The Power of Progressions: Untangling the Knotty Areas of Teaching and Learning Mathematics

Graham Fletcher

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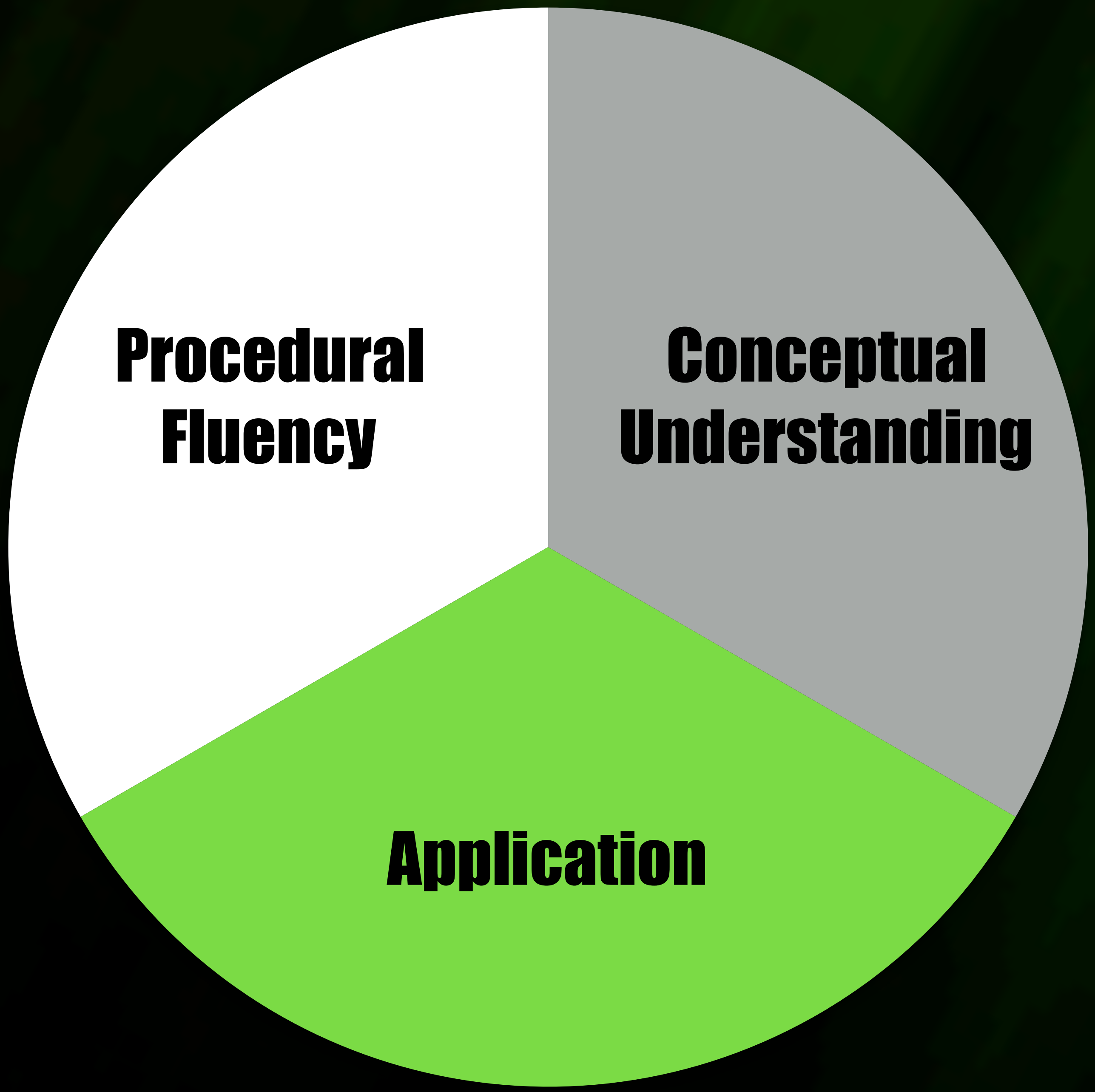
NEXT TIME YOU RE AFRAID
TO SHARE IDEAS
REMEMBER SOMEONE
ONCE SAID IN A MEETING
LETS MAKE A FILM WITH A
TORNADO FULL OF SHARKS

Morning's Goals

- Understand the structure of 3-act task and see how they fit into the scope and sequence of a unit.

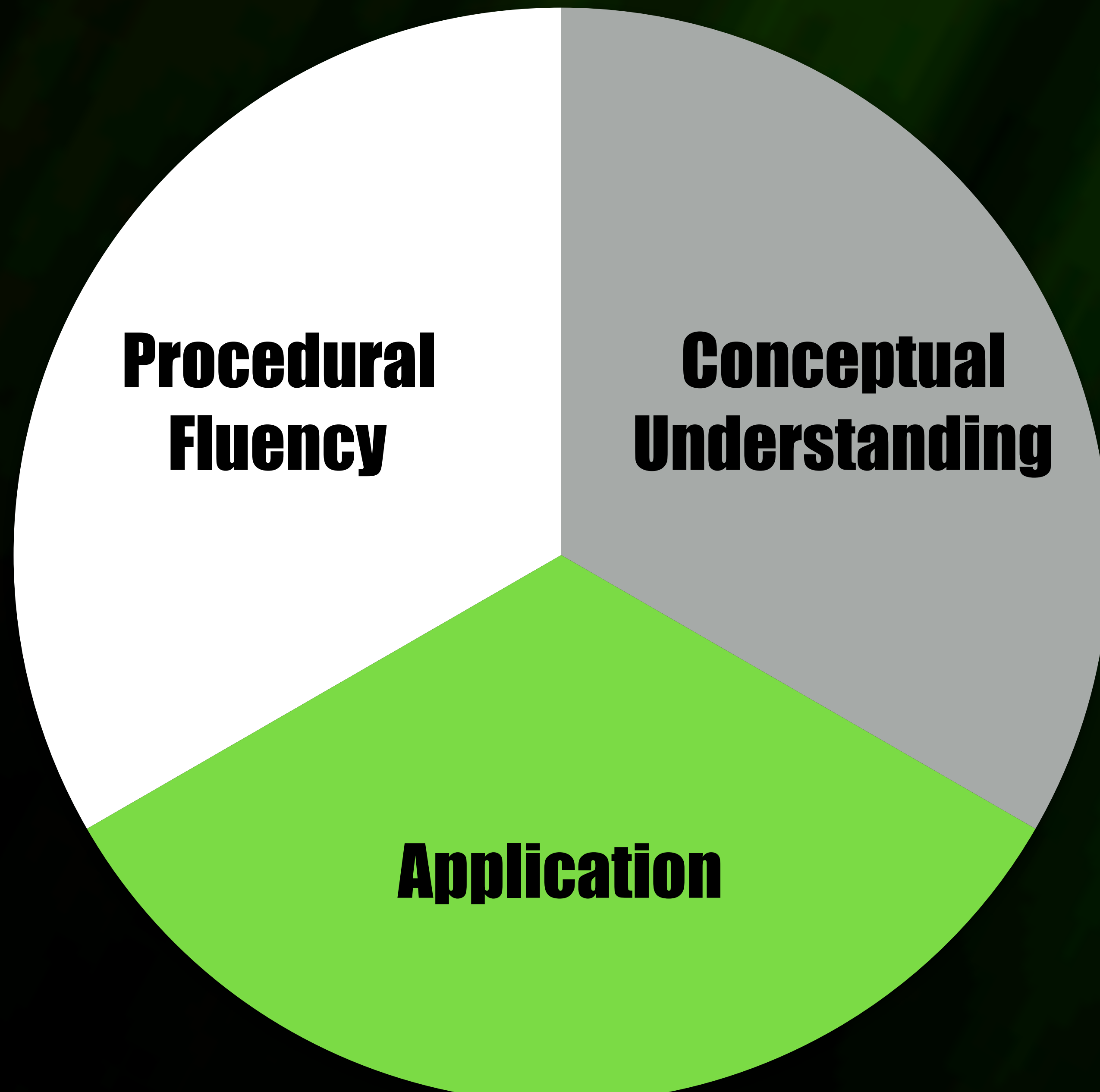
- Explore the importance of progressional understanding and how a good task can be used as formative assessment.
    - Early Number and Counting
    - Addition and Subtraction

- Understand the importance of an effective closing and the role it plays in deciding our next move.



http://www.corestandards.org/other-resources/key-shifts-in-mathematics/

Procedural
Fluency



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How many Skittles does Alicia have?

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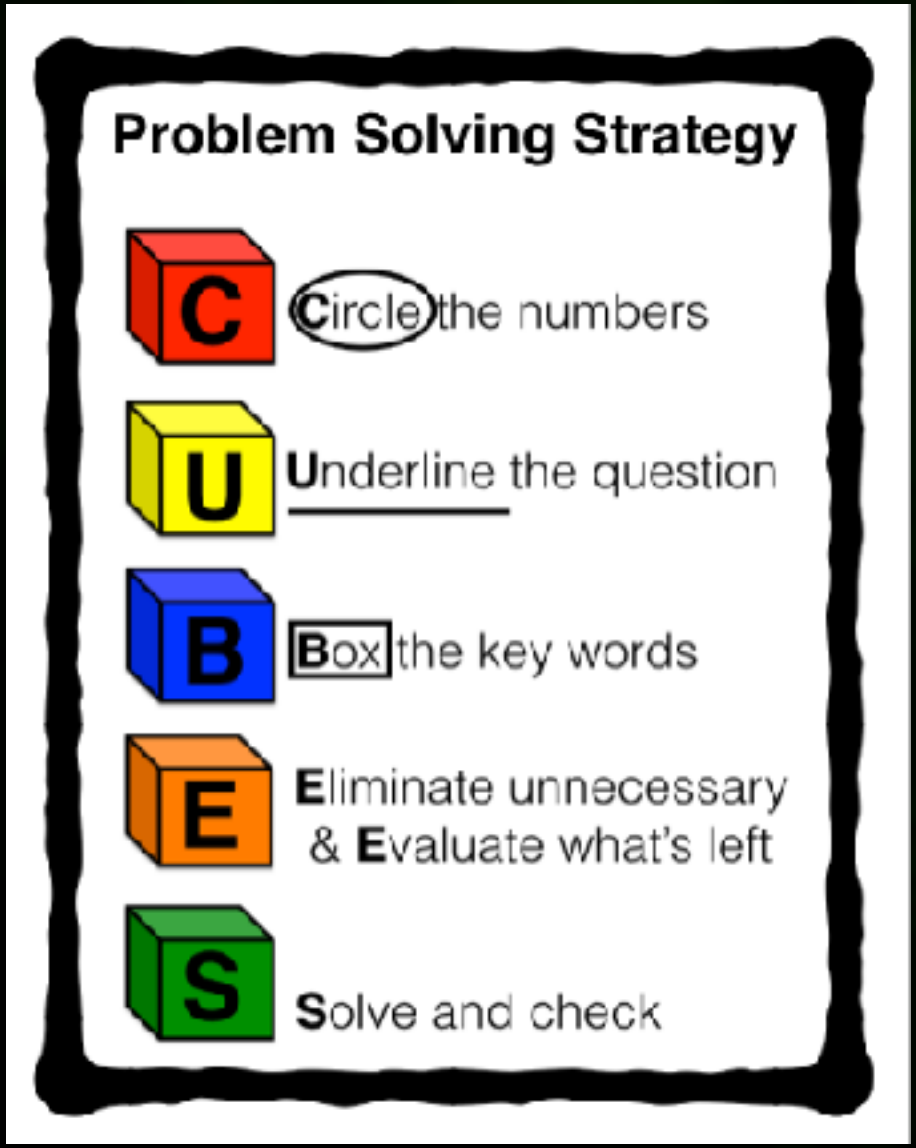

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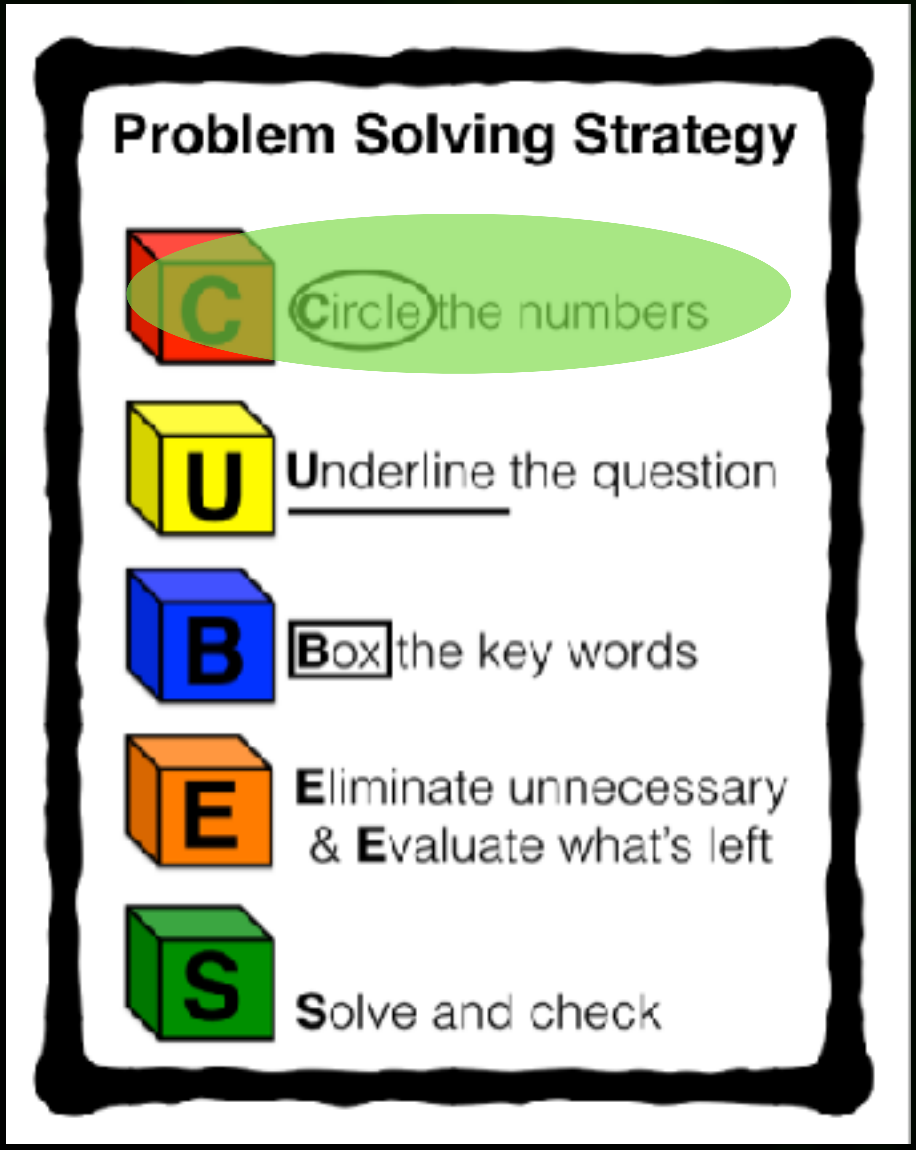

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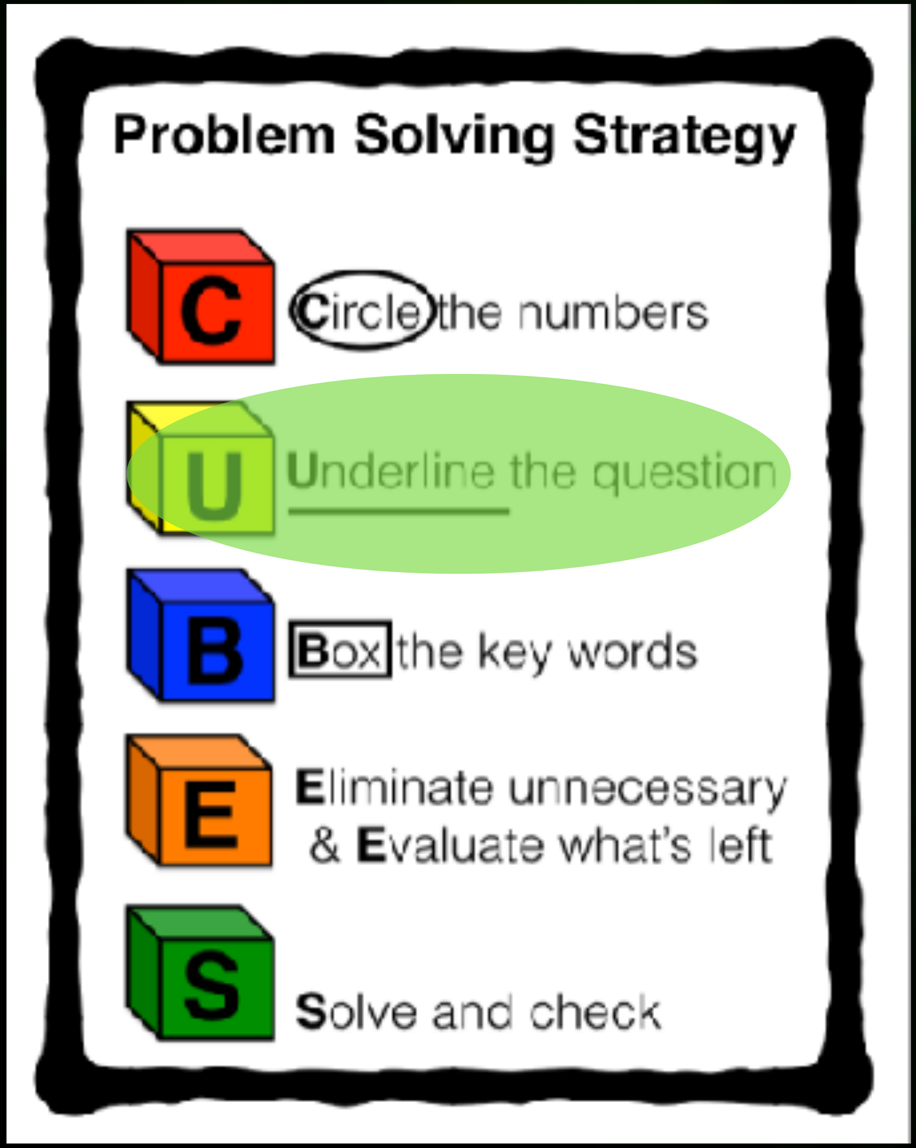

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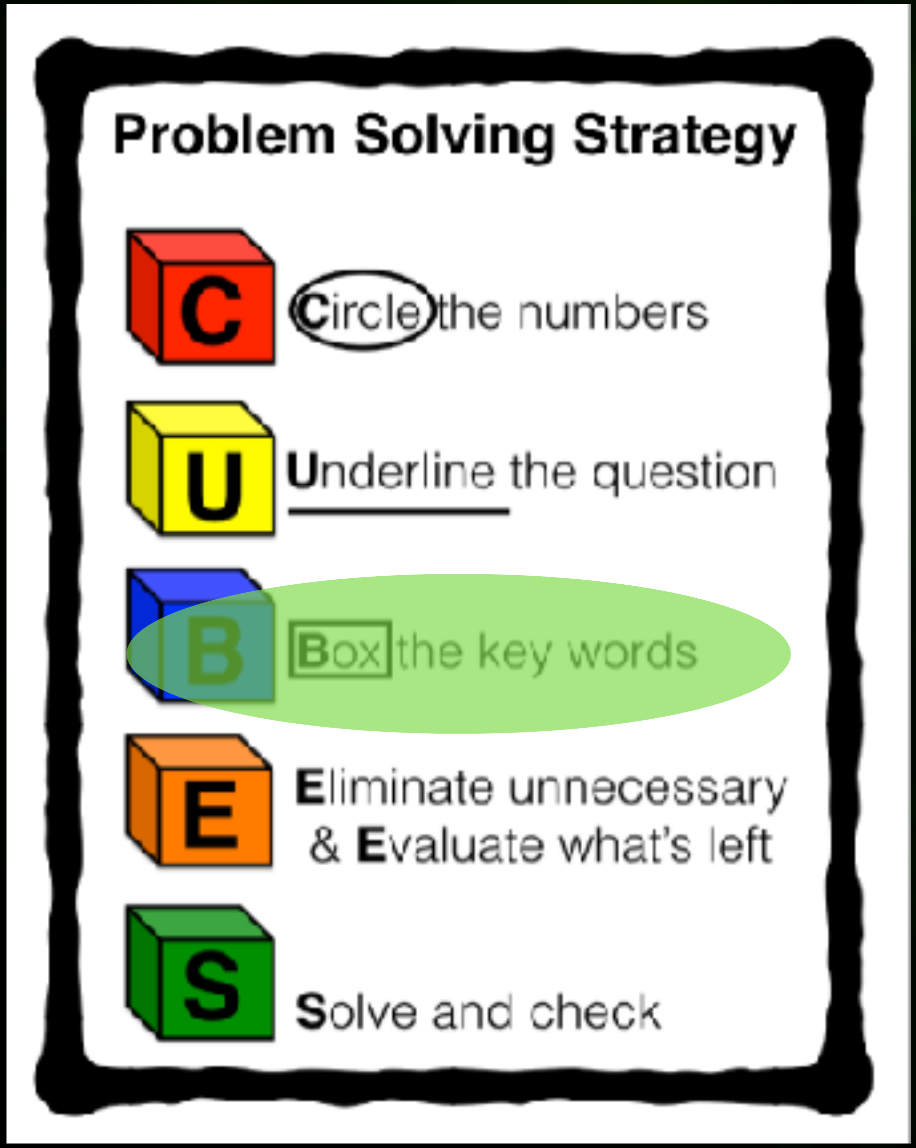
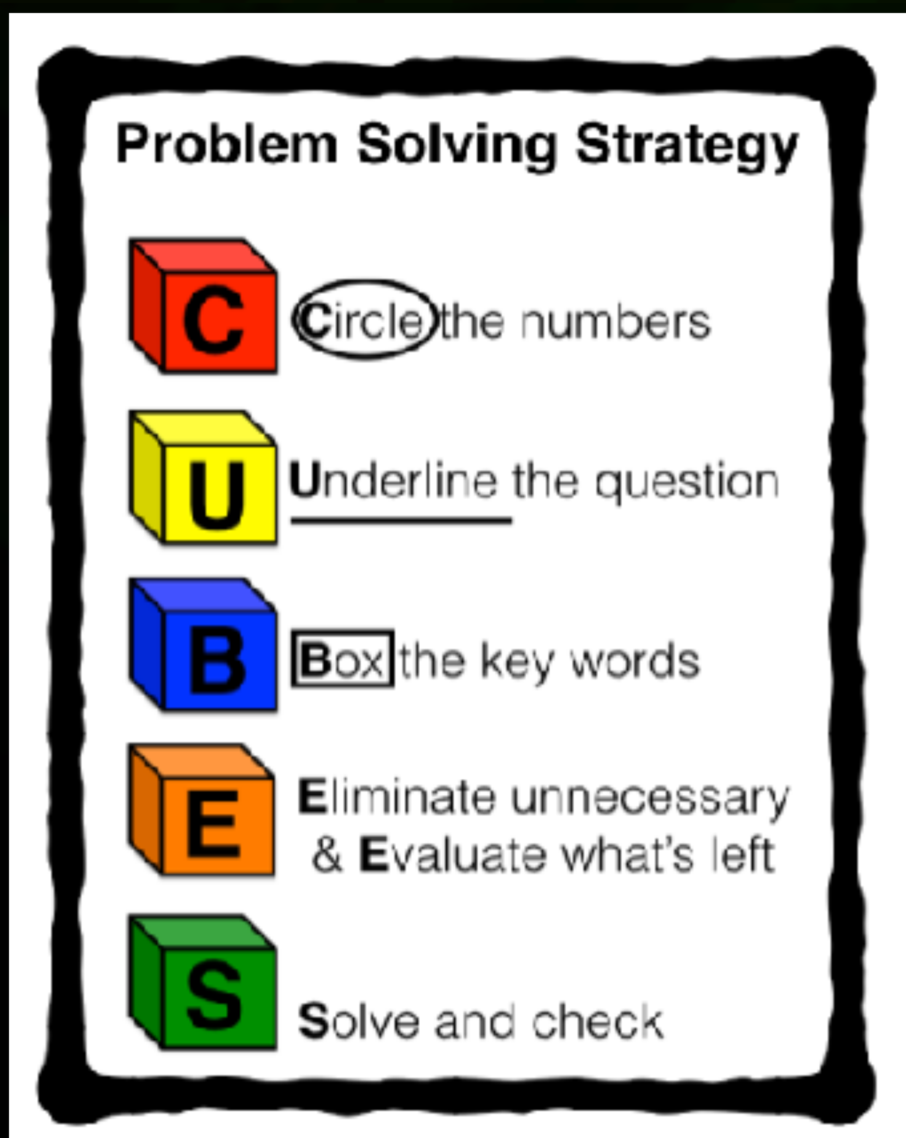
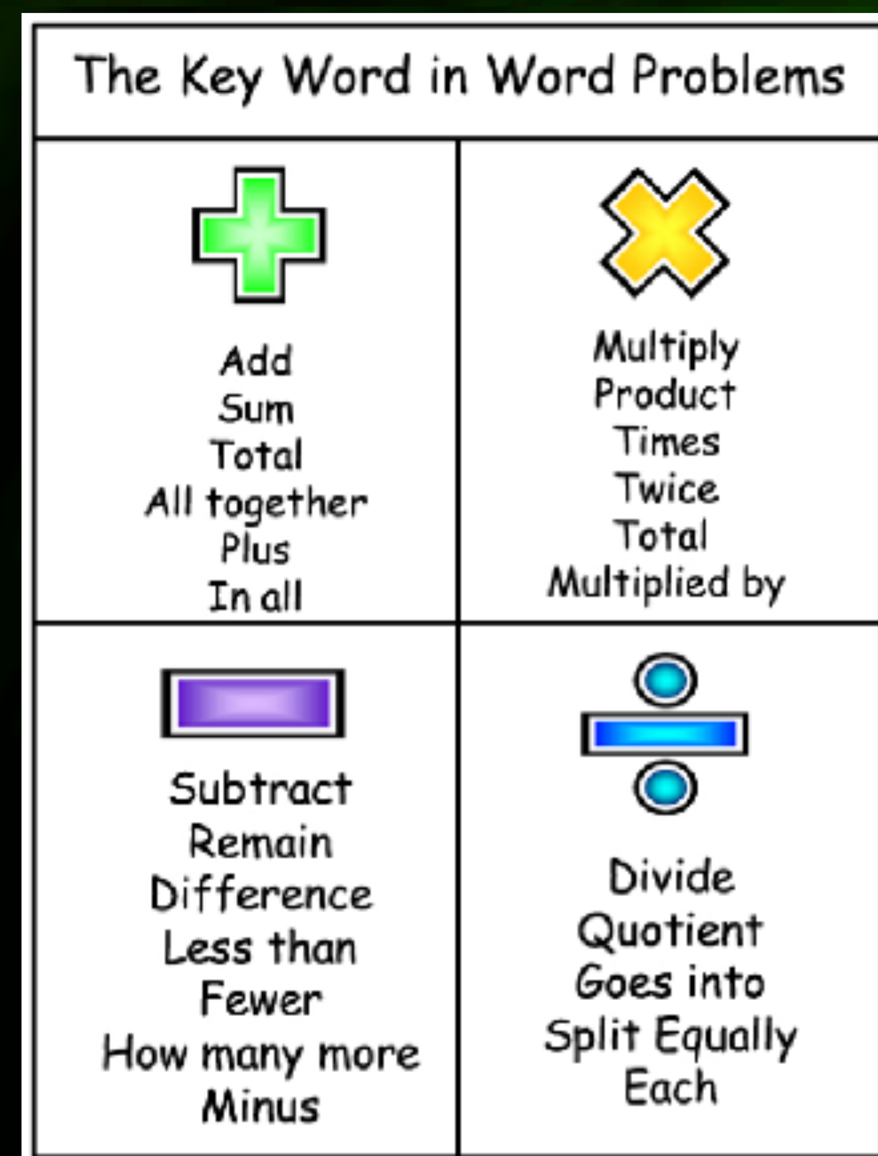


fewer

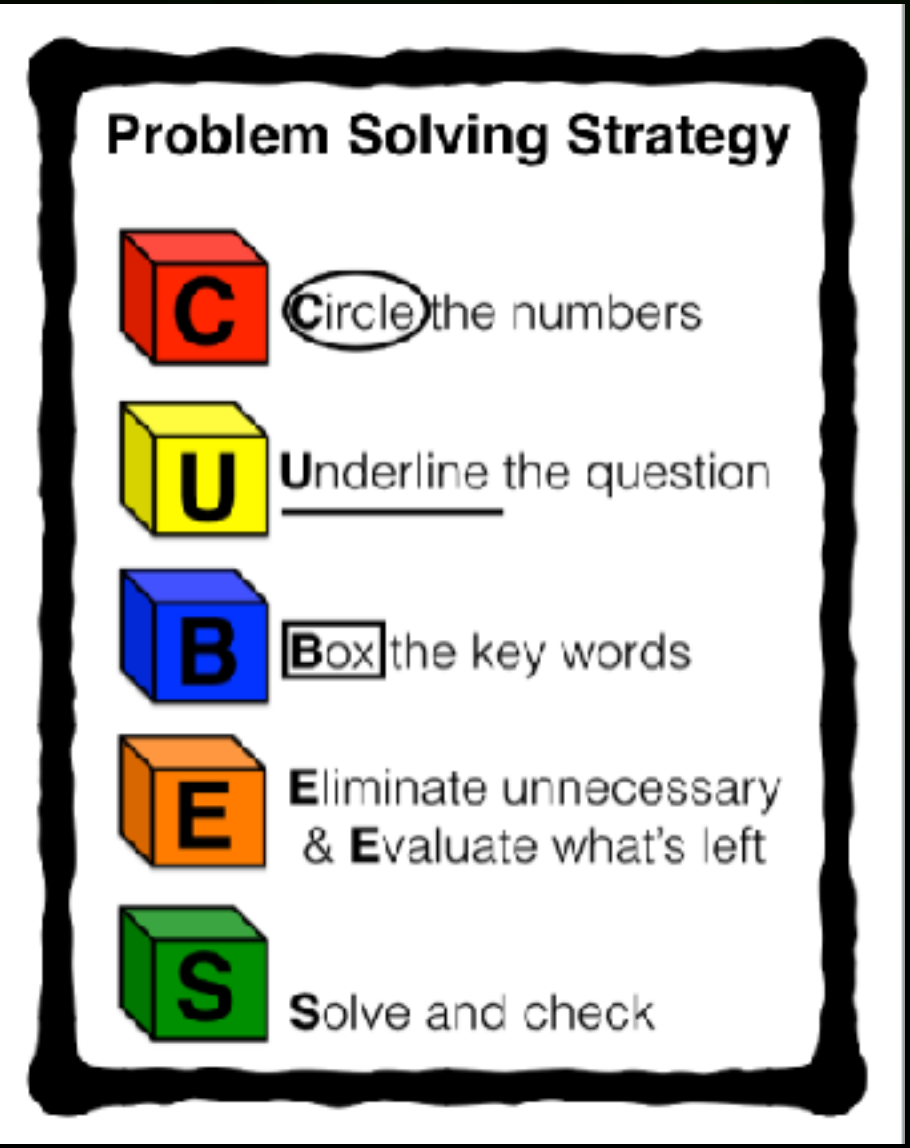
17

12


Problem Solving Strategy
C Circle the numbers
U Underline the question
B Box the key words
E Eliminate unnecessary & Evaluate what's left
S Solve and check


The Key Word in Word Problems
Add
Sum
Total
All together
Plus
In all
Multiply
Product
Times
Twice
Total
Multiplied by
Subtract
Remain
Difference
Less than
Fewer
How many more
Minus
Divide
Quotient
Goes into
Split Equally
Each

17 — 12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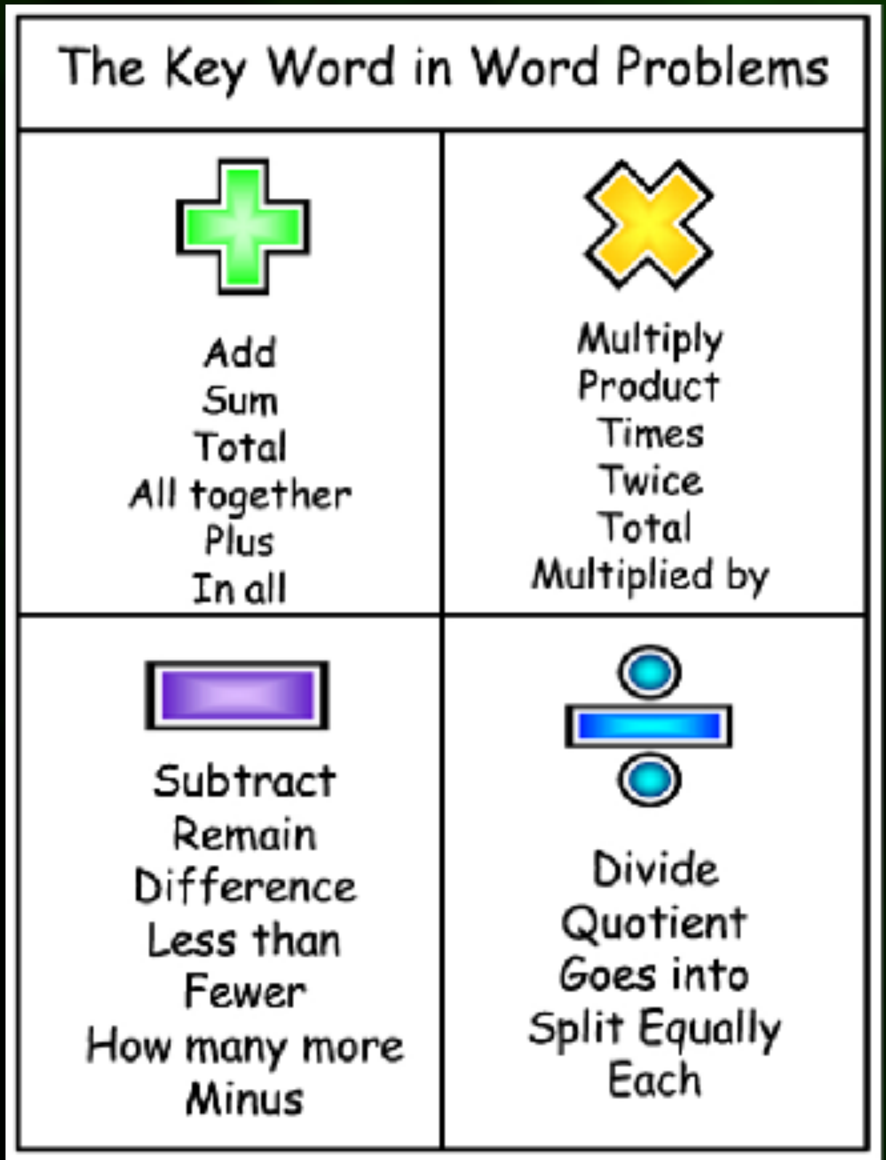

The Key Word in Word Problems
Add
Sum
Total
All together
Plus
In all
Multiply
Product
Times
Twice
Total
Multiplied by
Subtract
Remain
Difference
Less than
Fewer
How many more
Minus
Divide
Quotient
Goes into
Split Equally
Each

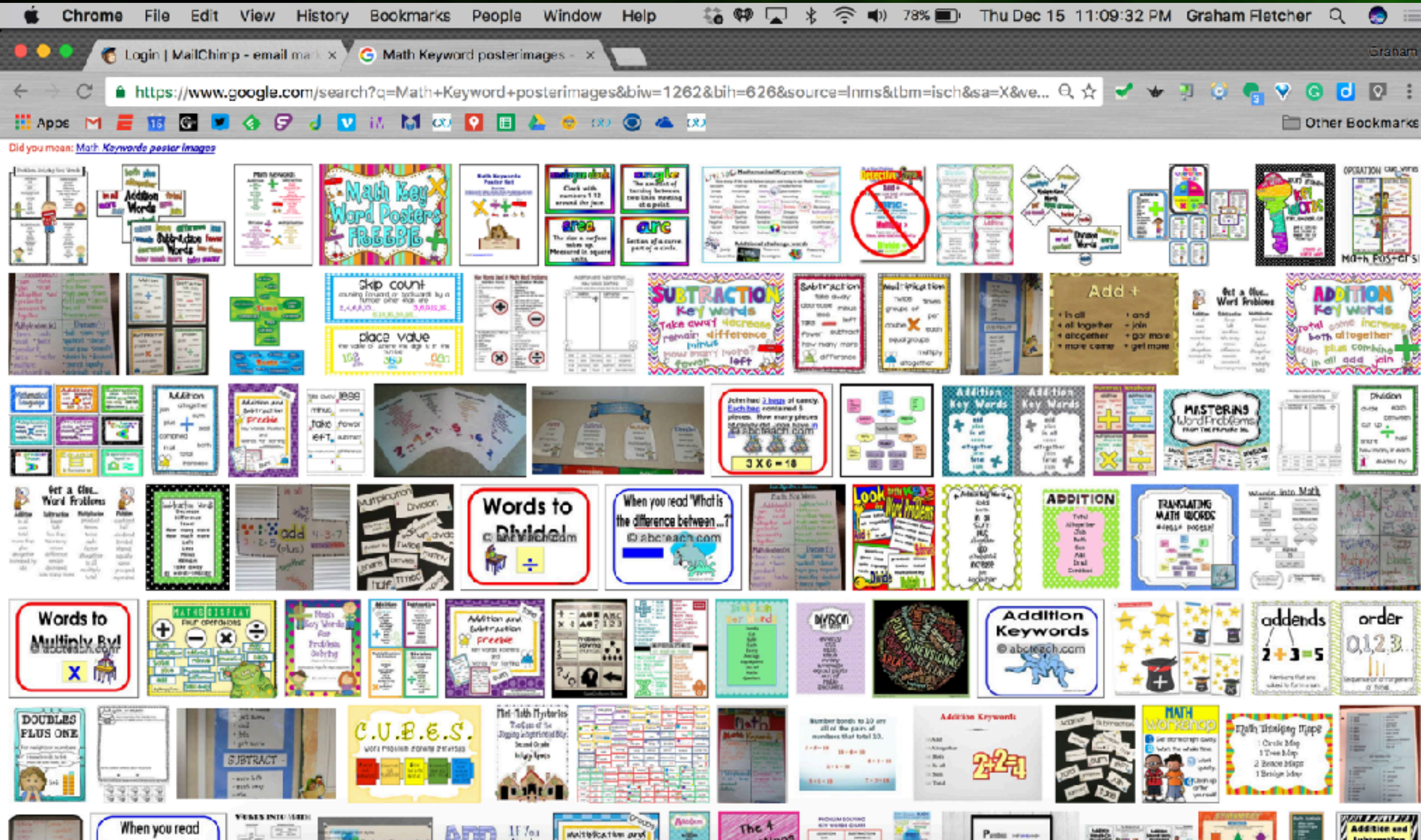
17 — 12

WTF?

17 — 12

W hat's T he F ive ?

Current Research



Name: ______________________ Date: ______________

1. What did you notice?

2. What do you wonder?

3. Main Question:

4. Make an estimate.

low estimate

high estimate

Place an "X" to represent your estimate on the number line.

5. What information do you need?

6. Construct a viable argument or share a reflection:

Answer

Name: ______________________

Estimate

Draw a picture to show your thinking:

Use numbers to show your thinking:

Answer:

?


-19 yellow
-15 orange
-19 green
-17 purple
-21 red

The Big Reveal

Graham had some Skittles. He had 19 yellow, 15 orange, 19 green, 17 purple, and 21 red. How many Skittles did Graham have?

3-Act Tasks

Act 1:
- Real world problem or scenario presented
- What do you notice? What do you wonder?
- Make estimates

Act 2:
- Identify missing variables and missing variables to solve
- Define solution path using variables

Act 3:
- Solve and interpret results of the solution
- Validate answer

Most asked questions:

- How often should we use 3-Act Tasks?
- When should we use 3-Act tasks? How do they fit into the scope of a unit?
- How long does one task usually take?
- What if we don’t have the time?

Orchestrating Discussions

Five practices constitute a model for effectively using student responses in whole-class discussions that can potentially make teaching with high-level tasks more manageable for teachers.

Margaret S. Smith, Elizabeth K. Hughes, Randi A. Engle, and Mary Kay Stein

Margaret S. Smith, pegs@pitt.edu, is an associate professor of mathematics education at the University of Pittsburgh. Over the past decade, she has been developing research-based materials for use in the professional development of mathematics teachers and studying what teachers learn from the professional development in which they engage. **Elizabeth K. Hughes**, elizabeth.hughes@uni.edu, recently finished her doctorate in mathematics education at the University of Pittsburgh. Her areas of interest include preservice secondary mathematics teacher education and the use of practice-based materials in developing teachers' understanding of what it means to teach and learn mathematics. **Randi A. Engle**, raengle@berkeley.edu, is an assistant professor of mathematics education and the social context of learning at the University of California Berkeley. She is interested in developing practical theories for how mathematics teachers can create discussion-based learning environments that promote strong student engagement, learning, and transfer. **Mary Kay Stein**, mkstein@pitt.edu, is a professor of learning sciences and policy and the director of the Learning Policy Center at the University of Pittsburgh. Her research focuses on instructional practice and the organizational and policy conditions that shape it.

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Although discussions about high-level tasks provide important

The 5 practices are:

1. **Anticipating** student responses to challenging mathematical tasks;
2. **Monitoring** students' work on and engagement with the tasks;
3. **Selecting** particular students to present their mathematical work;
4. **Sequencing** the student responses that will be displayed in a specific order and;
5. **Connecting** different students' responses and connecting the responses to key mathematical ideas.

MTMS: Vol. 14, No. 9, May 2009-5 Practices for Orchestrating Productive Mathematics Discussions

Task Planning Page

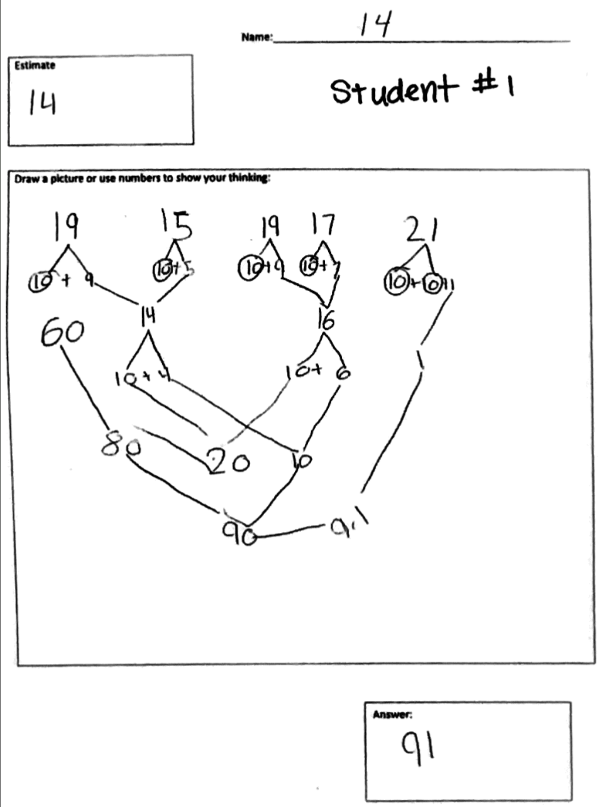
Learning Target:

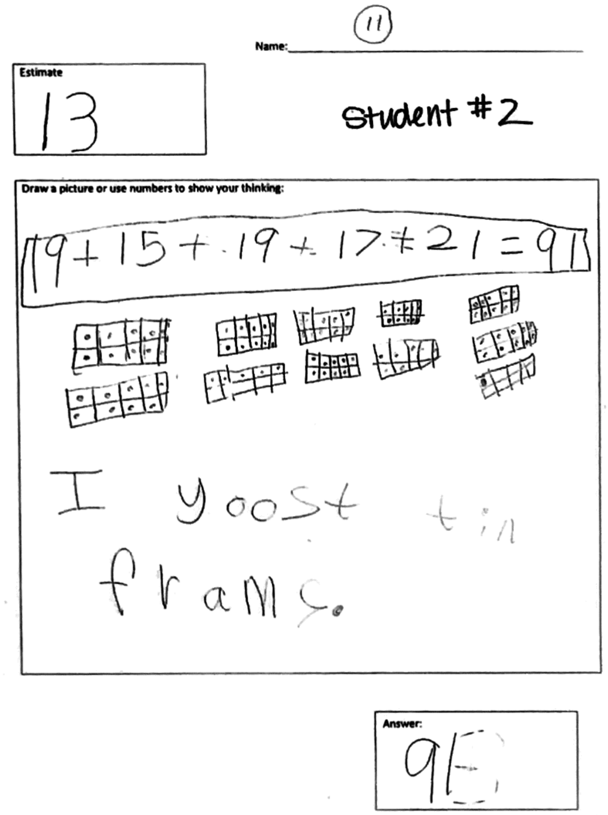
Questions and Look-Fors:

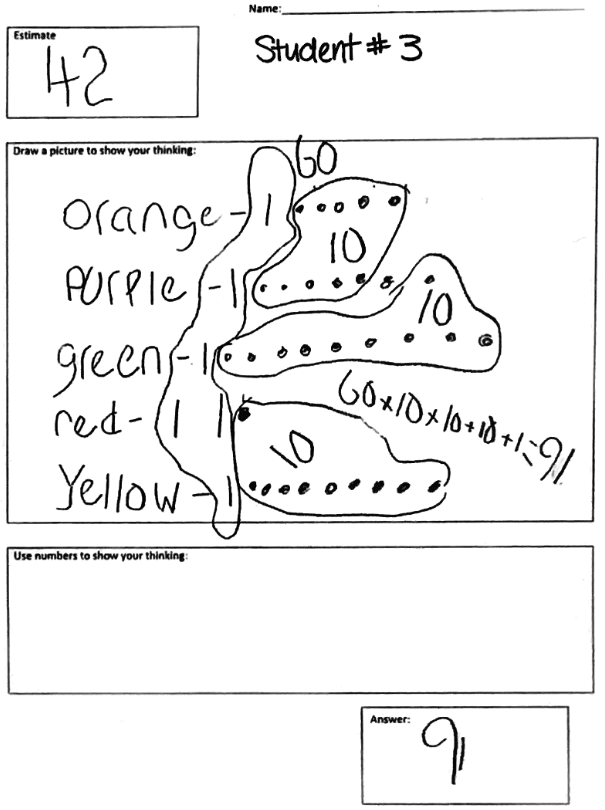
Strategy	Who and What	Order

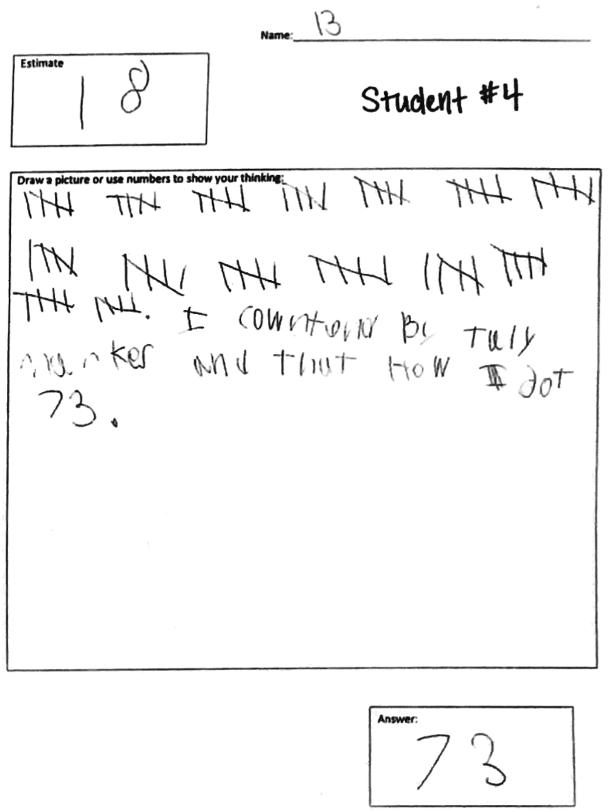
Notes:

Anticipating → Monitoring → Selecting → Sequencing → Connecting

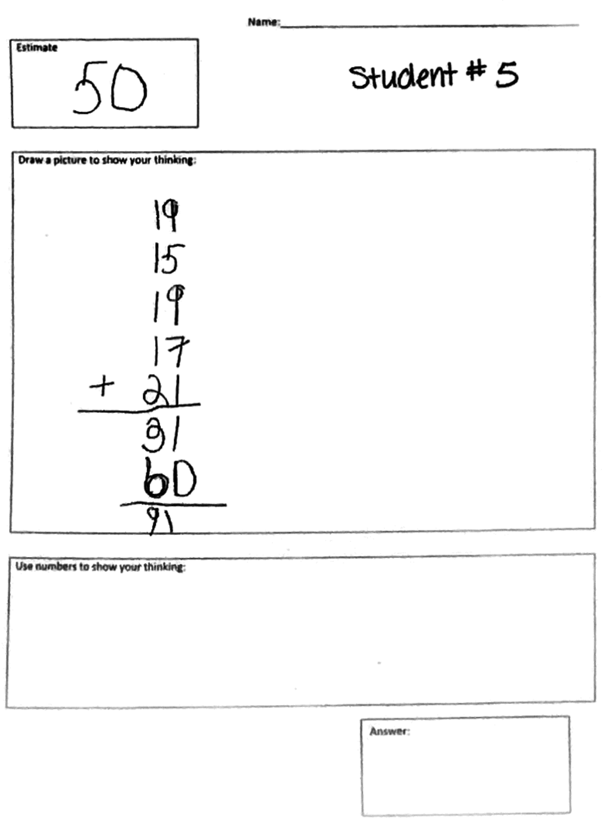


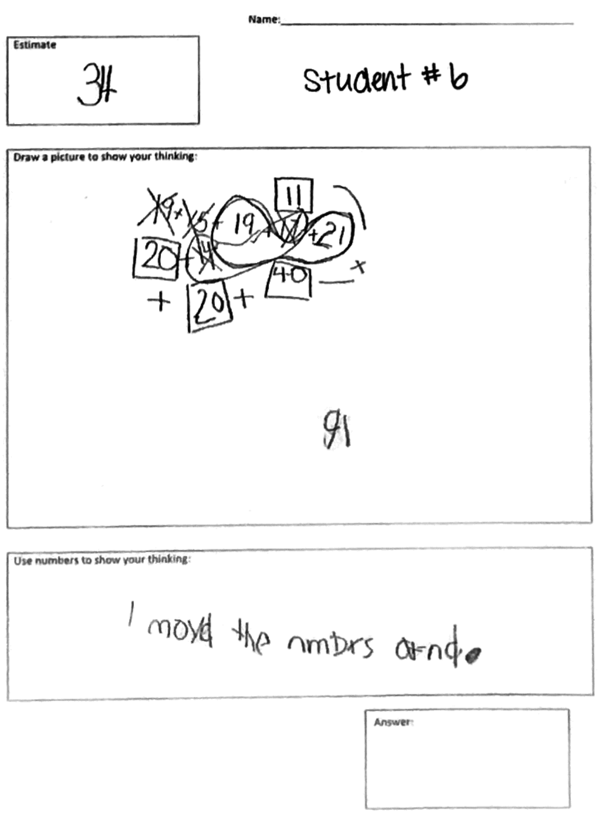


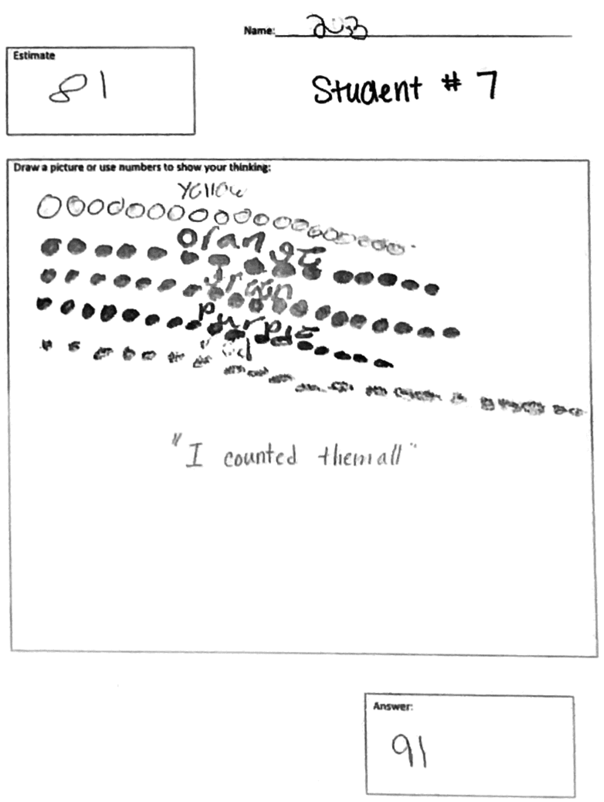


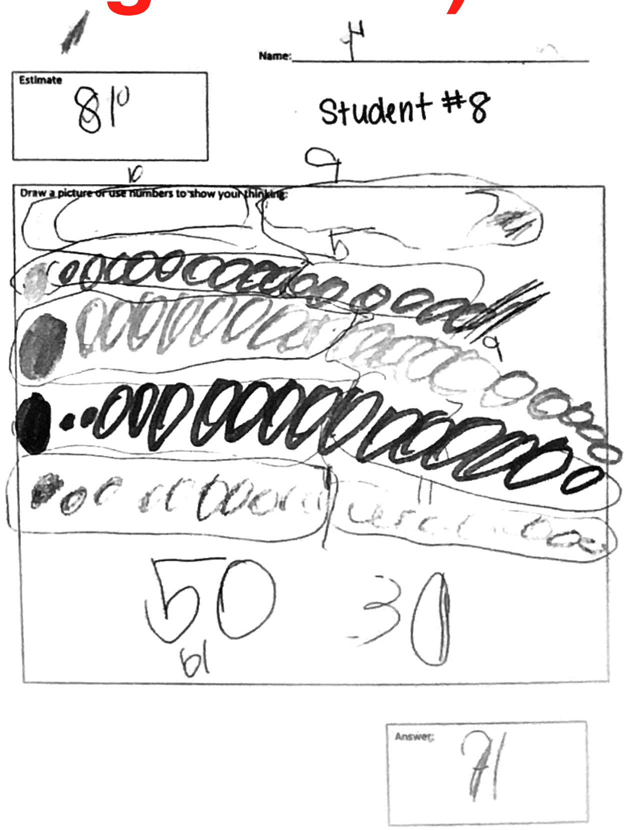


Identify and name the strategy used, then place the student work in order in terms of efficiency (least to greatest)









Making Sense Series

the progression of addition AND subtraction
the standard traditional algorithm

created by Graham Fletcher
@gfletchy
www.gfletchy.com

3-Act Tasks

5 Practices

Progressions

1 sheet for 4 people

Cut up all the numbers and symbols and create one equation. All the numbers and symbols must be used in the equation.

Nothing should be leftover except for the black square.

0	5	6	9
1	4	7	8
2	3	+	+
+	-	=	=
=	=	+	DO NOT USE DISCARD

Using the digits 1-9 at most one time each, create 4 numbers that have a sum of 91.

□□ + □ + □□ + □□

You can use the 9 & 1 from the cards

Kindergarten?


CHICLE
TABS
CHEWING GUM
25¢


4-orange
2-red
2-yellow
2-white
0-pink

Name: [redacted]

S5

Estimate

3

Draw a picture to show your thinking:

4 + 2 + 2 + 2 + 0 =
10

Use numbers to show your thinking:

Answer:

10

Estimate

6

S1

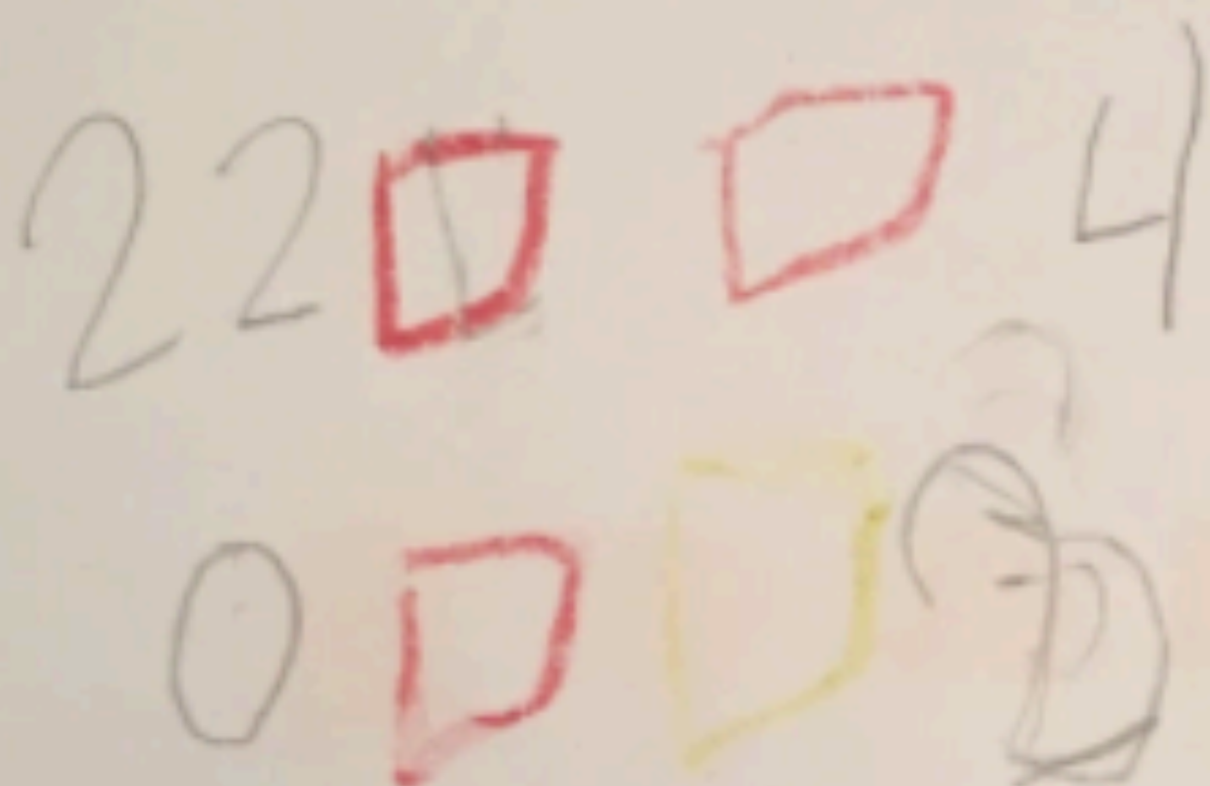
Draw a picture to show your thinking:

Estimate

1

86

Draw a picture to show your thinking:



Name: [redacted]

S3

Draw a picture to show your thinking

Use numbers to show your thinking

Name: [redacted]

S8

Estimate

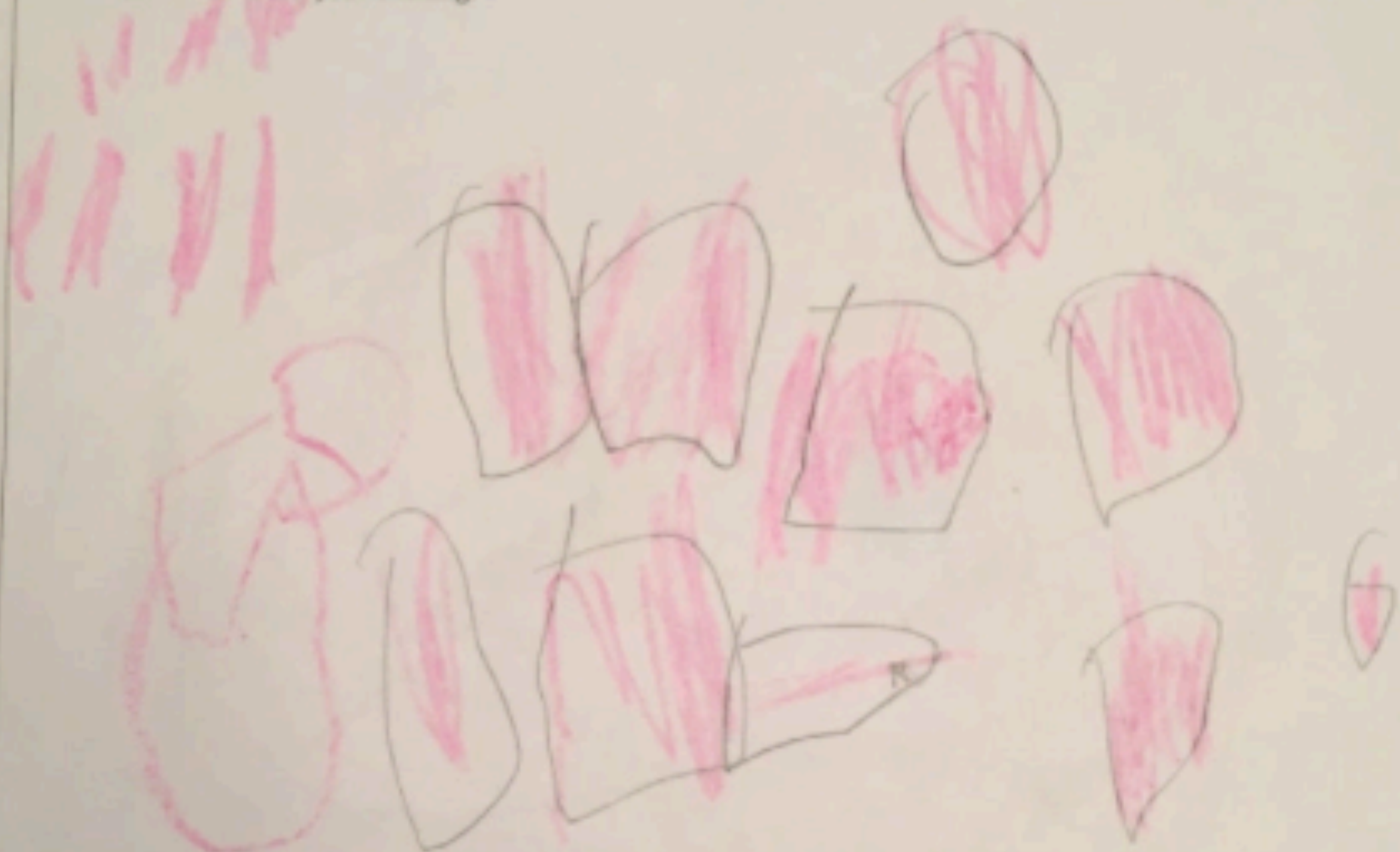
9

Draw a picture to show your thinking:

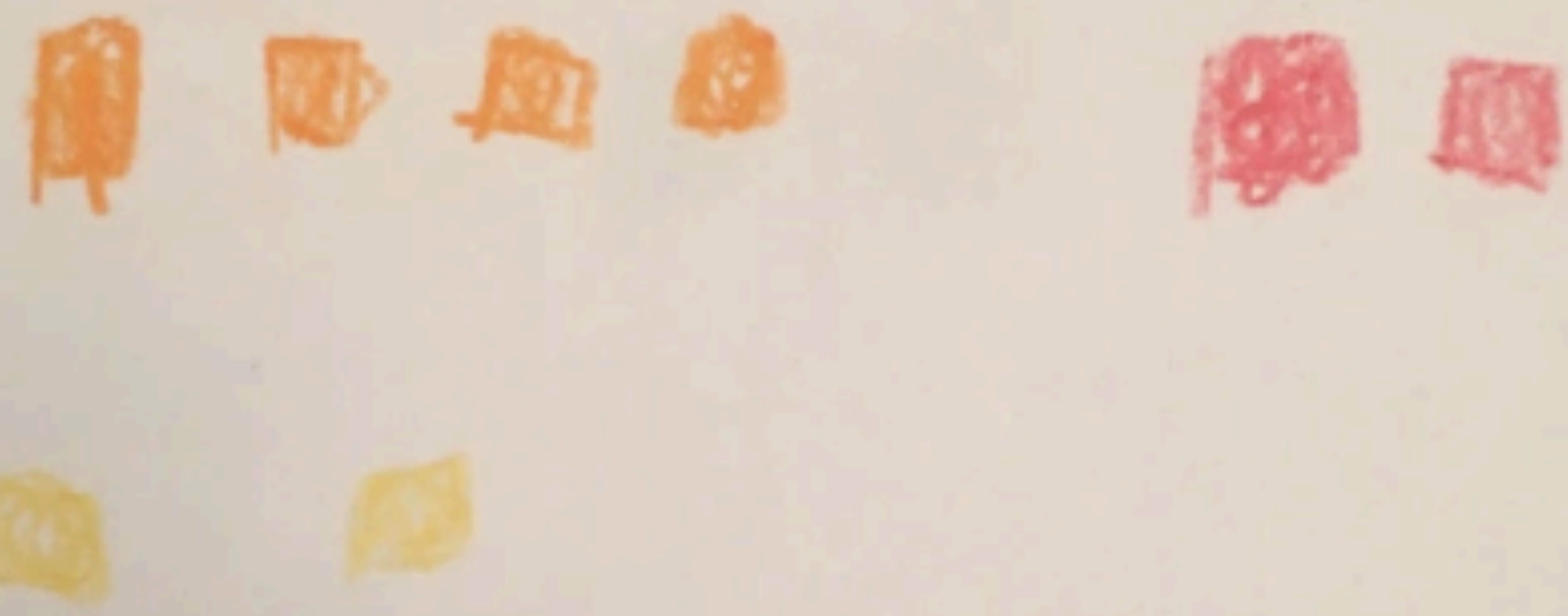
Use numbers to show your thinking:

12342678916

Draw a picture to show your thinking:



Use numbers to show your thinking:



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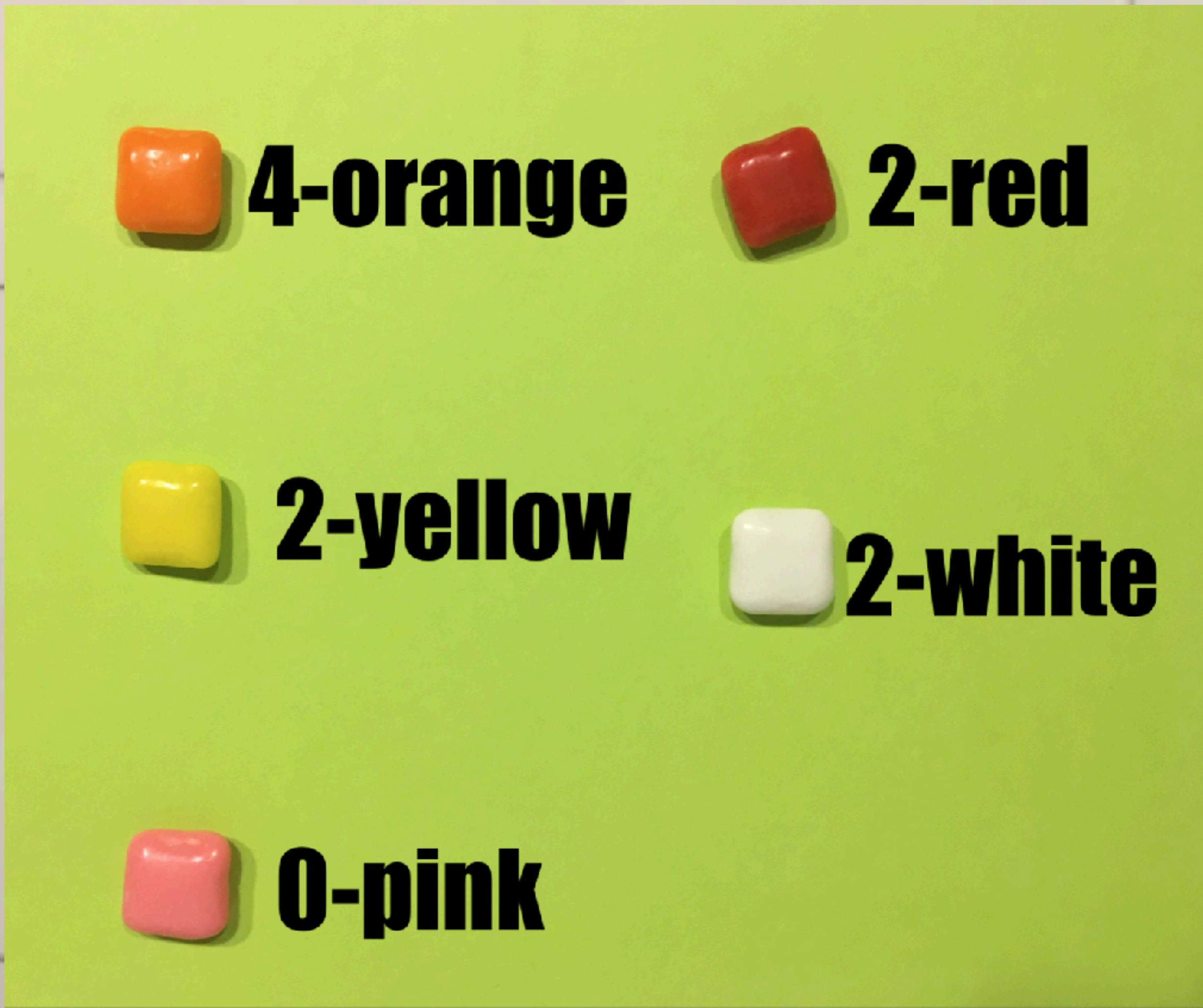


Number Sense Trajectory

Counting

Comparison

Hierarchical Inclusion

Subitizing

Cardinality

Number Conservation

1-to-1 Correspondence

Number Sense Trajectory

Subitizing

Comparison

Rote Counting

1-to-1 Correspondence

Cardinality

Hierarchical Inclusion

Number Conservation

Number Sense Trajectory –*Putting It All Together*

	Subitizing	Comparison	Counting	One-to-One Correspondence	Cardinality	Hierarchical Inclusion	Number Conservation
Trajectory	Being able to visually recognize a quantity of 5 or less.	Being able to compare quantities by identifying which has more and which has less.	Rote procedure of counting. The meaning attached to counting is developed through one-to-one correspondence.	Students can connect one number with one object and then count them with understanding.	Tells how many things are in a set. When counting a set of objects, the last word in the counting sequence names the quantity for that set.	Numbers are nested inside of each other and that the number grows by one each count. 9 is inside 10 or 10 is the same as 9 + 1.	The number of objects remains the same when they are rearranged spatially. 5 is 4&1 OR 3&2.

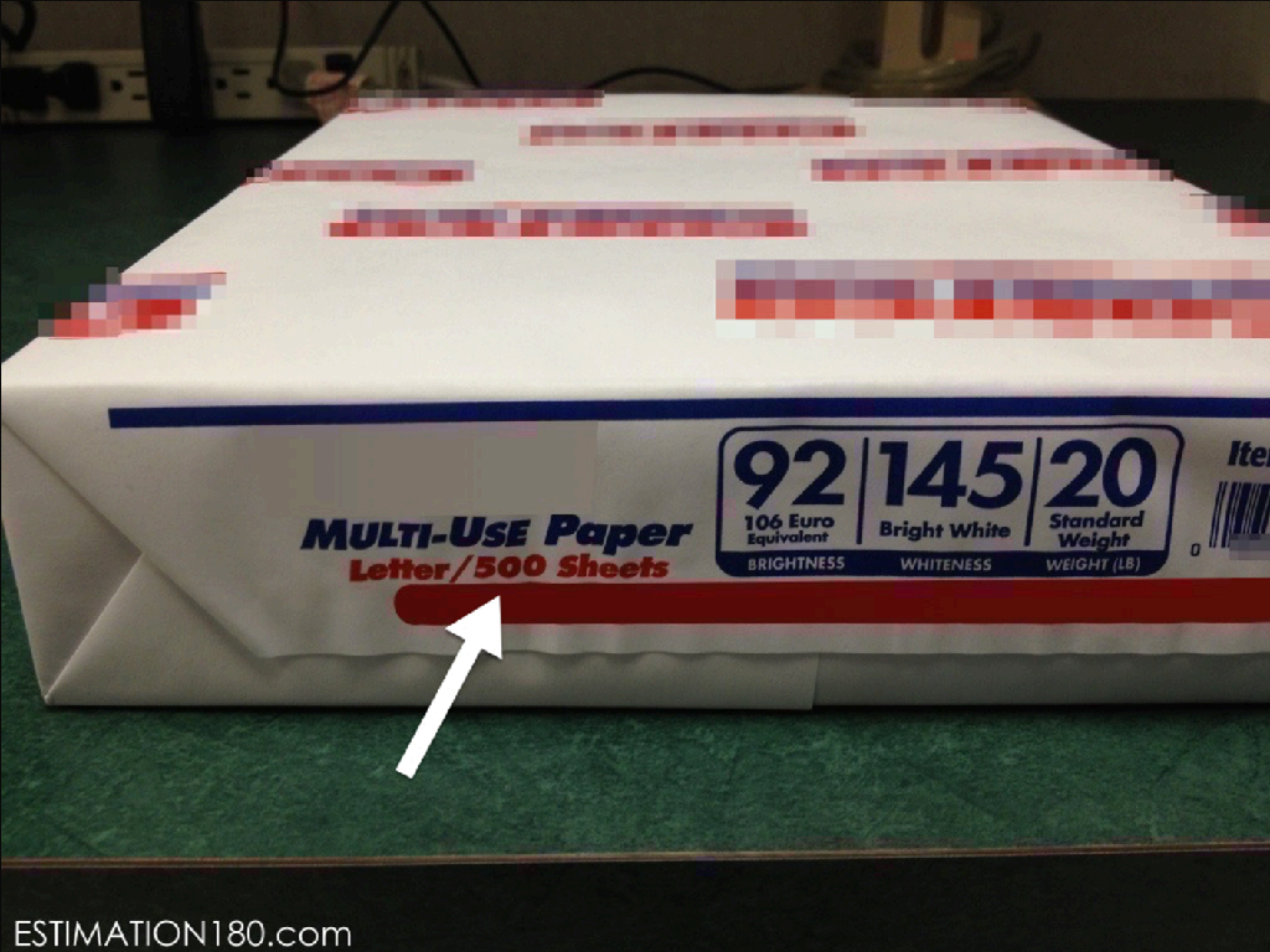
Each concept builds on the previous idea and students should explore and construct concepts in such a sequence

	Spatial Relationship Patterned Set Recognition	One and Two-More or Less	Understanding Anchors	Part-Part-Whole Relationship
Number Relationships	Students can learn to recognize sets of objects in patterned arrangements and tell how many without counting.	Students need to understand the relationship of number as it relates to +/- one or two. Here students should begin to see that 5 is 1 more than 4 and that it is also 2 less than 7.	Students need to see the relationship between numbers and how they relate to 5s and 10s. 3 is 2 away from 5 and 7 away from 10.	Students begin to conceptualize a number as being made up from two or more parts.

Perceptual

&

Conceptual


MULTI-USE Paper
Letter/500 Sheets
92
106 Euro Equivalent
BRIGHTNESS
145
Bright White
WHITENESS
20
Standard Weight
WEIGHT (LB)
ESTIMATION180.com

Making Sense Series

the progression of early number & counting

created by Graham Fletcher
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$$6 \times \frac{5}{8}$$

Fletchy

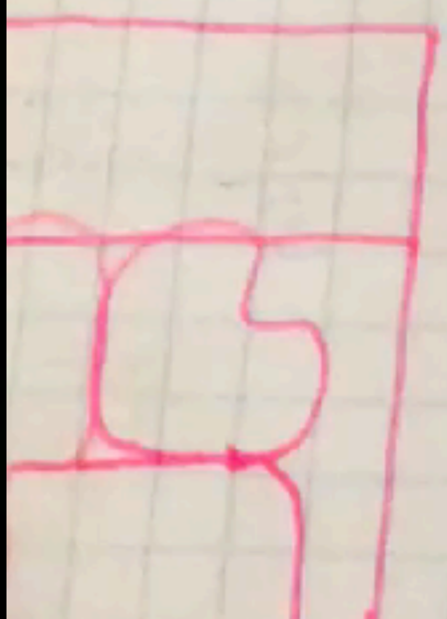
$$6 \times \frac{5}{8}$$

	10 +	3 +	5/8
60	600	180	

$6 * 5/8 = 3\ 6/8$

	10	+ 3	+ 5/8
0	600	180	

$6 * 5/8 = 3\ 6/8$



$6 \times \frac{5}{8}$

8

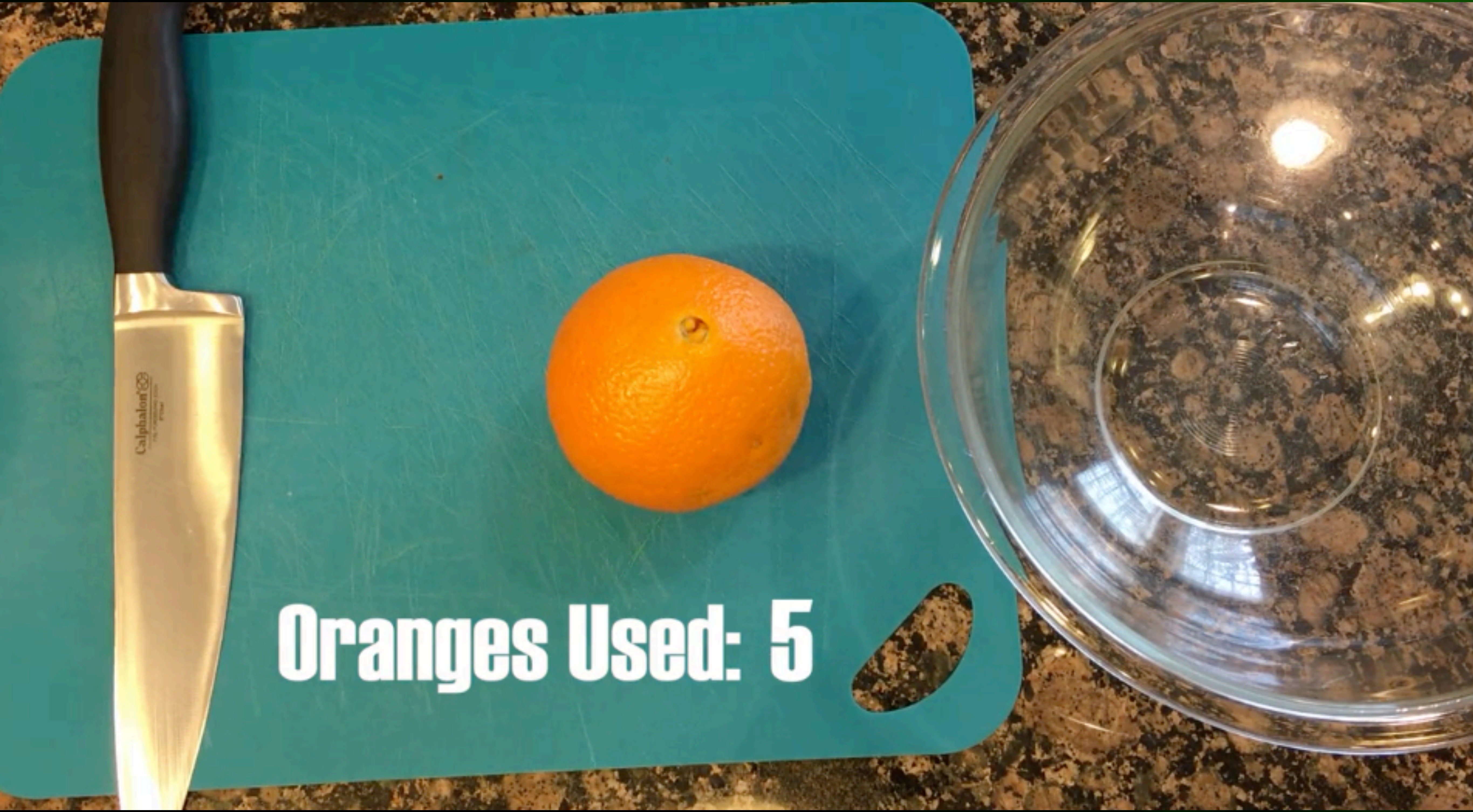
How many orange wedges are in the bowl?

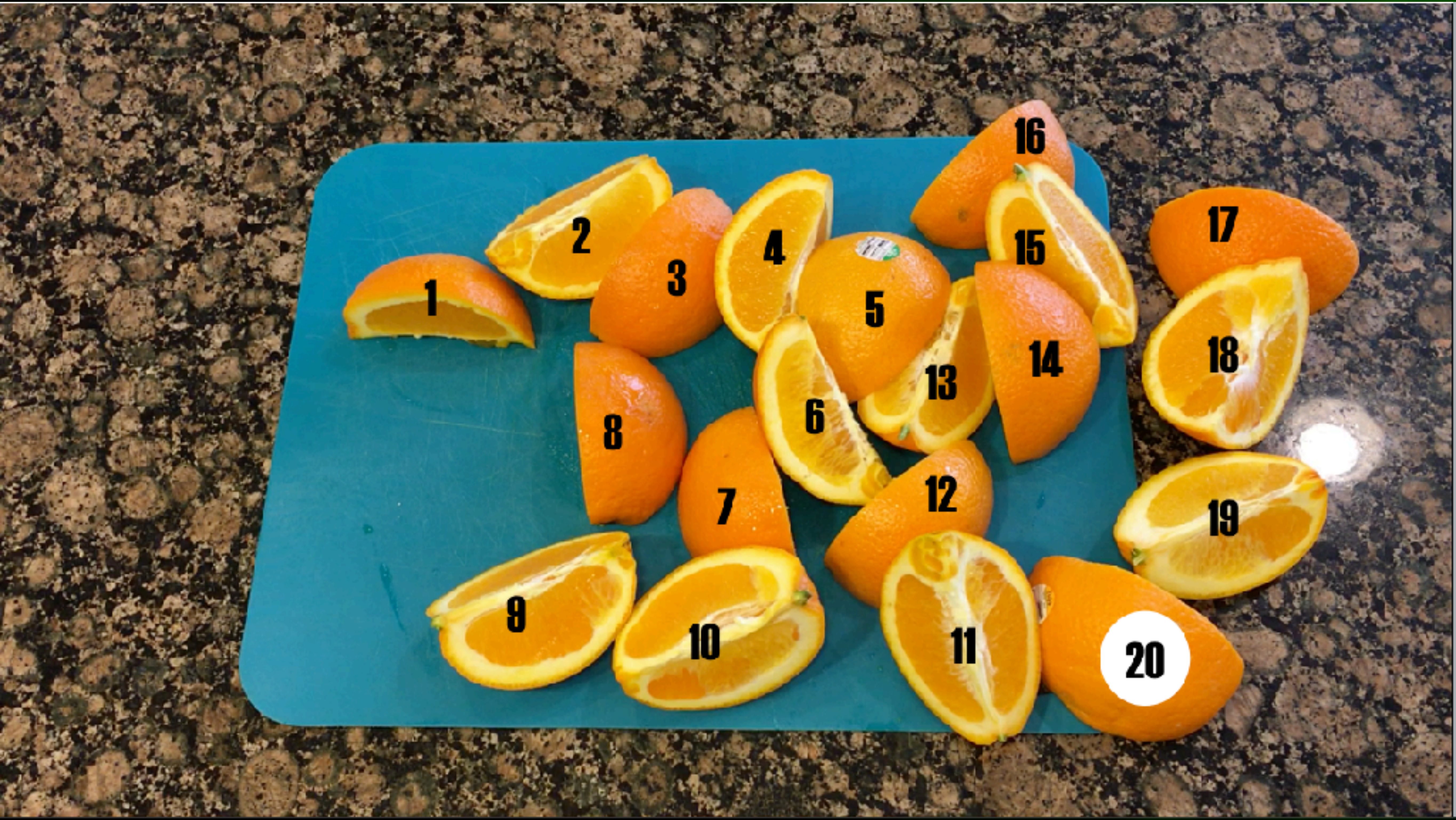
Estimate

How many orange wedges are in the bowl?

What information do you need to know?

Each orange wedges is a quarter.


Oranges Used: 5


1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

Graham had 5 oranges and cut them into quarters.

How many orange wedges did Graham have?

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Notes:

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5 oranges

Each wedge is a quarter

S1

Name: [redacted]

Estimate
20

Draw a picture to show your thinking:

$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1$

$\frac{4}{4} + \frac{8}{4} + \frac{12}{4} + \frac{16}{4} + \frac{20}{4}$

$\frac{20}{4}$ = wedges

Use numbers to show your thinking:

Answer
5

S2

Name: [redacted] Date:

1. What did you notice?
owingis glass Bowl

2. What do you wonder?
how money orengis?

3. Main Question
how mene wedges?

4. Make an estimate.
low estimate — 4 — high estimate
Place an "X" to represent your estimate on the number line.

5. What information do you need?
How Big is the peisis

6. Construct a viable argument or share a reflection.
5 qor+r oingis
4 8 12 16 20

Answer
20

S3

Name: [redacted] Date:

1. What did you notice?
Bowls oranges oranges wedges

2. What do you wonder?
how many whole oranges?

3. Main Questions
How many wedges?

4. Make an estimate.
low estimate — high estimate
Place an "X" to represent your estimate on the number line.

5. What information do you need?
what are the sizes of the wedges?

6. Construct a viable argument or share a reflection.
5 Oranges
Orange cut in fourths
5 x 4 = 20

Answer
20

S4

Name: [redacted]

Estimate
6

Draw a picture to show your thinking:
1 2 3 4
5 7 6 8
9 10 11 12
13 14 15 16
17 18 19 20

Use numbers to show your thinking:
4+4+4+4+4=20

Answer:
20

S5

Name: [redacted] Date:

1. What did you notice?
bowl orange

2. What do you wonder?
Why?

3. Main Questions
How many wedges

4. Make an estimate.
low estimate — 5 — high estimate
Place an "X" to represent your estimate on the number line.

5. What information do you need?

6. Construct a viable argument or share a reflection.
IIII IIII IIII IIII IIII

Answer
20

S6

Name: [redacted] Date:

1. What did you notice?
Oranges, wedges, bowl, counter, hand.
Two wedges outside the bowl.

2. What do you wonder?
How many wedges?

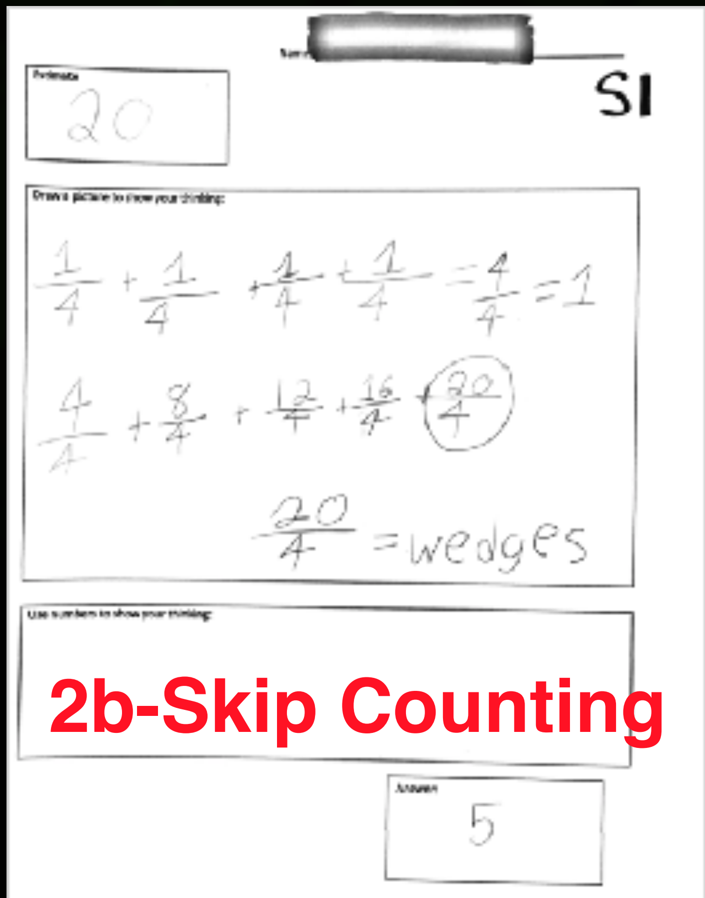
3. Main Question
How many wedges?

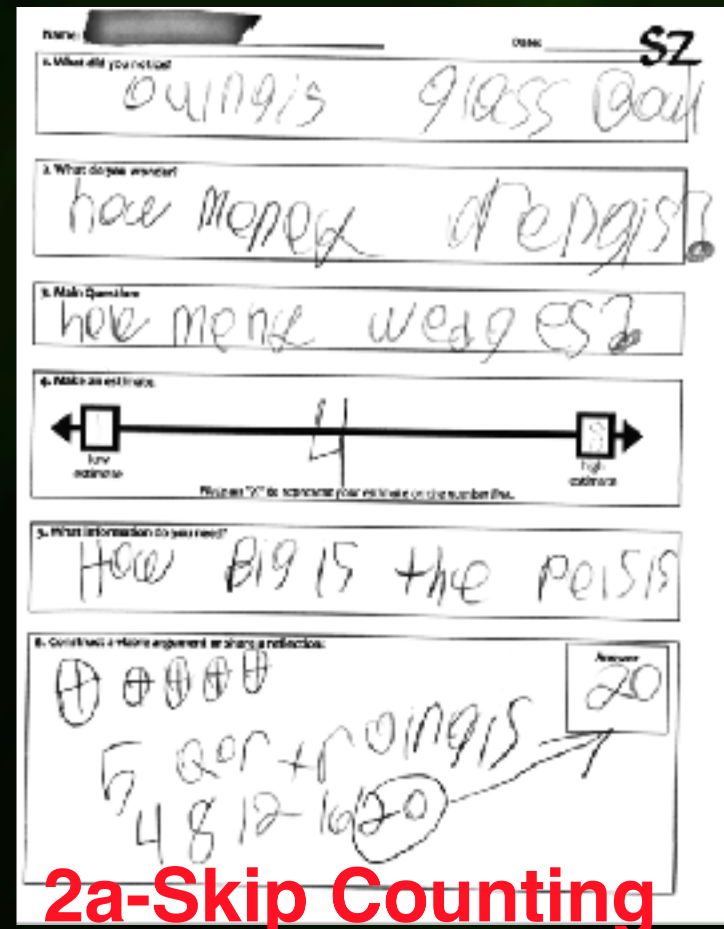
4. Make an estimate.
low estimate — 20 — high estimate
Place an "X" to represent your estimate on the number line.

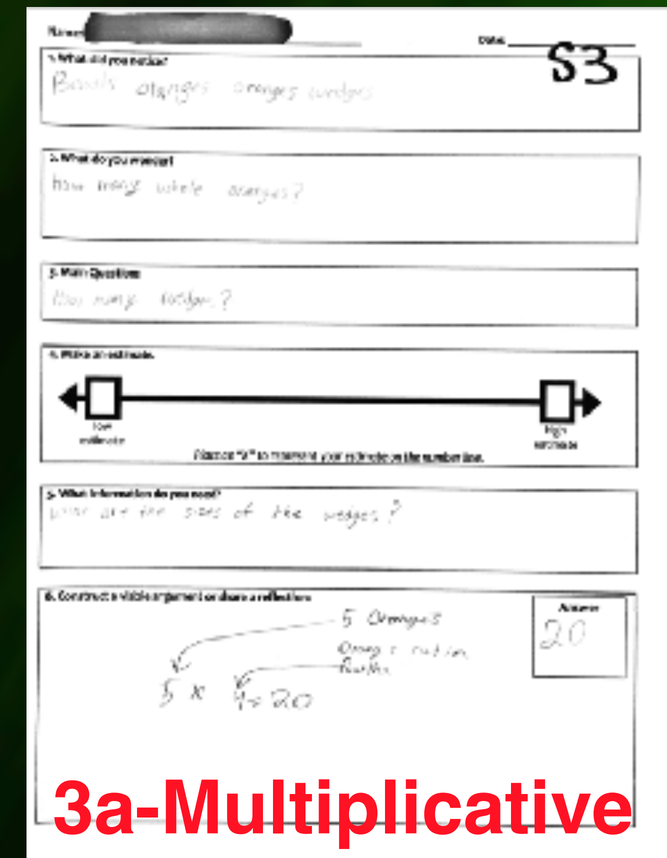
5. What information do you need?
Size of wedges? Number of oranges?

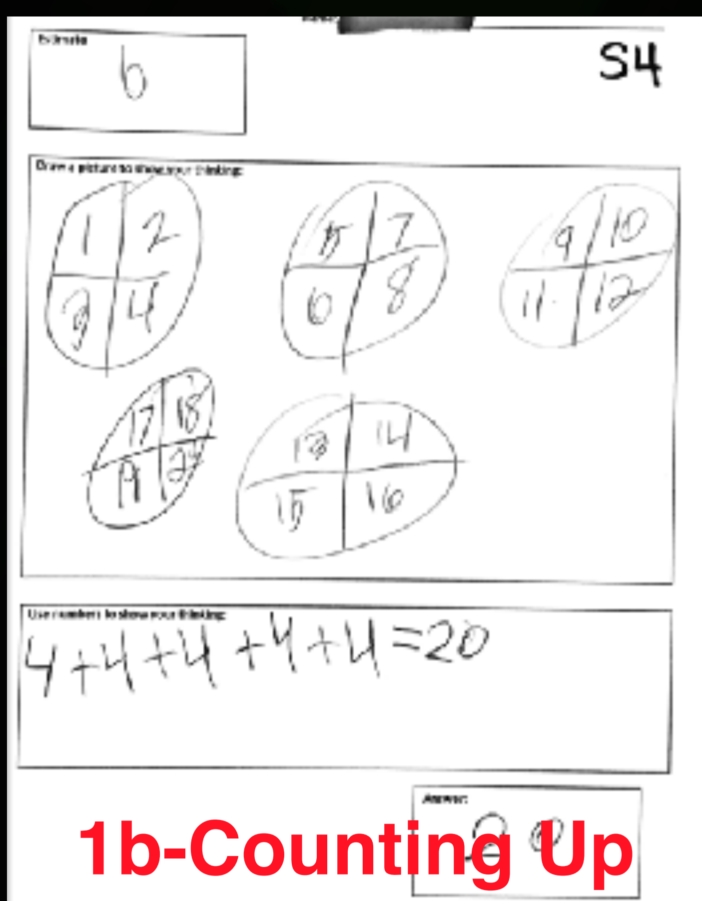
6. Construct a viable argument or share a reflection.
$20 \times 1/4 = 20/4 = 5$

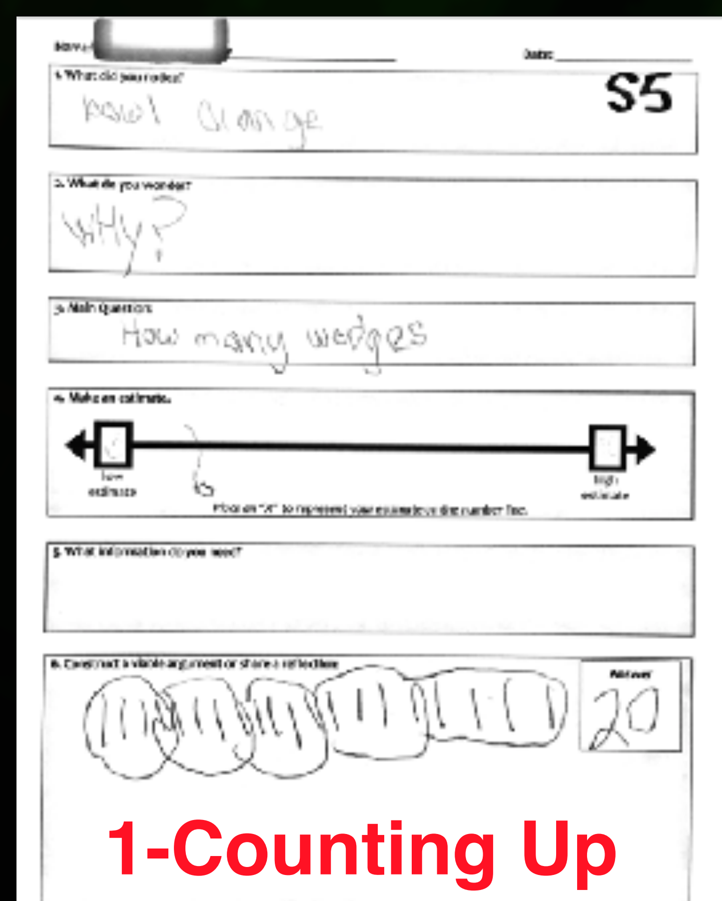
Answer
20

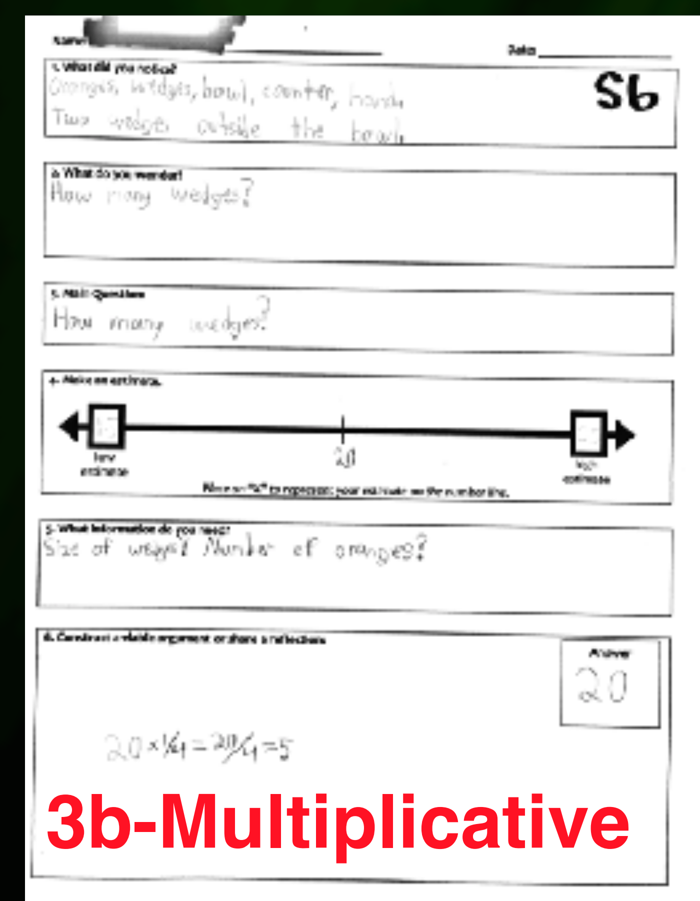

S1
2b-Skip Counting


S2
2a-Skip Counting


S3
3a-Multiplicative

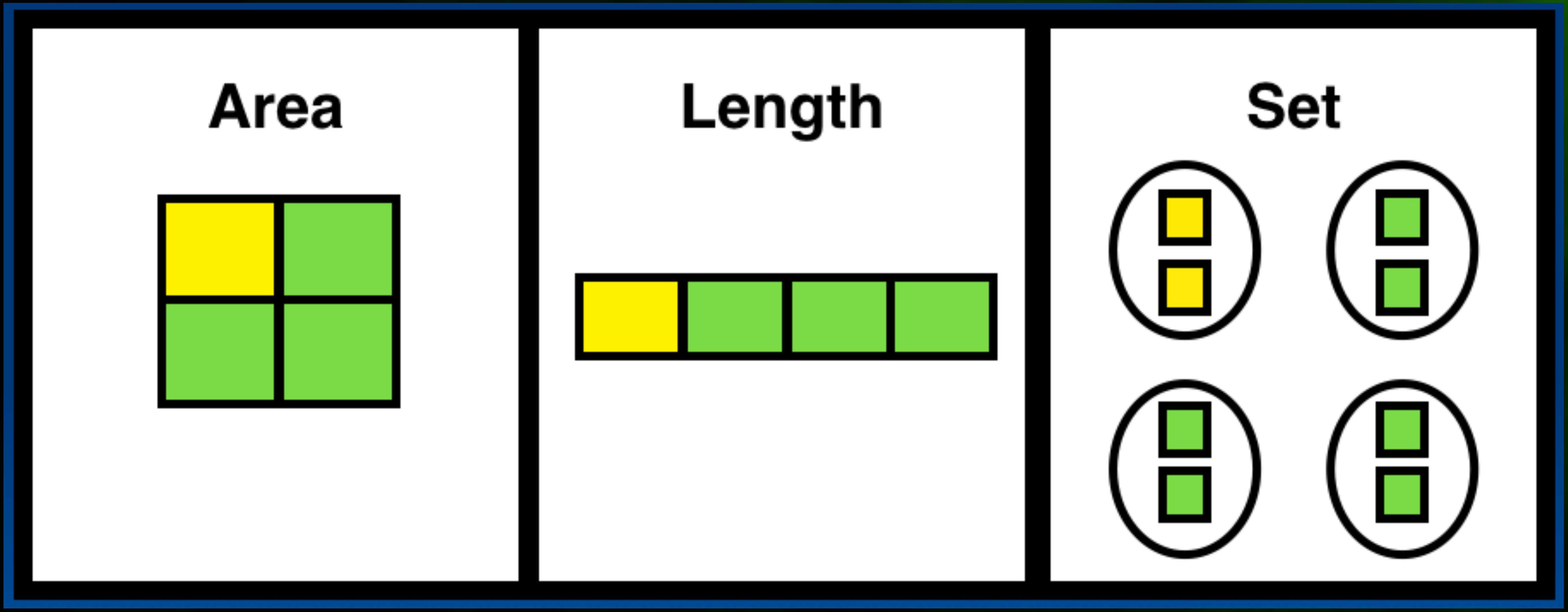

S4
1b-Counting Up


S5
1-Counting Up


S6
3b-Multiplicative

Unit Fractions

Representation of a Fraction



unit fraction $\frac{1}{a}$

Say this fraction

$$\frac{3}{4}$$

Say this fraction

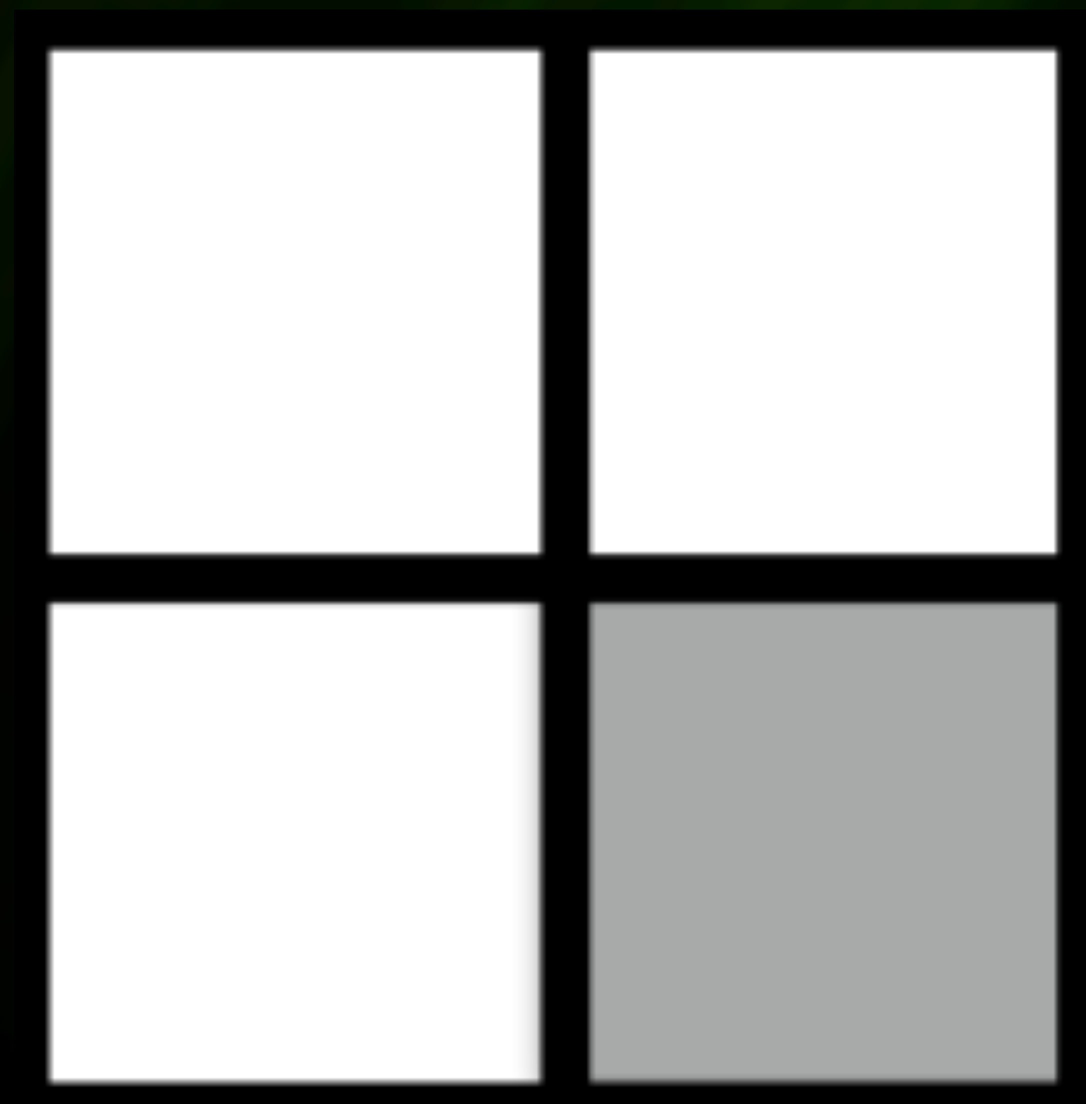
$$\frac{3}{4}$$

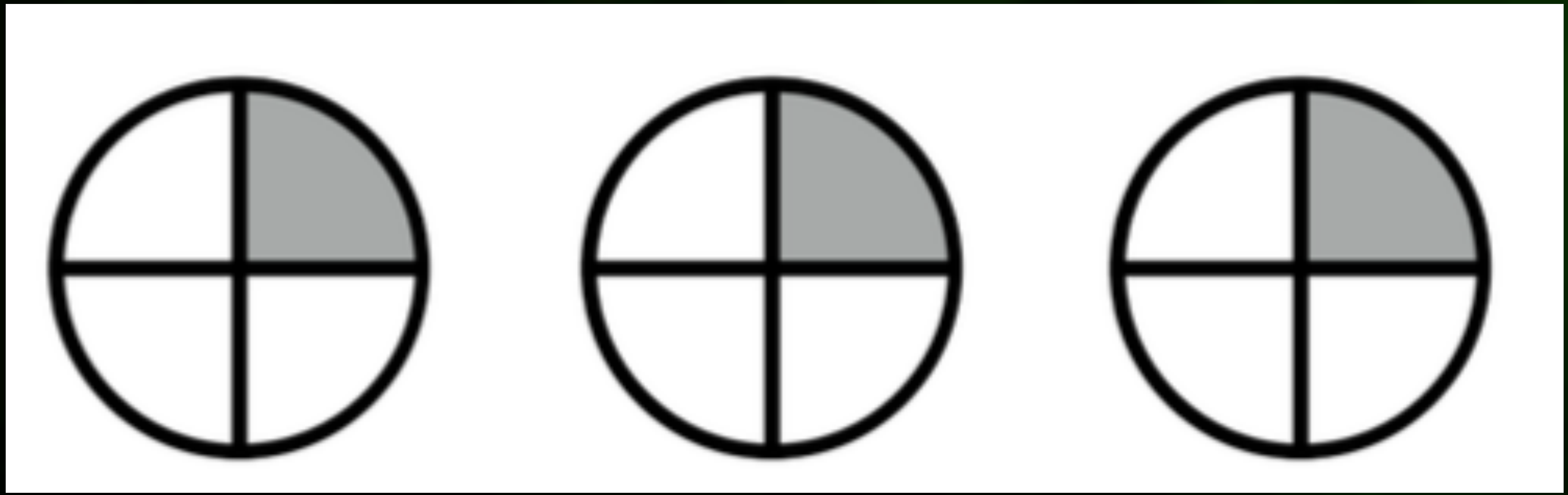
three one-fourths

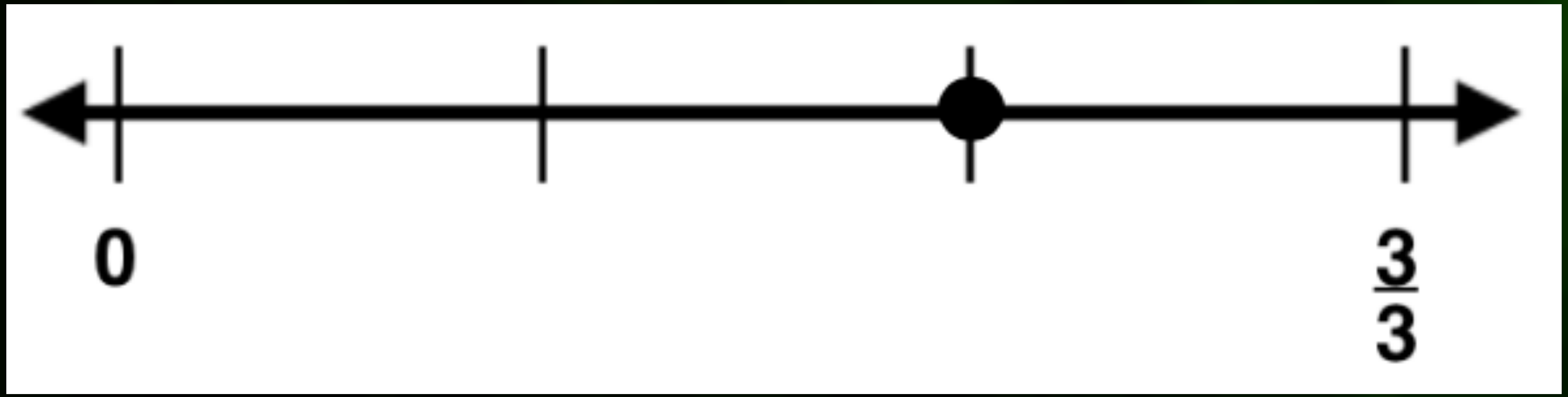
$$3 = 1 + 1 + 1$$

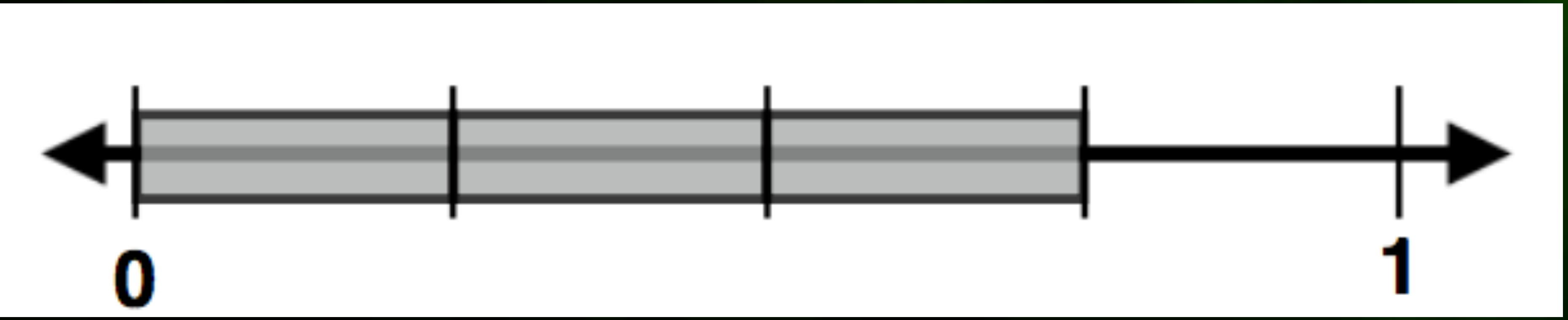
$$3 = 1 + 1 + 1$$

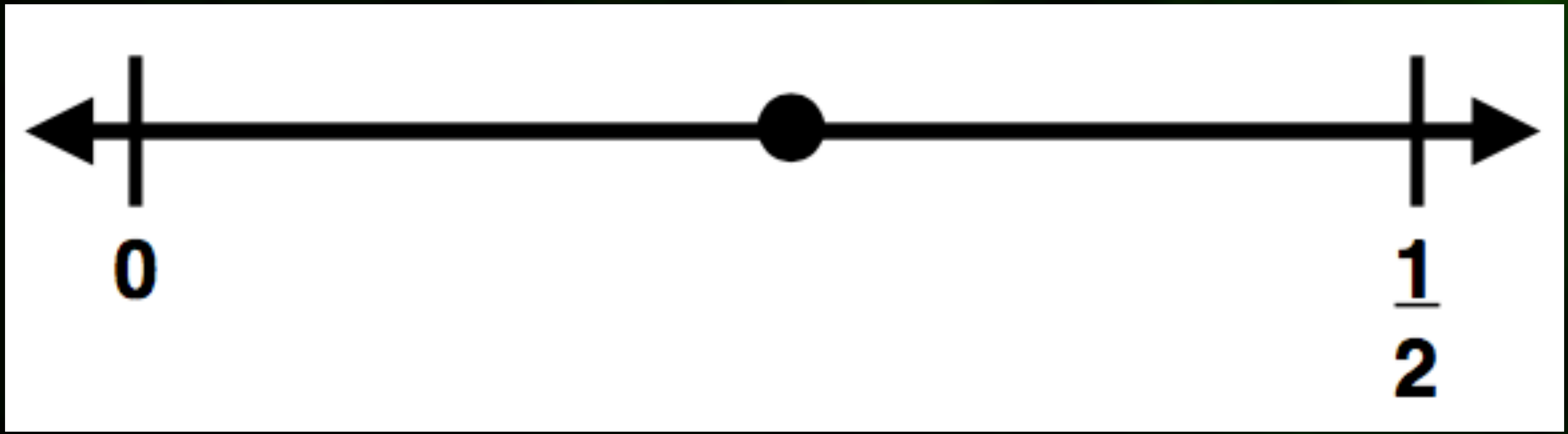
$$\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$

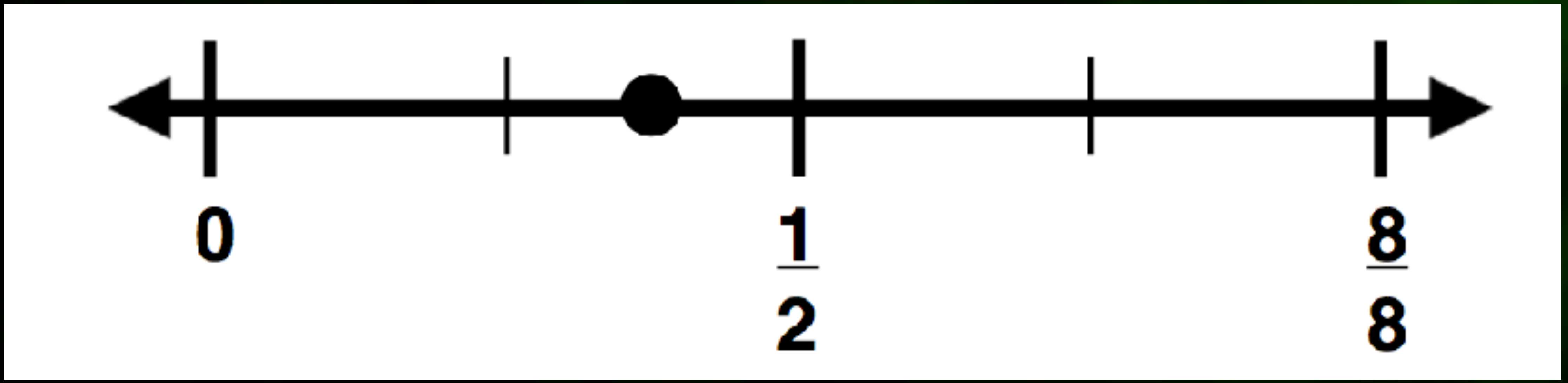


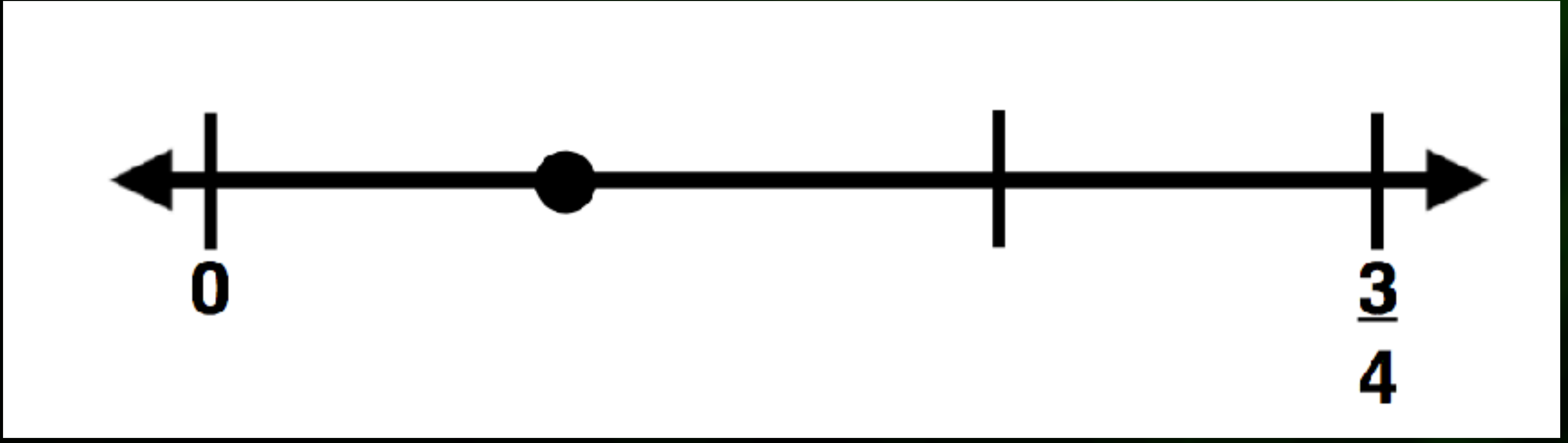



0
$\frac{3}{3}$


0
1

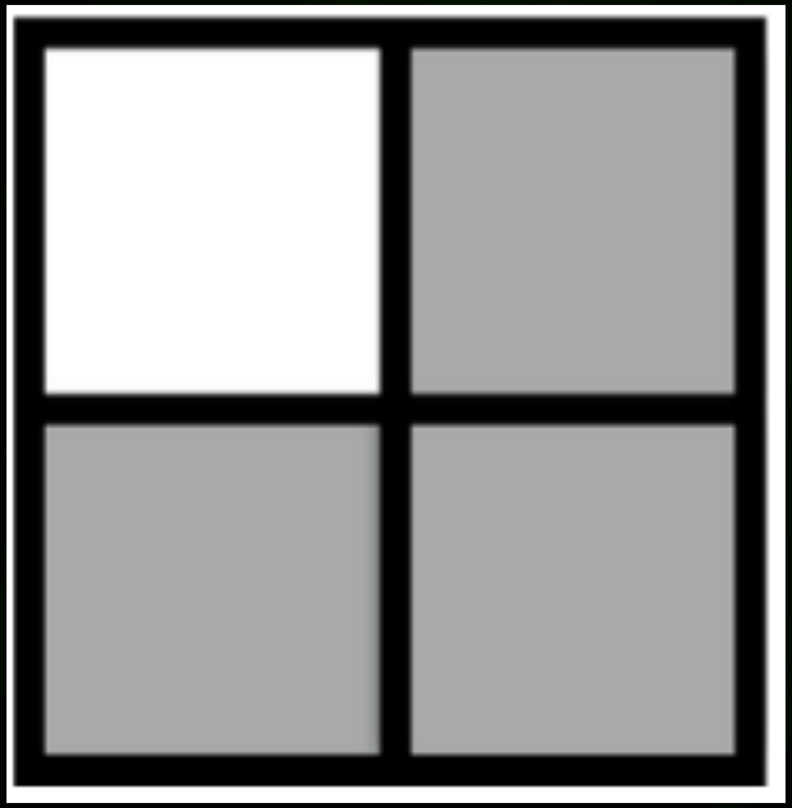

0
1
2


0
1
2
8
8

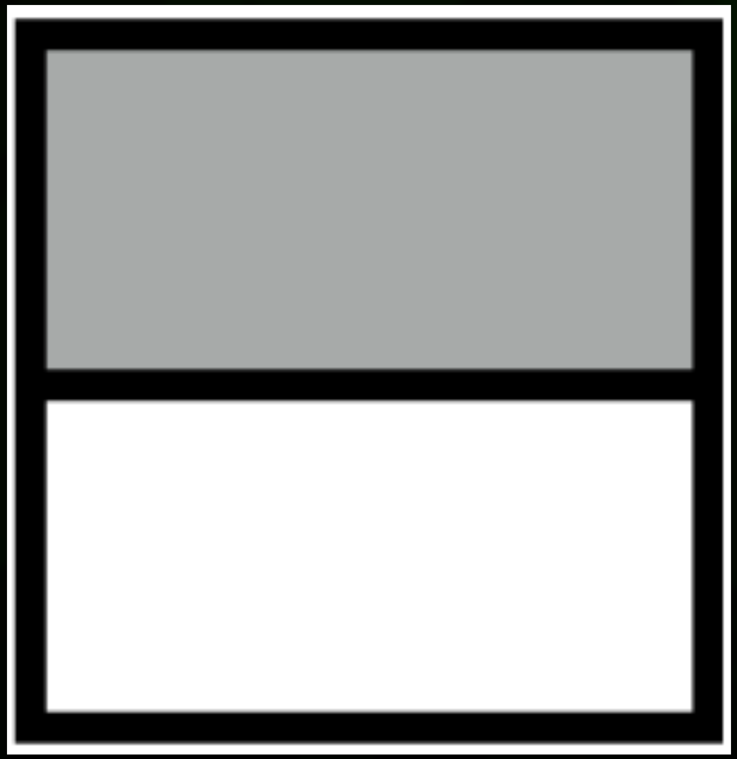

0
$\frac{3}{4}$

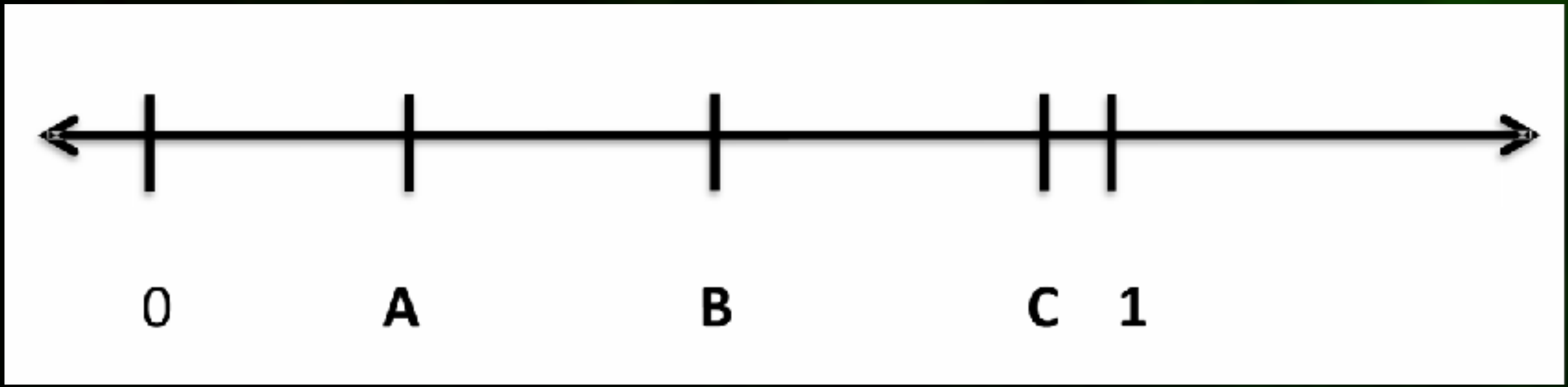
What’s the Sum?

What’s the Sum?



What's the Sum?

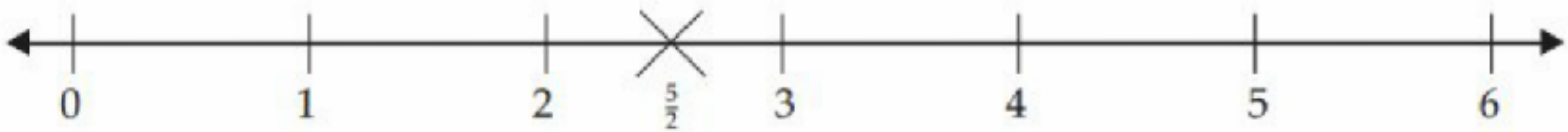



0
A
B
C
1


½
A
2
B

Dotty Pairs Game

The students play in pairs. One student takes dots, the other takes crosses. Place the cards (cards 1–6, two lots, see Material Master 4-1) face down in a pile. The players take turns turning over two cards. The numbers are used to form a fraction, e.g., 2 and 5 are turned over, so $\frac{5}{2}$ or $\frac{2}{5}$ can be made. One fraction is chosen, made with the fraction pieces, if necessary, and marked on a 0–6 number line with the player's identifying mark (dot or cross).



Players take turns. The aim of the game is to get three of their marks uninterrupted by their opponent's marks on the number line. If a player chooses a fraction that is equivalent to a mark that is already there, they miss that turn.

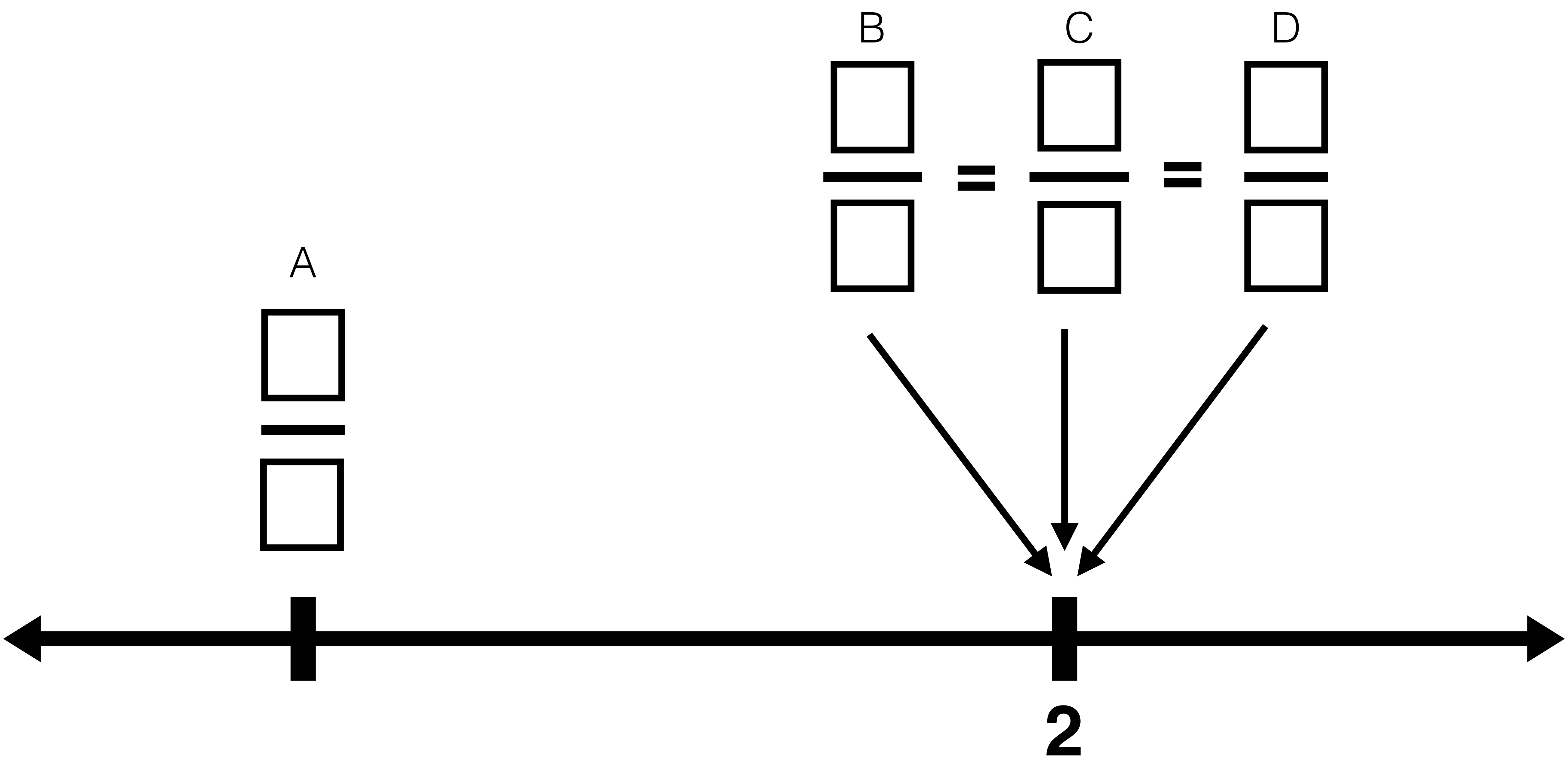
NB: A fraction such as $\frac{4}{1}$ can be made using the cards. Students may not be familiar with fractions in this form and the meaning of the numerator and denominator will need to be explored with the fraction circles.

→ **random dice roller**

Open Middle

Open Middle

Directions: Using the whole numbers 1-9 no more than one time each, create and place 4 fractions on the number line in the correct order. A is less than 2. Fractions B, C, and D equal 2.

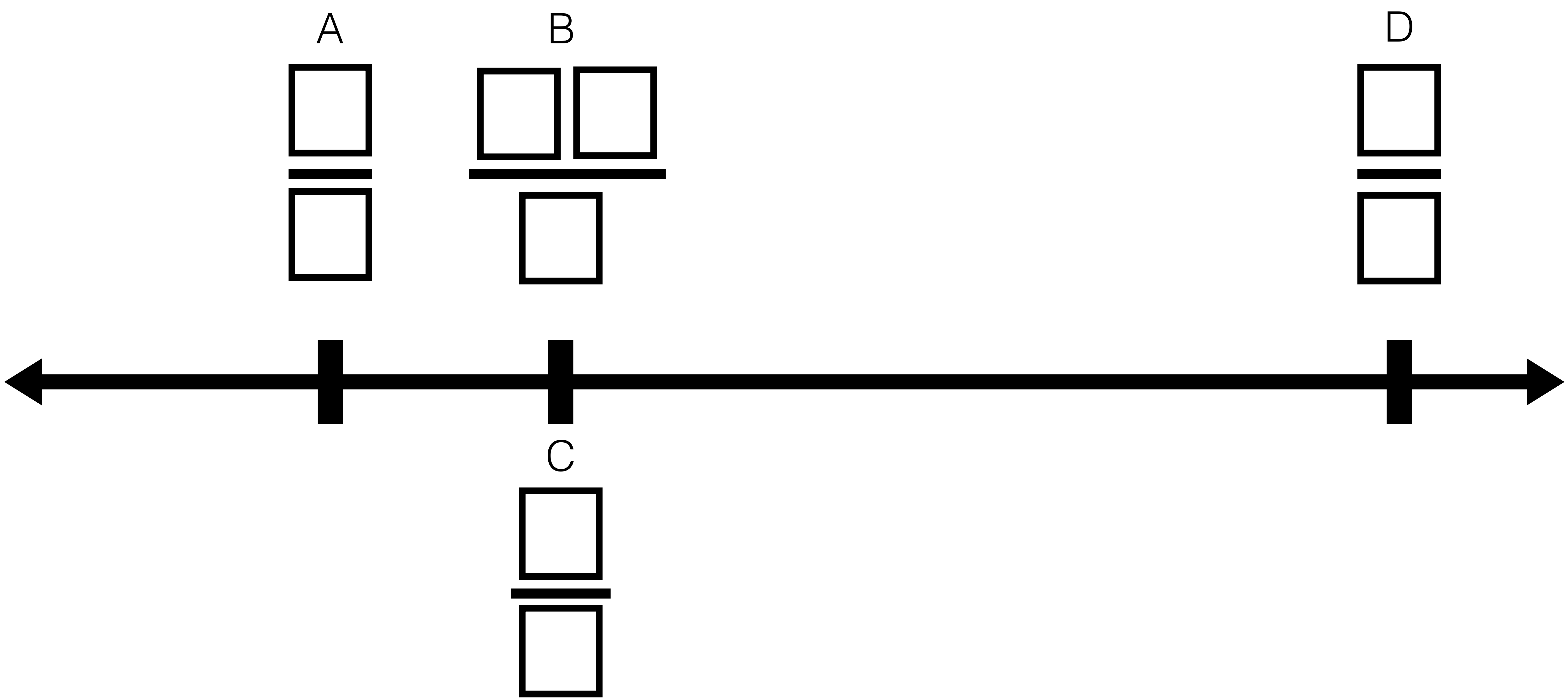


@gfletchy

Open Middle

CCSS.MATH.CONTENT.4.NF.A.2

Directions: Using the whole numbers 1-9 once each, create and place 4 fractions *greater than 1* on the number line in the correct order. *(fractions B & C are equal)*



@gfletchy

$\frac{1}{20}$	$\frac{20}{25}$
$\frac{2}{3}$	$\frac{5}{4}$

http://wodb.ca/numbers.html

Equivalent Fractions

Equal Fraction

$$\frac{2}{3} = \frac{\text{[redacted]}}{\text{[redacted]}} \qquad \frac{3}{4} = \frac{\text{[redacted]}}{\text{[redacted]}}$$

$$\frac{2}{6} = \frac{\text{[redacted]}}{\text{[redacted]}}$$

Equal Fraction

$$\frac{2}{3} = \frac{5}{6} \qquad \frac{3}{4} = \frac{7}{8}$$

$$\frac{2}{6} = \frac{5}{9}$$

Name ______________________________ Date ________________

1. Draw a rectangular fraction model to find the sum. Simplify your answer, if possible.

a. $\frac{1}{4}+\frac{1}{3}=$

b. $\frac{1}{4}+\frac{1}{5}=$

It is possible to over-emphasize the importance of simplifying fractions in this way. There is no mathematical reason why fractions must be written in simplified form, although it may be convenient to do so in some cases.

://commoncoretools.me/wp-content/uploads/2011/08/ccss_progression_nf_35_2013_09_19.pdf

What about “the test”?

$\frac{3}{6} + \frac{1}{6}$ is equal to which of the following?

a. $\frac{4}{12}$

b. $\frac{8}{12}$

c. $\frac{3}{6}$

d. None of the above

Simplifying

Equivalence

Comparing Fractions


RUN

Which girl ate more apple?

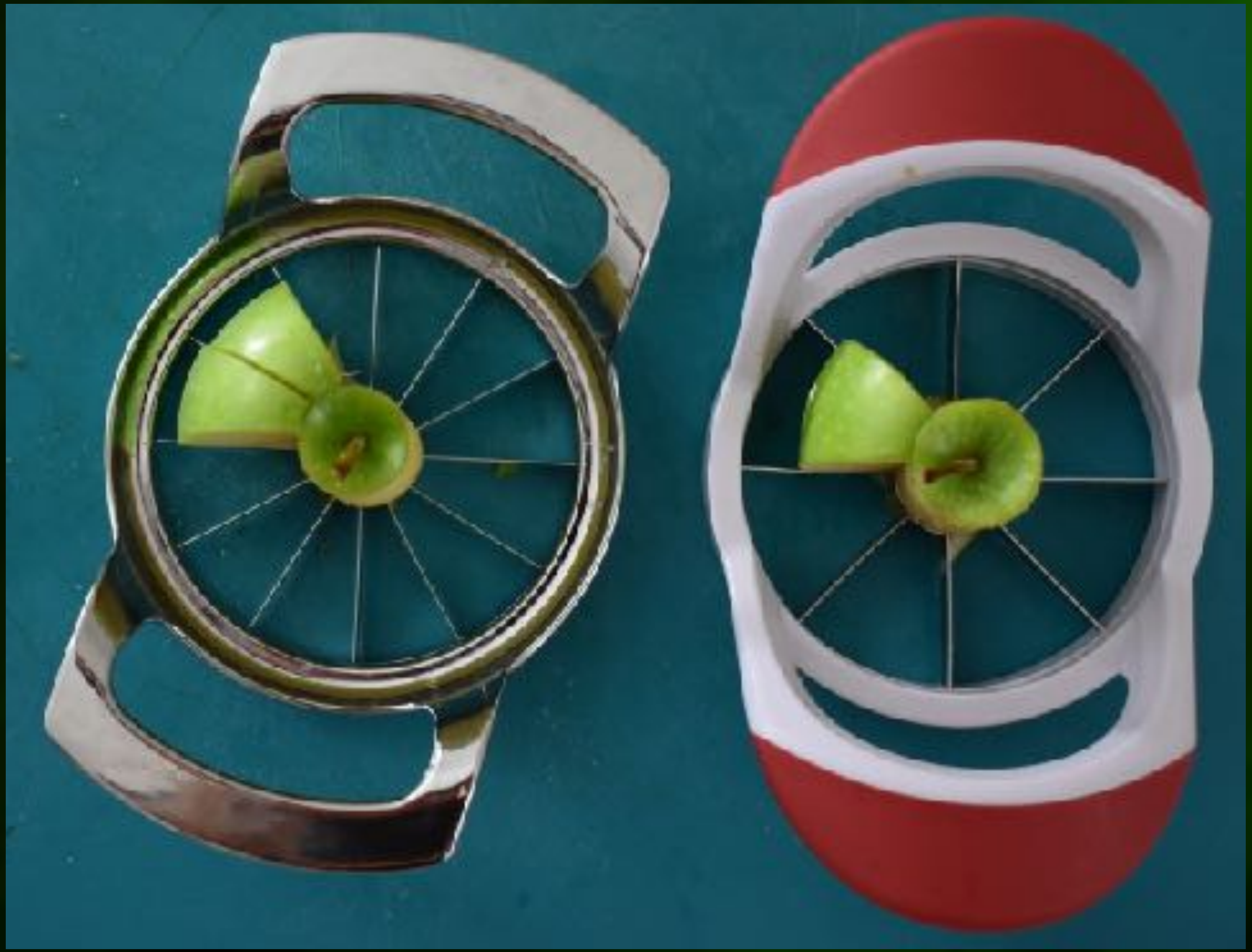
twelfths

Pause

| |

eighths

Apple Eat Off
Act-3

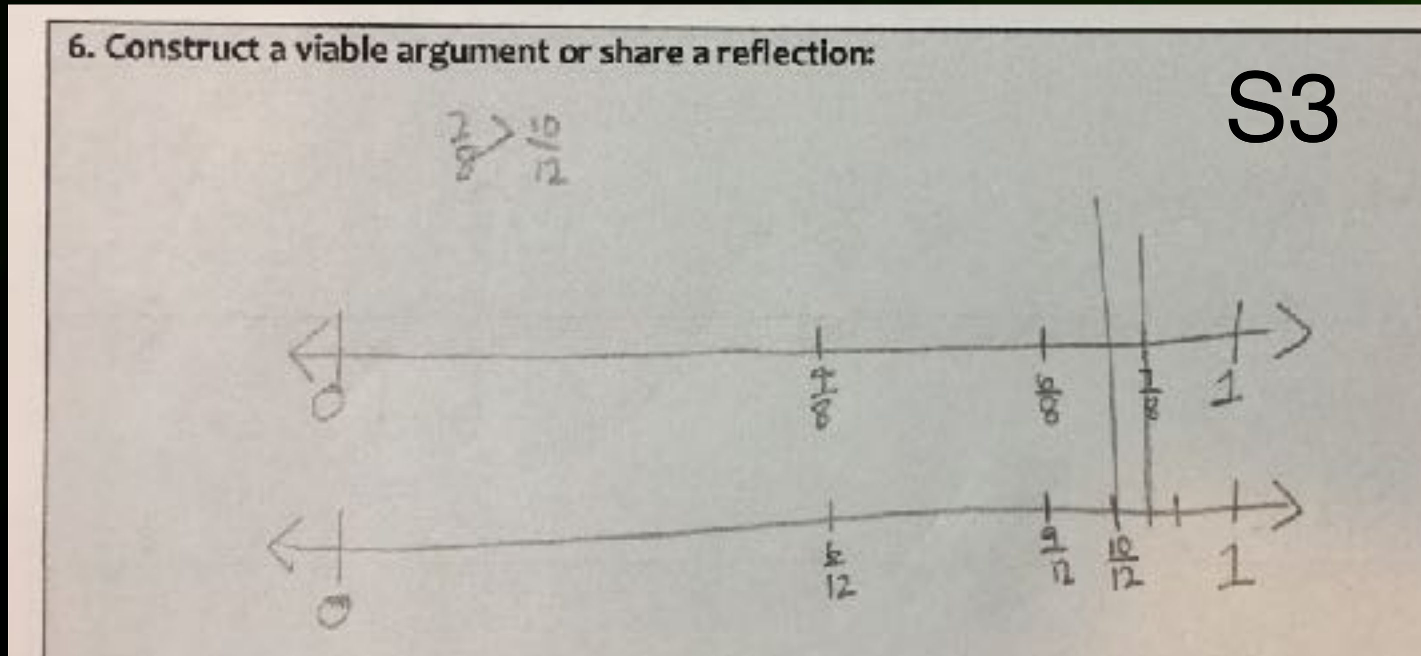


Big sister ate of an apple and

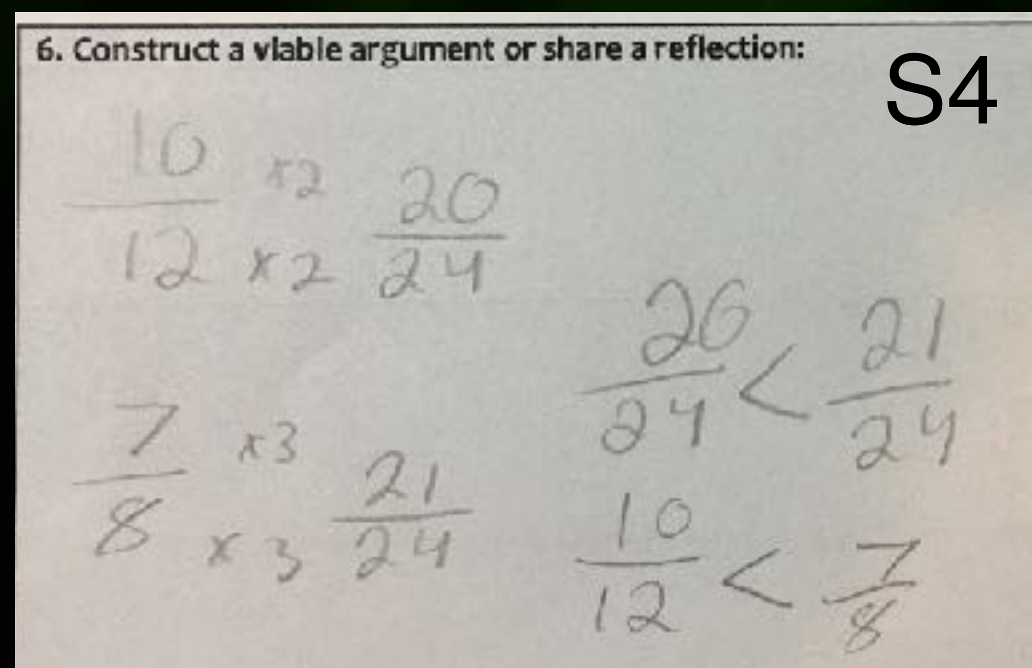
little sister $\frac{10}{12}$ ate

$\frac{7}{8}$ of an apple. Which sister ate

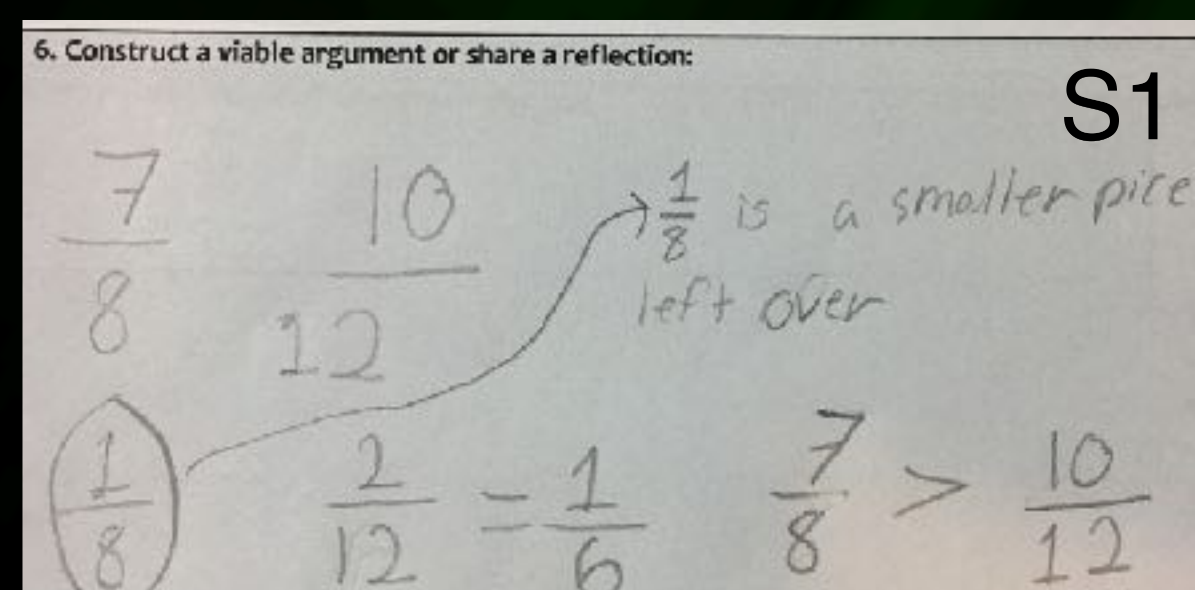
more apple?



6. Construct a viable argument or share a reflection:
S3
7/8 > 10/12
0
4/8
6/8
7/8
1
0
6/12
9/12
10/12
1

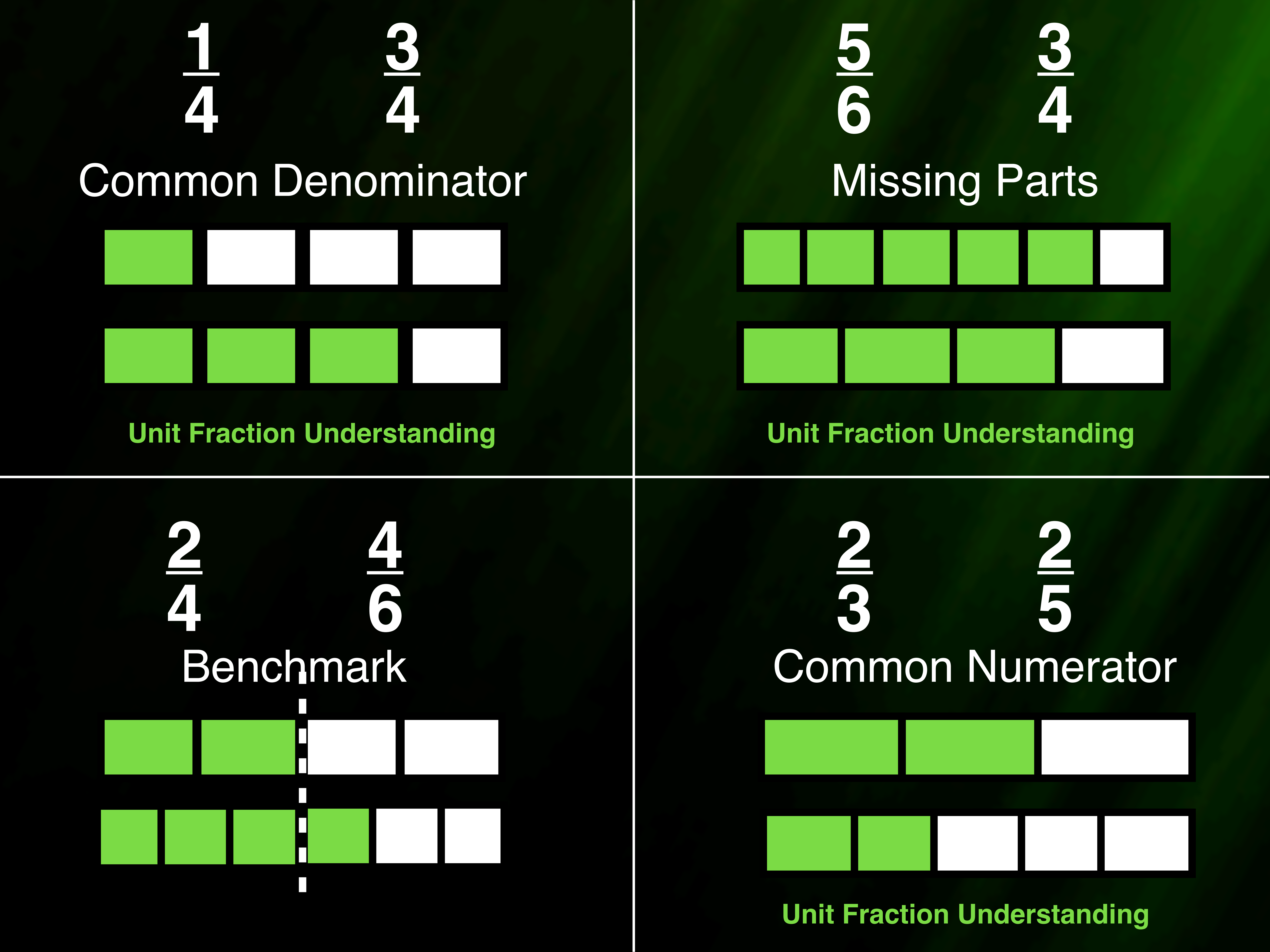


6. Construct a viable argument or share a reflection:
S4
10/12 ×2 20/24
7/8 ×3 21/24
20/24 < 21/24
10/12 < 7/8



6. Construct a viable argument or share a reflection:
S1
7/8
10/12
1/8 is a smaller pice left over
1/8
2/12 = 1/6
7/8 > 10/12

It Takes 3 to Prove it to Me


$\frac{1}{4}$ $\frac{3}{4}$
Common Denominator
Unit Fraction Understanding
$\frac{5}{6}$ $\frac{3}{4}$
Missing Parts
Unit Fraction Understanding
$\frac{2}{4}$ $\frac{4}{6}$
Benchmark
$\frac{2}{3}$ $\frac{2}{5}$
Common Numerator
Unit Fraction Understanding

$$\frac{8}{11} \qquad \frac{4}{7}$$

$\frac{8}{11}$ $\frac{8}{14}$

Comparing Fractions

CCSS.MATH.CONTENT.3.NF.A.3.D
Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

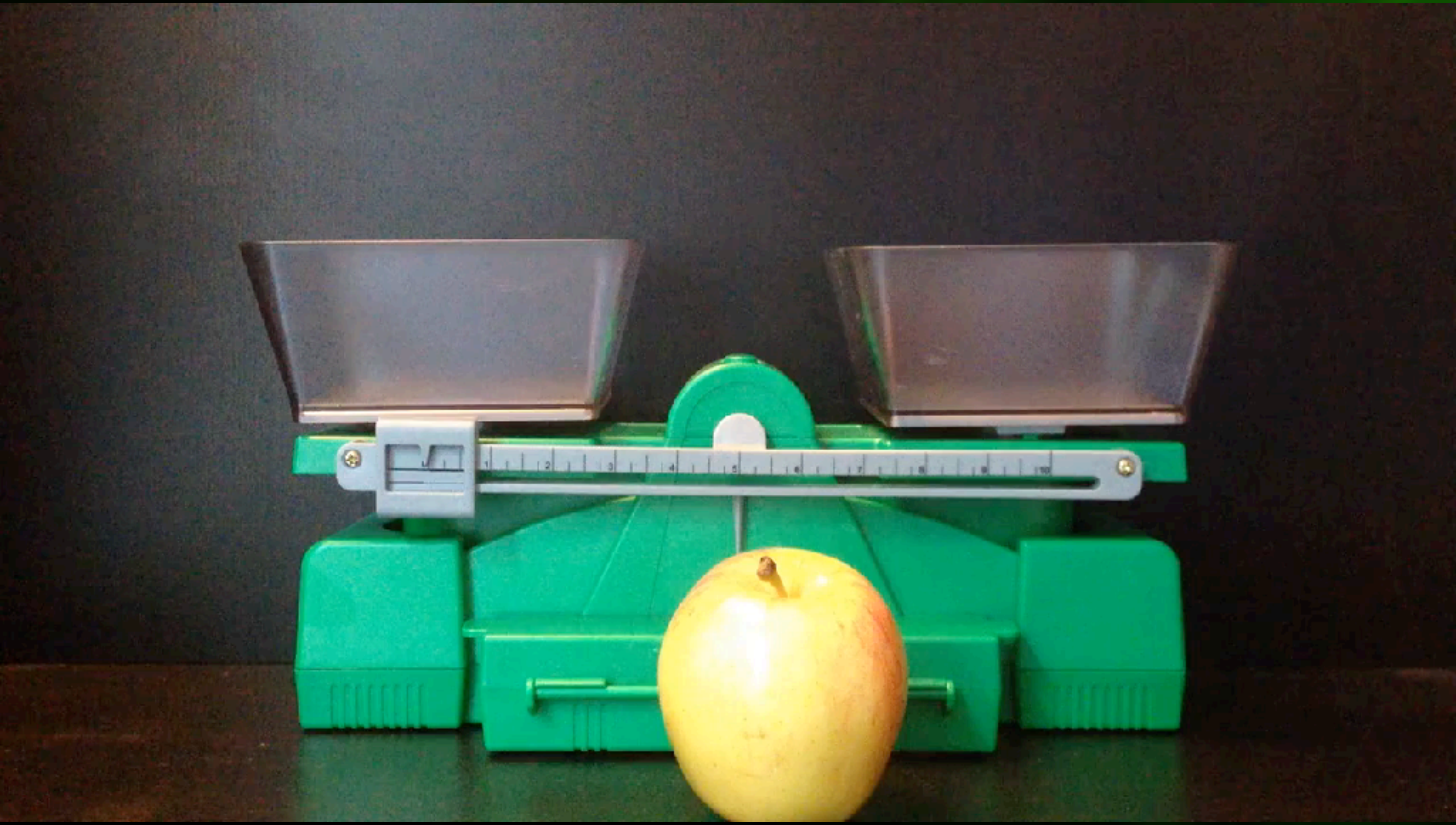
CCSS.MATH.CONTENT.4.NF.A.2
Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

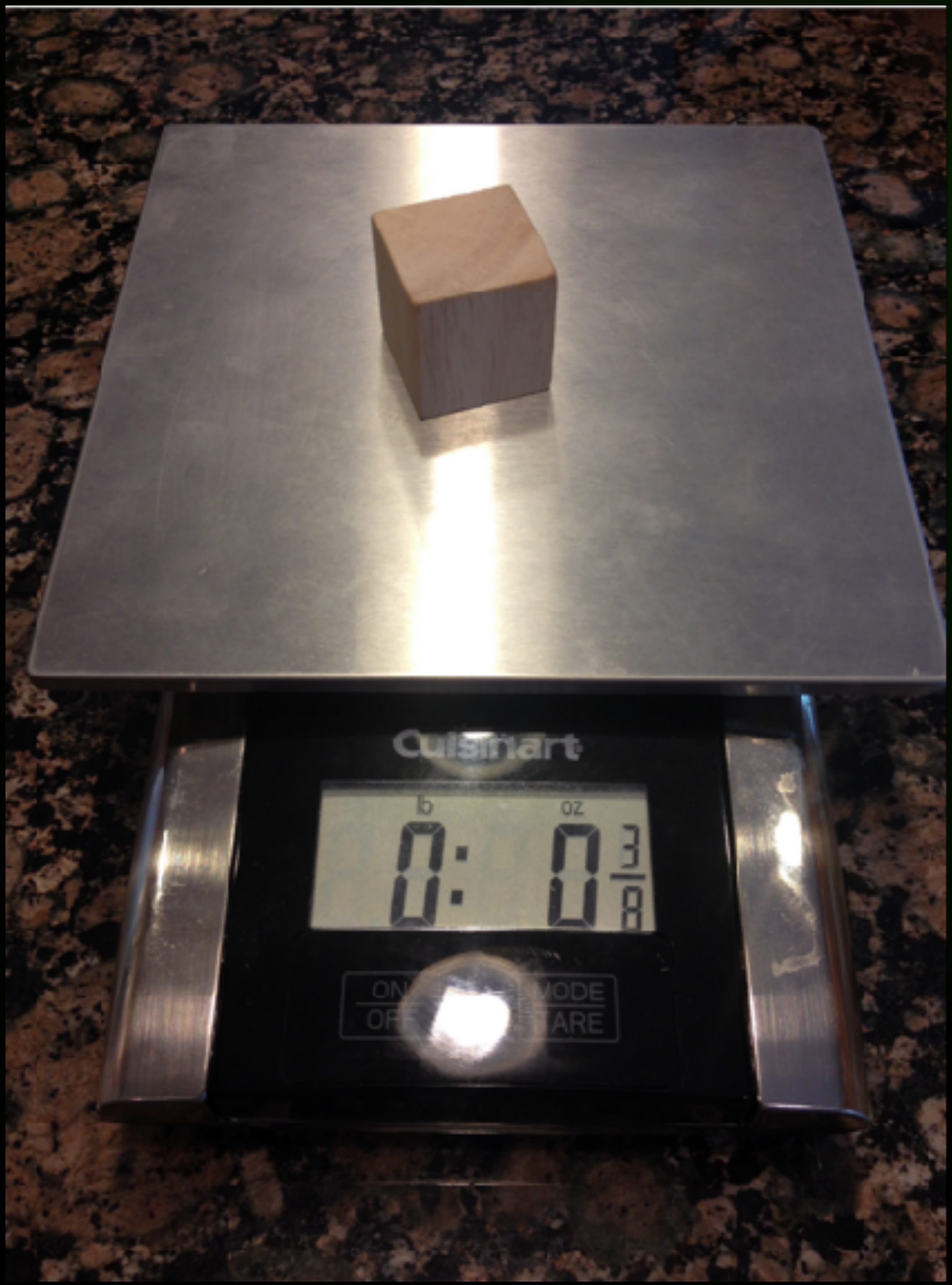
Making Sense Series

The Progression of Fractions
Meaning, Equivalence, & Comparison

created by Graham Fletcher
@gfletchy
www.gfletchy.com

A Task




Cuisinart
lb oz
0: 0 3/8
ON OFF MODE TARE

Cuisinart
lb oz
0: 5 1/4
ON OFF MODE TARE


0:00 / 0:14

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