aMS.</p> <h1>1-counting all</h1>	
Answer:	91

1-counting all

Estimate	42
Name: _____	
Student # 3	
Draw a picture to show your thinking:	
$60 \times 10 \times 10 + 10 + 1 = 911$	
Use numbers to show your thinking:	
<h1>4-making tens from ones</h1>	

4-making tens from ones

Name: <u>13</u>	Student #4
Estimate	18
<p>Draw a picture or use numbers to show your thinking:</p> <p>TEN ELEVEN TWELVE.</p> <p>I counted by 1's and that how I got 73.</p>	
<h1>2-counting all with skip counting</h1>	
Answer:	<u>73</u>

2-counting all with skip counting

Estimate	50	Student # 5
Draw a picture to show your thinking:		
$ \begin{array}{r} 19 \\ 15 \\ 19 \\ 17 \\ + 21 \\ \hline 81 \\ \hline 60 \\ \hline 91 \end{array} $		
Use numbers to show your thinking:		
		Answer:

<p>Estimate</p> <p><u>34</u></p>	<p>Name: _____</p> <p>Student # b</p>
<p>Draw a picture to show your thinking:</p>	<p>Use numbers to show your thinking:</p> <p>I moved the numbers around.</p>
	<p>Answer:</p> <p><u>91</u></p>

I moved the numbers around.

<p>Estimate</p> <div style="border: 1px solid black; padding: 5px; width: fit-content;">81</div>	<p>Name: <u>203</u></p> <p>Student # 7</p> <p>Draw a picture or use numbers to show your thinking:</p> <p>"I counted them all"</p> <h1>1-counting all</h1>
--	--

1-counting all

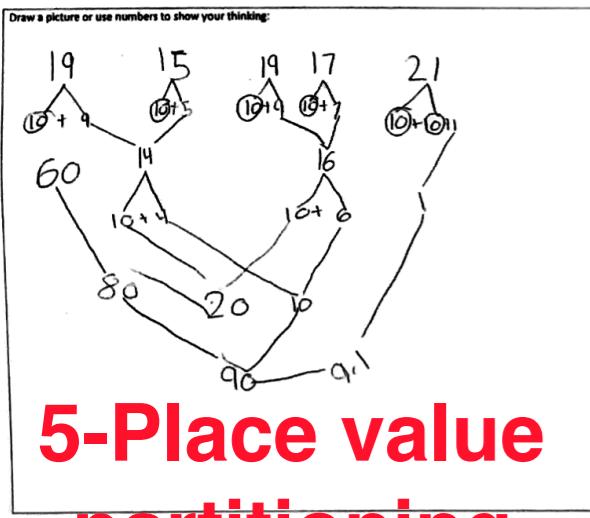
Estimate	1 810	Name: H Student #8
Draw a picture or use numbers to show your thinking:		
50 61		30

3-making tens from ones

Name: 14

Student #1

Estimate
14



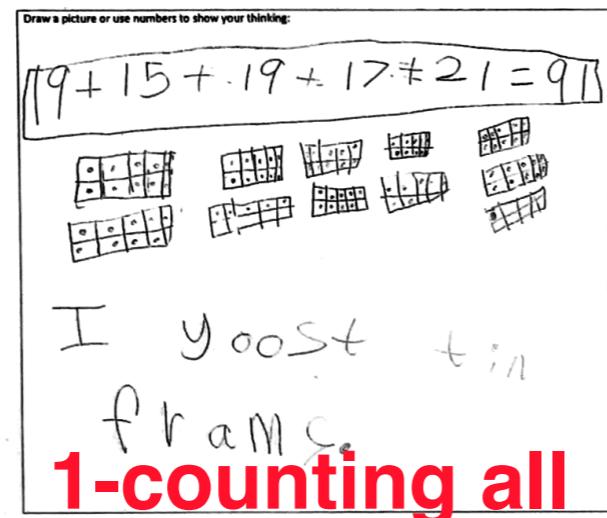
5-Place value partitioning

Answer:
91

Name: 11

Student #2

Estimate
13



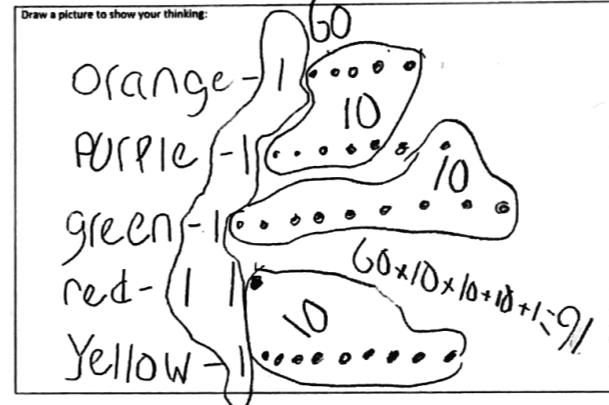
1-counting all

Answer:
91

Name: 12

Student #3

Estimate
42



Use numbers to show your thinking:

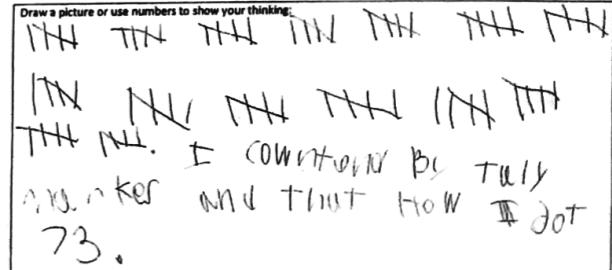
4-making tens from ones

Answer:
91

Name: 13

Student #4

Estimate
18



2-counting all with skip counting

Answer:
73

Name: _____

Student #5

Estimate
50

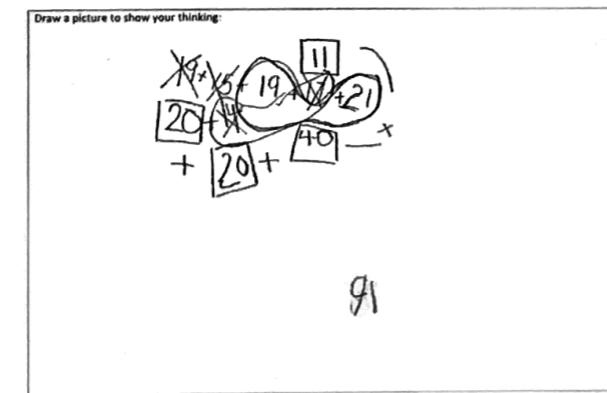
$$\begin{array}{r} 19 \\ 15 \\ 19 \\ 17 \\ + 21 \\ \hline 91 \\ 60 \\ \hline 91 \end{array}$$

Draw a picture to show your thinking:

Name: _____

Student #6

Estimate
34



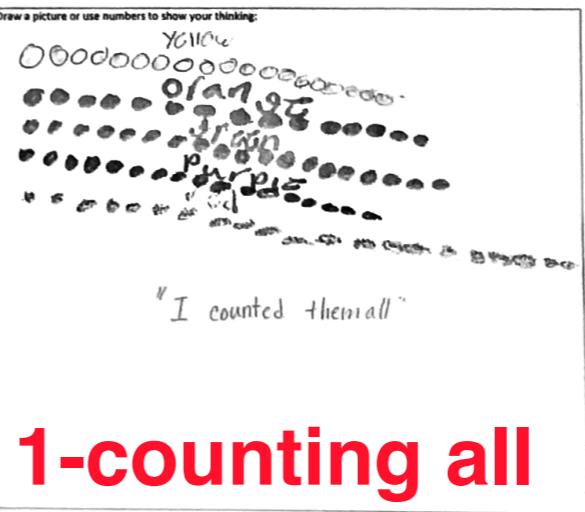
Student #6

Answer:
91

Name: 203

Student #7

Estimate
81



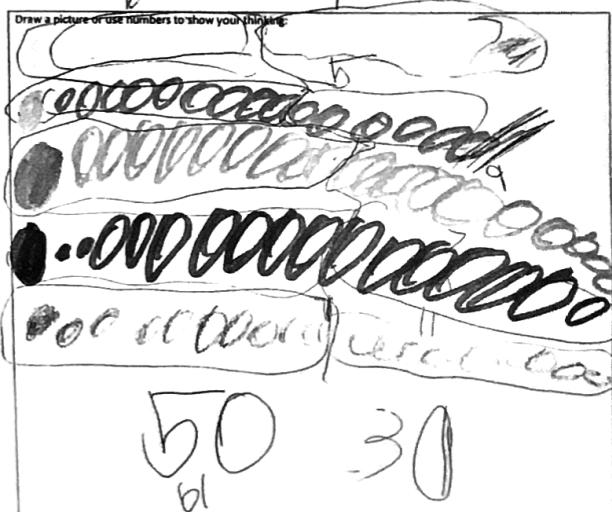
1-counting all

Answer:
91

Name: 4

Student #8

Estimate
81



3-making tens from ones

Answer:
41

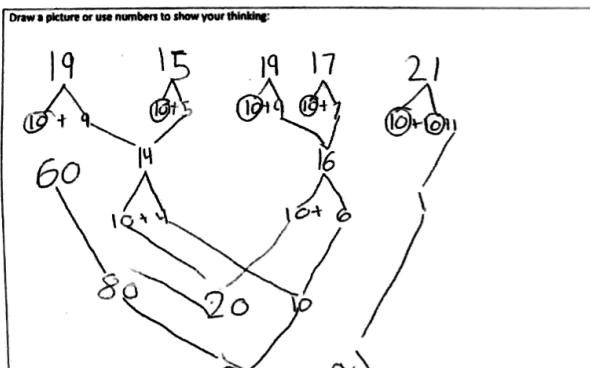
6-Partial Sums

Answer:
91

Name: 14

Student #1

Estimate
14



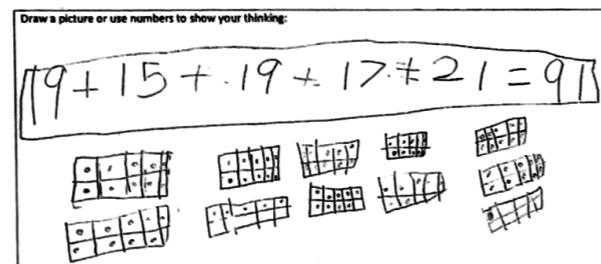
5-Place value partitioning

Answer:
91

Name: 11

Student #2

Estimate
13



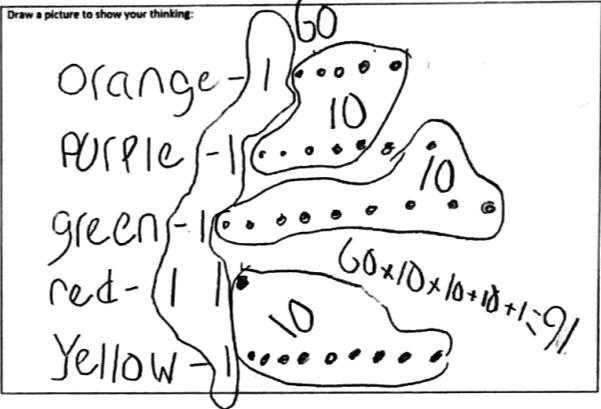
1-counting all

Answer:
91

Name: 12

Student #3

Estimate
12

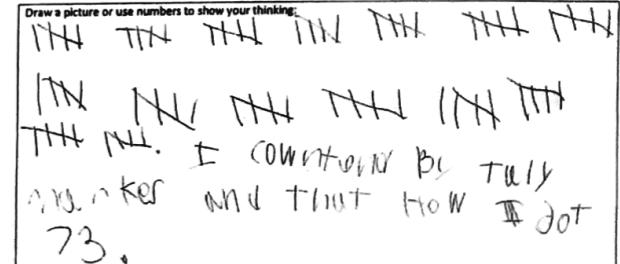


Use numbers to show your thinking:
4-making tens from ones
Answer:
91

Name: 13

Student #4

Estimate
18



2-counting all with skip counting

Answer:
73

Name: _____

Student #5

Estimate
50

Draw a picture to show your thinking:

$$\begin{array}{r} 19 \\ 15 \\ 19 \\ 17 \\ + 21 \\ \hline 91 \\ 60 \end{array}$$

Use numbers to show your thinking:

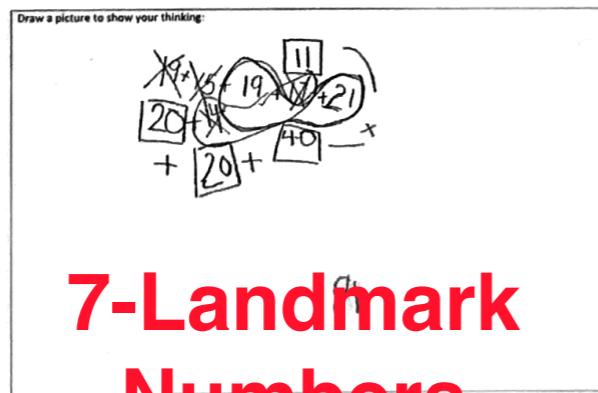
6-Partial Sums

Answer:
91

Name: _____

Student #6

Estimate
34



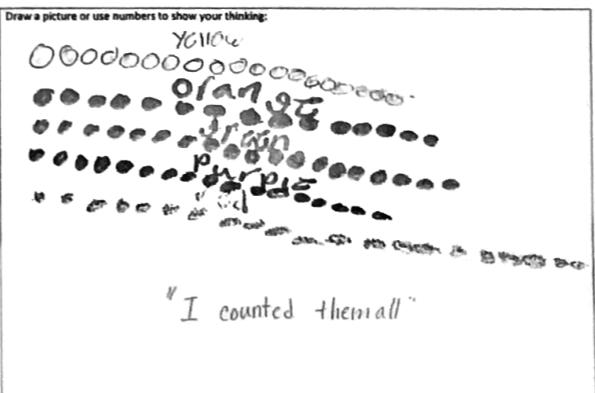
7-Landmark Numbers

Use numbers to show your thinking:
I mixed the numbers around.

Name: 203

Student #7

Estimate
81

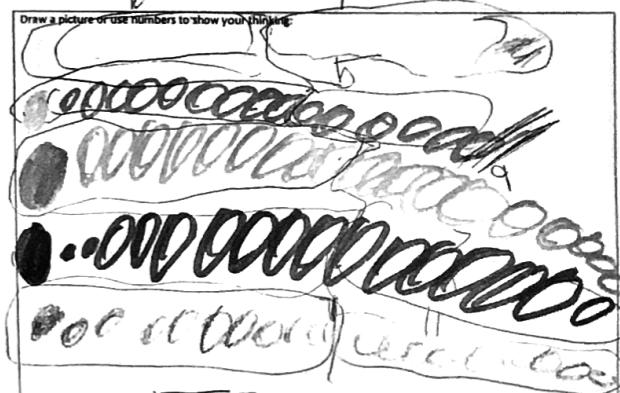


Use numbers to show your thinking:
1-counting all
Answer:
91

Name: 4

Student #8

Estimate
81



Answer:
50
30

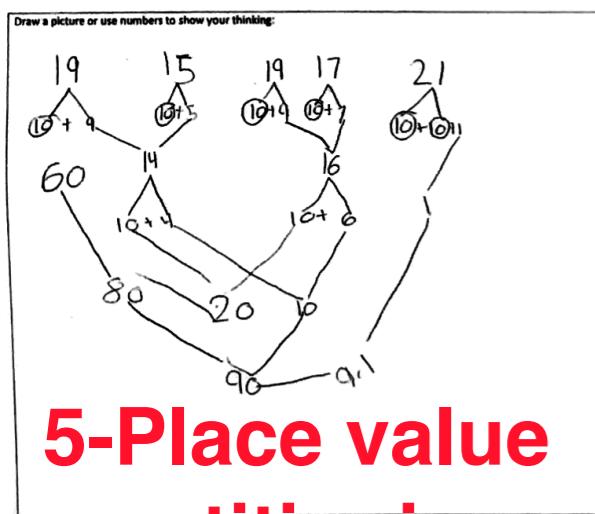
3-making tens from ones

Answer:
44

Name: 14

Estimate
14

Student #1



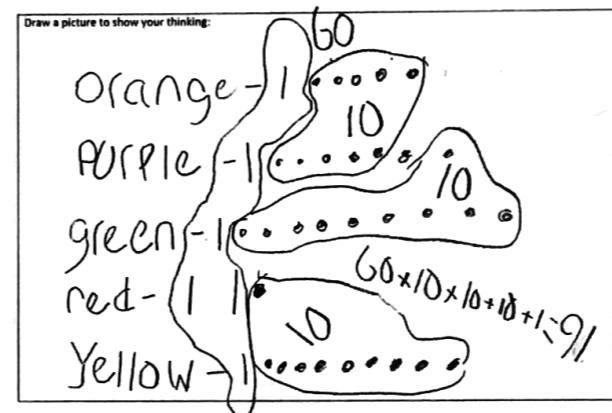
5-Place value partitioning

Answer:
91

Name: _____

Estimate
42

Student #3



Use numbers to show your thinking:

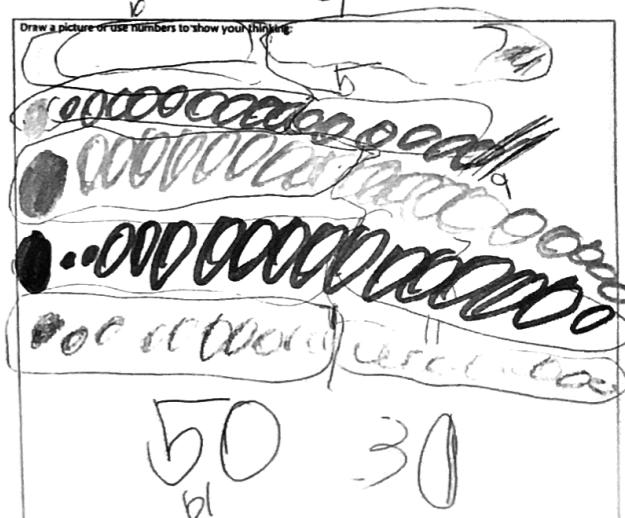
4-making tens from ones

91

Name: 4

Estimate
810

Student #8

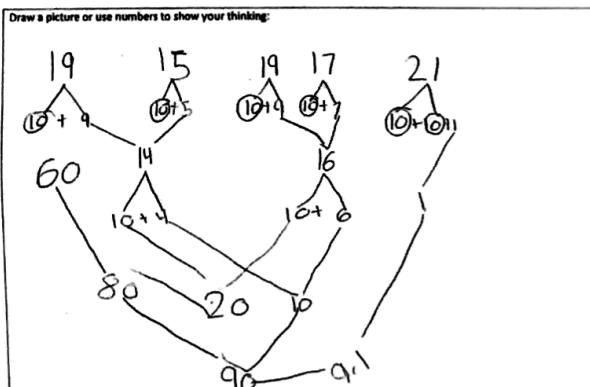


3-making tens from ones

Name: 14

Student #1

Estimate
14



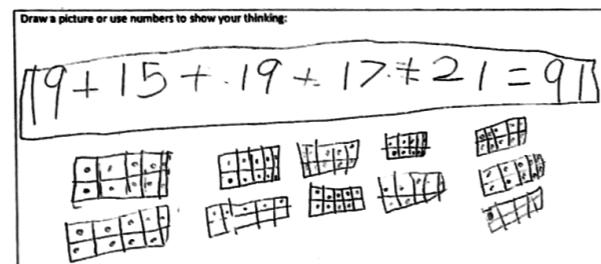
5-Place value partitioning

Answer:
91

Name: 11

Student #2

Estimate
13



I yooost + in
frames.

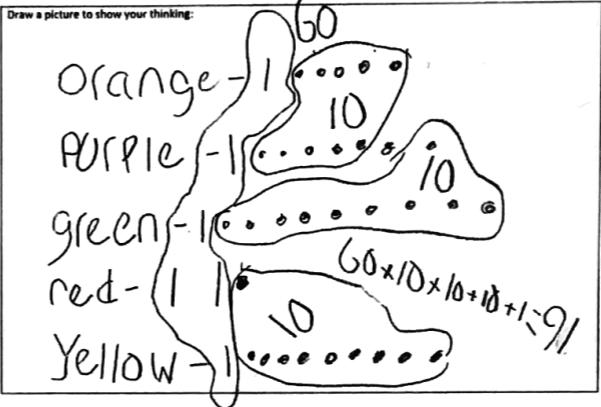
1-counting all

Answer:
91

Name: 12

Student #3

Estimate
12



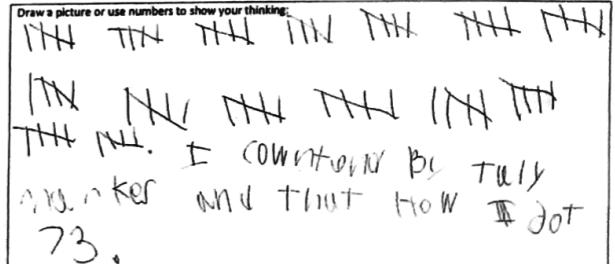
Use numbers to show your thinking:
4-making tens from ones

Answer:
91

Name: 13

Student #4

Estimate
18



2-counting all with skip counting

Answer:
73

Name: _____

Student #5

Estimate
50

Draw a picture to show your thinking:

$$\begin{array}{r} 19 \\ 15 \\ 19 \\ 17 \\ + 21 \\ \hline 91 \\ 60 \\ \hline 91 \end{array}$$

Use numbers to show your thinking:

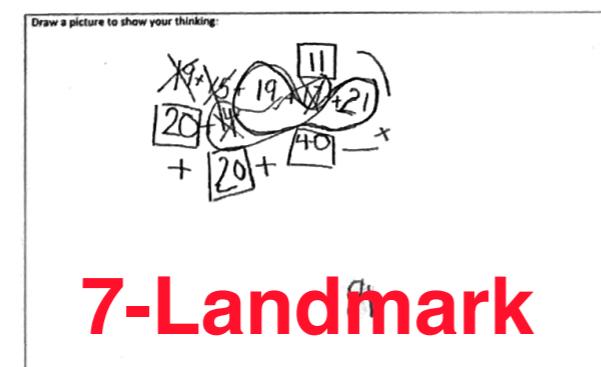
6-Partial Sums

Answer:
91

Name: _____

Student #6

Estimate
34



7-Landmark Numbers

I mixed the numbers around.

Use numbers to show your thinking:
Answer:
91

Name: 203

Student #7

Estimate
81



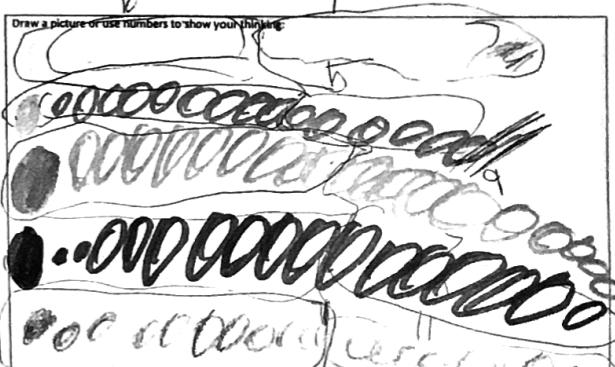
1-counting all

Answer:
91

Name: 4

Student #8

Estimate
81



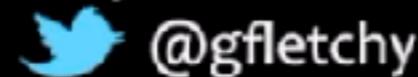
3-making tens from ones

Answer:
44

Making Sense Series

the progression of addition AND subtraction
the standard traditional algorithm

created by Graham Fletcher



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