

The Power of Progressions: Untangling the Knotty Areas of Teaching and Learning Mathematics

Graham Fletcher

gfletchy@gmail.com

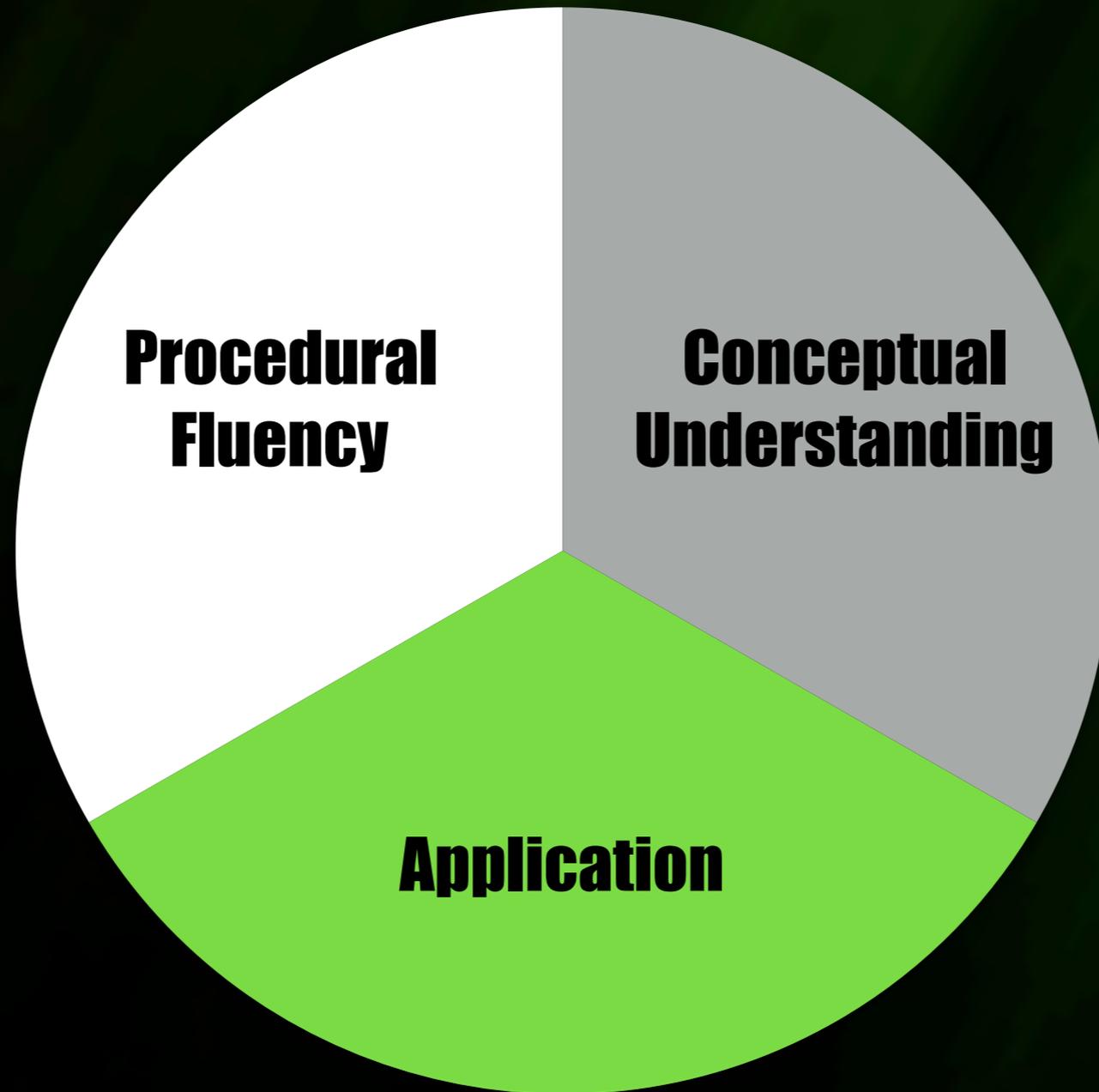


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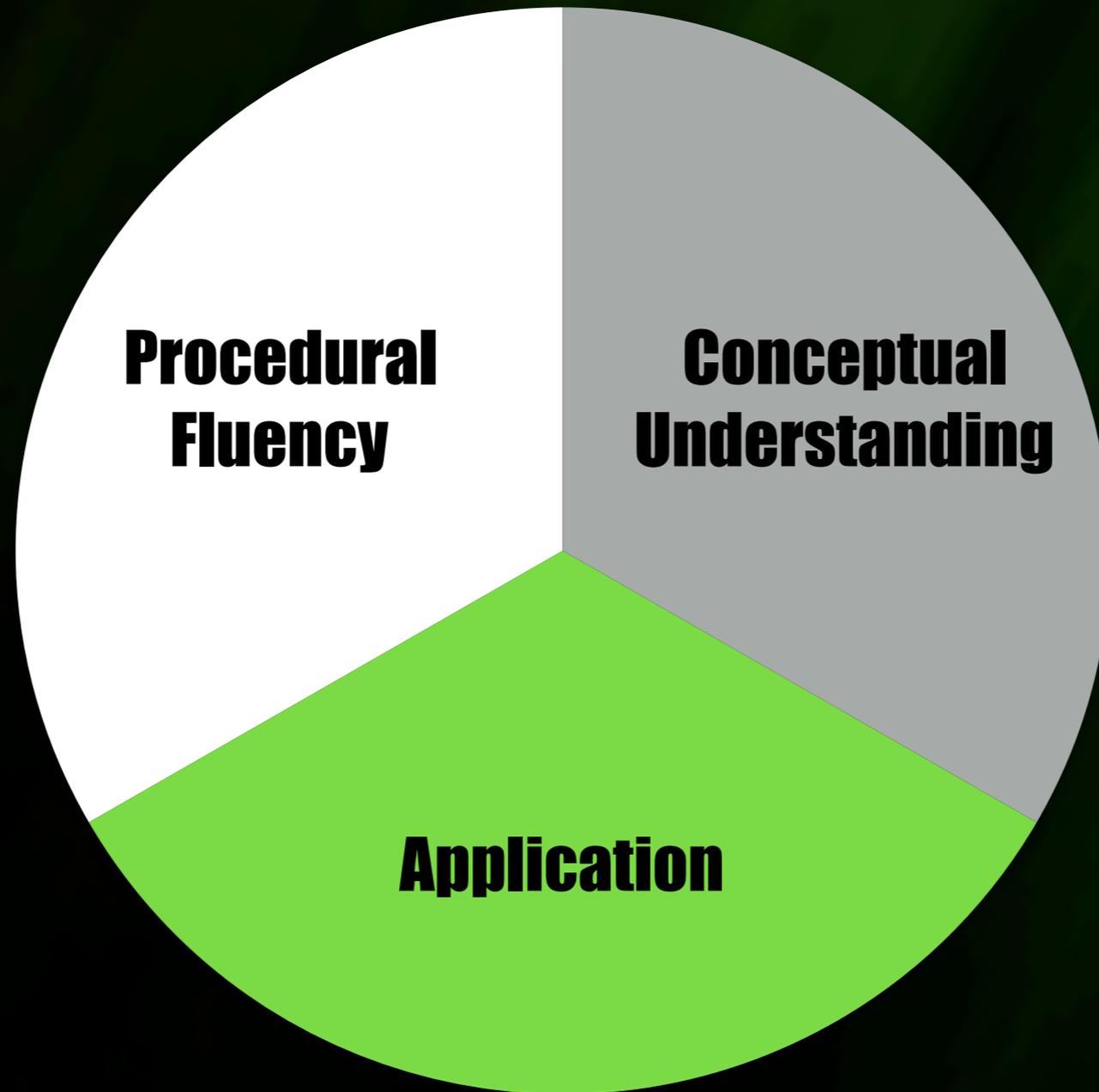


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**Procedural
Fluency**



Demetrius has 17 Skittles which is 12 fewer than Alicia.

How many Skittles does Alicia have?

Demetrius has 17 Skittles which is 12 fewer than Alicia.

How many Skittles does Alicia have?

Problem Solving Strategy

-  **C** Circle the numbers
-  **U** Underline the question
-  **B** **B**ox the key words
-  **E** Eliminate unnecessary & **E**valuate what's left
-  **S** Solve and check

Demetrius has 17 Skittles which is 12 fewer than Alicia.

How many Skittles does Alicia have?

Problem Solving Strategy

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Demetrius has 17 Skittles which is 12 fewer than Alicia.

How many Skittles does Alicia have?

Problem Solving Strategy

- C** Circle the numbers
- U** Underline the question
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Demetrius has **17** Skittles which is **12** fewer than Alicia.

How many Skittles does Alicia have?

Problem Solving Strategy

- C** Circle the numbers
- U** Underline the question
- B** Box the key words
- E** Eliminate unnecessary & Evaluate what's left
- S** Solve and check

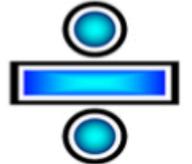
fewer

17

12

Problem Solving Strategy

- C** Circle the numbers
- U** Underline the question
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- E** Eliminate unnecessary & Evaluate what's left
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The Key Word in Word Problems	
 Add Sum Total All together Plus In all	 Multiply Product Times Twice Total Multiplied by
 Subtract Remain Difference Less than Fewer How many more Minus	 Divide Quotient Goes into Split Equally Each

17 – 12

Problem Solving Strategy

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17 – 12

W T F ?

17 – 12

What's **T**he **F**ive **?**

Current Research

Did you mean: [Math Keywords poster images](#)







65%

“65% of children entering primary school today will ultimately end up working in completely new job types that don’t yet exist.”

“Early mathematics competency predicts later reading achievement better than early literacy skills.”

A. Szekely. Unlocking Young Children’s Potential: Governors’ Role in Strengthening Early Mathematics Learning (Washington, D.C.: National Governors Association Center for Best Practices, October 28, 2014).

Joe had some playing cards in his bag. Ashley gave him 13 more cards. Joe now has 21 cards. How many cards did Joe have in his bag?

13

21

You little plucker!

13

21

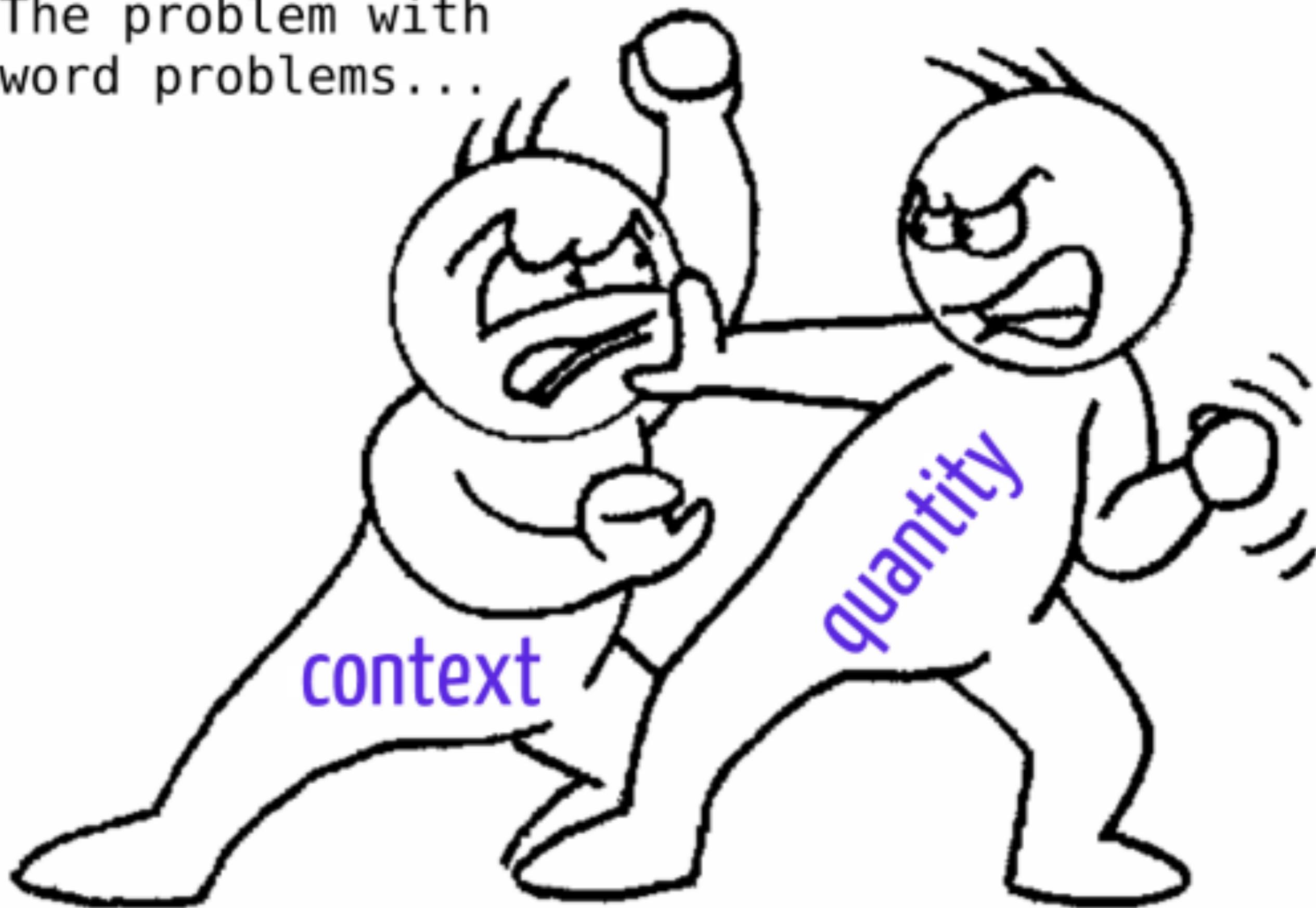
number
^

You little plucker!

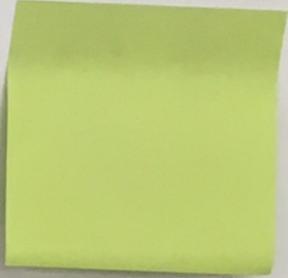
13

21

The problem with
word problems...



Joe had some playing cards in his bag. Ashley gave him 13 more cards. Joe now has 21 cards. How many cards did Joe have in his bag?

Joe had some playing cards
in his bag. Ashley gave him 
more cards. Joe now has 
cards. How many cards did Joe
have in his bag?



How many cards did Joe have in his bag?

Name: _____

Date: _____

1. What did you notice?

2. What do you wonder?

3. Main Question:

4. Make an estimate.



Place an "X" to represent your estimate on the number line.

5. What information do you need?

6. Construct a viable argument or share a reflection:

Answer

Name: _____

Estimate

Draw a picture to show your thinking:

Use numbers to show your thinking:

Answer:





?

 **-19 yellow**

 **-15 orange**

 **-19 green**

 **-17 purple**

 **-21 red**

The Big Reveal



Graham had some Skittles. He had 19 yellow, 15 orange, 19 green, 17 purple, and 21 red. How many Skittles did Graham have?

3-Act Tasks

Act 1:

- Real world problem or scenario presented
- What do you notice? What do you wonder?
- Make estimates

Act 2:

- Identify missing variables and missing variables to solve
- Define solution path using variables

Act 3:

- Solve and interpret results of the solution
- Validate answer

Most asked questions:

- How often should we use 3-Act Tasks?
- When should we use 3-Act tasks? How do they fit into the scope of a unit?
- How long does one task usually take?
- What if we don't have the time?

Orchestrating Discussions

Five practices constitute a model for effectively using student responses in whole-class discussions that can potentially make teaching with high-level tasks more manageable for teachers.

Margaret S. Smith, Elizabeth K. Hughes, Randi A. Engle, and Mary Kay Stein



Margaret S. Smith, pegs@pitt.edu, is an associate professor of mathematics education at the University of Pittsburgh. Over the past decade, she has been developing research-based materials for use in the professional development of mathematics teachers and studying what teachers learn from the professional development in which they engage. **Elizabeth K. Hughes**, elizabeth.hughes@uni.edu, recently finished her doctorate in mathematics education at the University of Pittsburgh. Her areas of interest include preservice secondary mathematics teacher education and the use of practice-based materials in developing teachers' understanding of what it means to teach and learn mathematics. **Randi A. Engle**, raengle@berkeley.edu, is an assistant professor of mathematics education and the social context of learning at the University of California Berkeley. She is interested in developing practical theories for how mathematics teachers can create discussion-based learning environments that promote strong student engagement, learning, and transfer. **Mary Kay Stein**, mkstein@pitt.edu, is a professor of learning sciences and policy and the director of the Learning Policy Center at the University of Pittsburgh. Her research focuses on instructional practice and the organizational and policy conditions that shape it.

Discussions that focus on cognitively challenging mathematical tasks, namely, those that promote thinking, reasoning, and problem solving, are a primary mechanism for promoting conceptual understanding of mathematics (Hatano and Inagaki 1991; Michaels, O'Connor, and Resnick forthcoming). Such discussions give students opportunities to share ideas and clarify understandings, develop convincing arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things from other perspectives (NCTM 2000).

Although discussions about high-level tasks provide important

The **5** practices are:

1. **Anticipating** student responses to challenging mathematical tasks;
2. **Monitoring** students' work on and engagement with the tasks;
3. **Selecting** particular students to present their mathematical work;
4. **Sequencing** the student responses that will be displayed in a specific order and;
5. **Connecting** different students' responses and connecting the responses to key mathematical ideas.

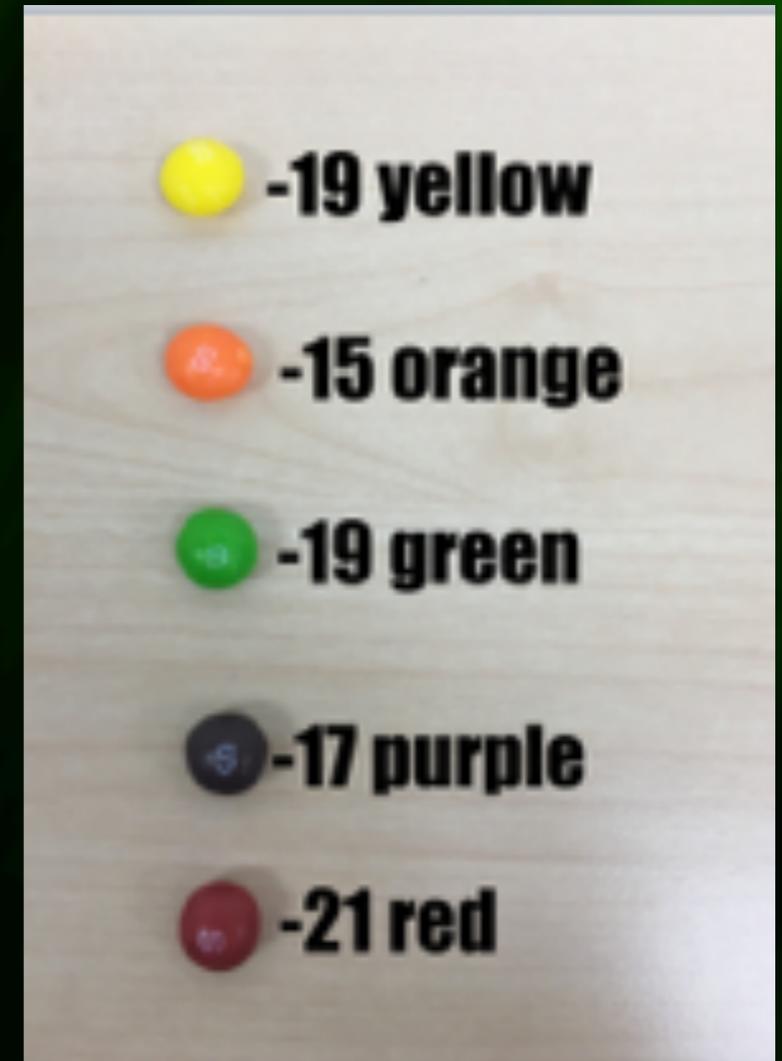
Task Planning Page

Learning Target:

Questions and Look-Fors:

Strategy	Who and What	Order

Notes:



Name: 14

Student #1

Estimate
14

Draw a picture or use numbers to show your thinking:

Answer:
91

Name: 11

Student #2

Estimate
13

Draw a picture or use numbers to show your thinking:

$19 + 15 + 19 + 17 + 21 = 91$

I yooost t in
frams.

Answer:
91

Name:

Student #3

Estimate
42

Draw a picture to show your thinking:

Orange - 10
Purple - 10
green - 10
red - 10
Yellow - 10

$60 + 10 + 10 + 10 + 10 + 10 = 91$

Use numbers to show your thinking:

Answer:
91

Name: 13

Student #4

Estimate
18

Draw a picture or use numbers to show your thinking:

I counted by tally
marker and that how I got
73.

Answer:
73

Identify and name the strategy used, then place the student work in order in terms of efficiency (least to greatest)

Name:

Student #5

Estimate
50

Draw a picture to show your thinking:

$$\begin{array}{r} 19 \\ 15 \\ 19 \\ 17 \\ + 21 \\ \hline 91 \\ 60 \\ \hline 91 \end{array}$$

Use numbers to show your thinking:

Answer:

Name:

Student #6

Estimate
34

Draw a picture to show your thinking:

91

Use numbers to show your thinking:
I moved the numbers around

Answer:

Name: 23

Student #7

Estimate
81

Draw a picture or use numbers to show your thinking:

yellow
orange
green
purple

"I counted them all"

Answer:
91

Name: 4

Student #8

Estimate
81

Draw a picture or use numbers to show your thinking:

50 30
61

Answer:
91

Task Planning Page

Learning Target:		
Questions and Look-Fors:		
Strategy	Who and What	Order
Notes:		



Name: 14

Student #1

Estimate
14

Draw a picture or use numbers to show your thinking:

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Estimate
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Orange - 10
Purple - 10
green - 10
red - 10
Yellow - 10

$60 + 10 + 10 + 10 + 10 = 91$

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$$\begin{array}{r} 19 \\ 15 \\ 19 \\ 17 \\ + 21 \\ \hline 91 \\ 60 \\ \hline 91 \end{array}$$

Use numbers to show your thinking:

Answer:

Name:

Student #6

Estimate
34

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Use numbers to show your thinking:
I moved the numbers around

Answer:

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Estimate
81

Draw a picture or use numbers to show your thinking:

yellow
orange
green
purple

"I counted them all"

Answer:
91

Name: 4

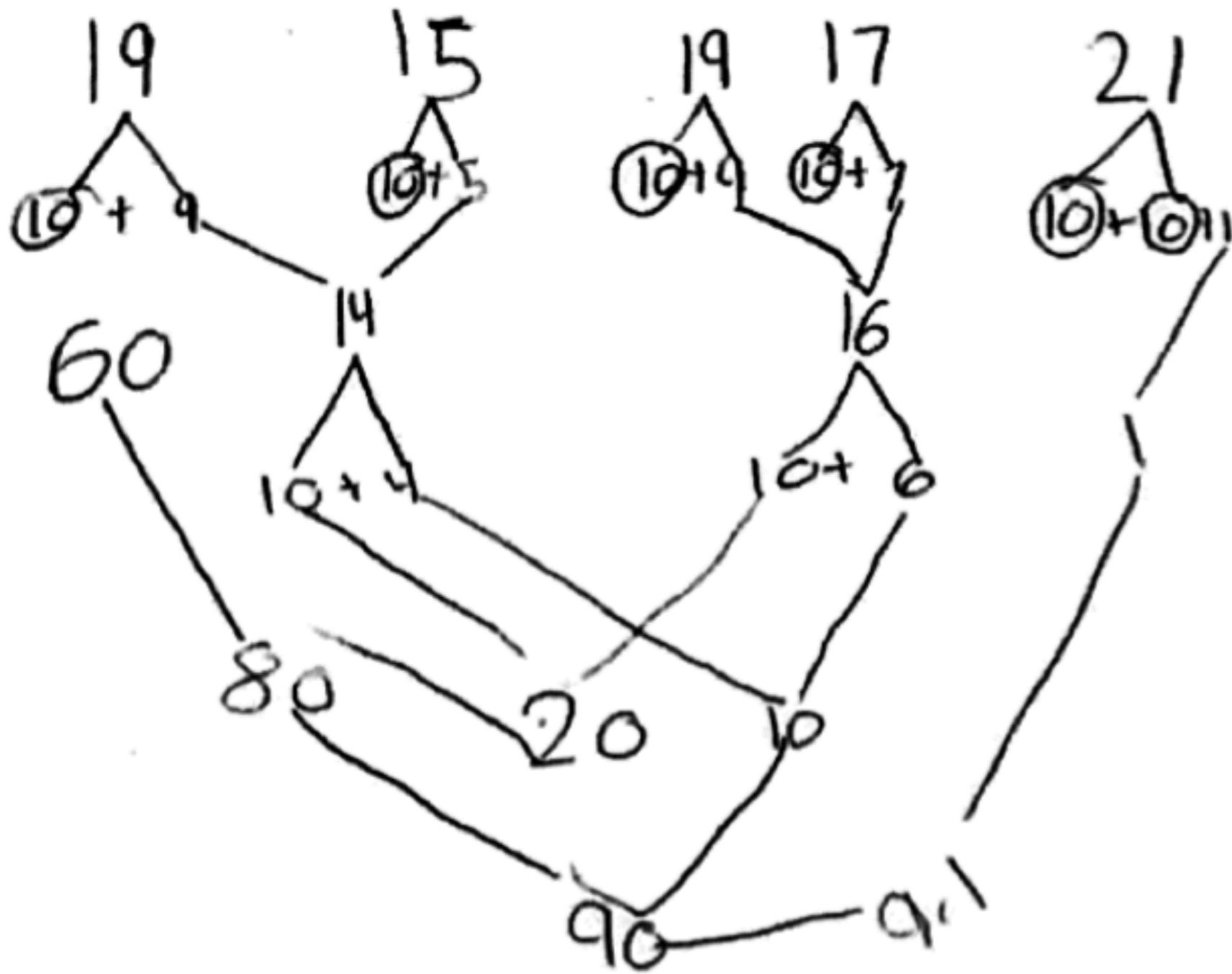
Student #8

Estimate
81

Draw a picture or use numbers to show your thinking:

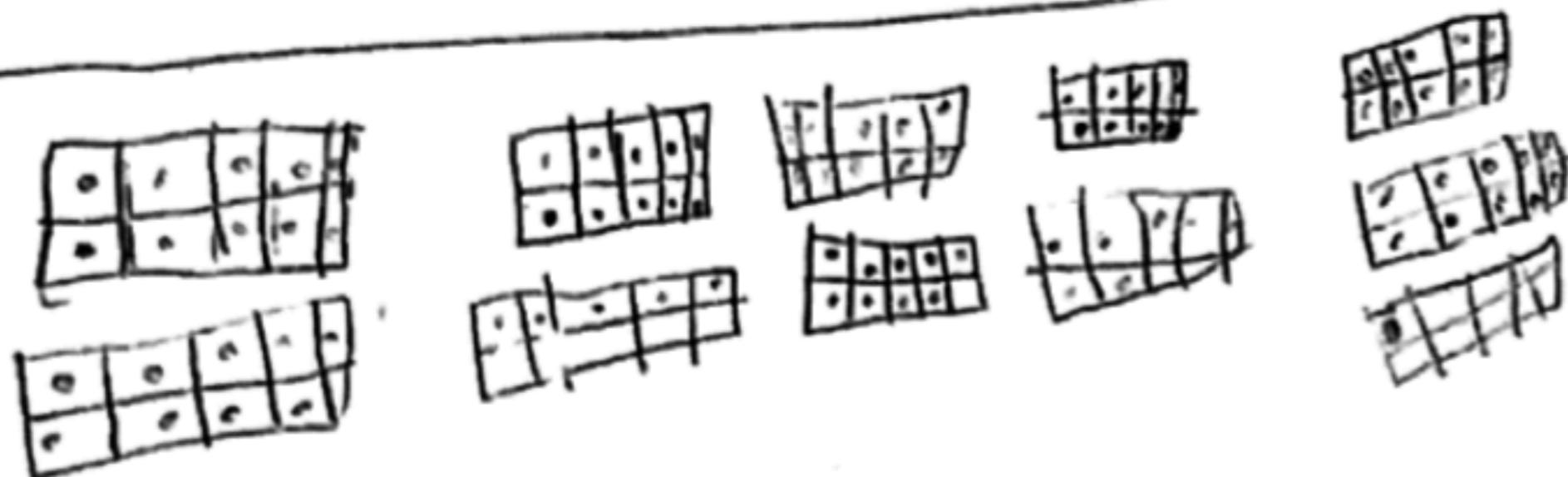
Answer:
71

Draw a picture or use numbers to show your thinking:



Draw a picture or use numbers to show your thinking:

$$19 + 15 + 19 + 17 = 21 = 911$$



I yooSt + in
fram 9.

Draw a picture to show your thinking:

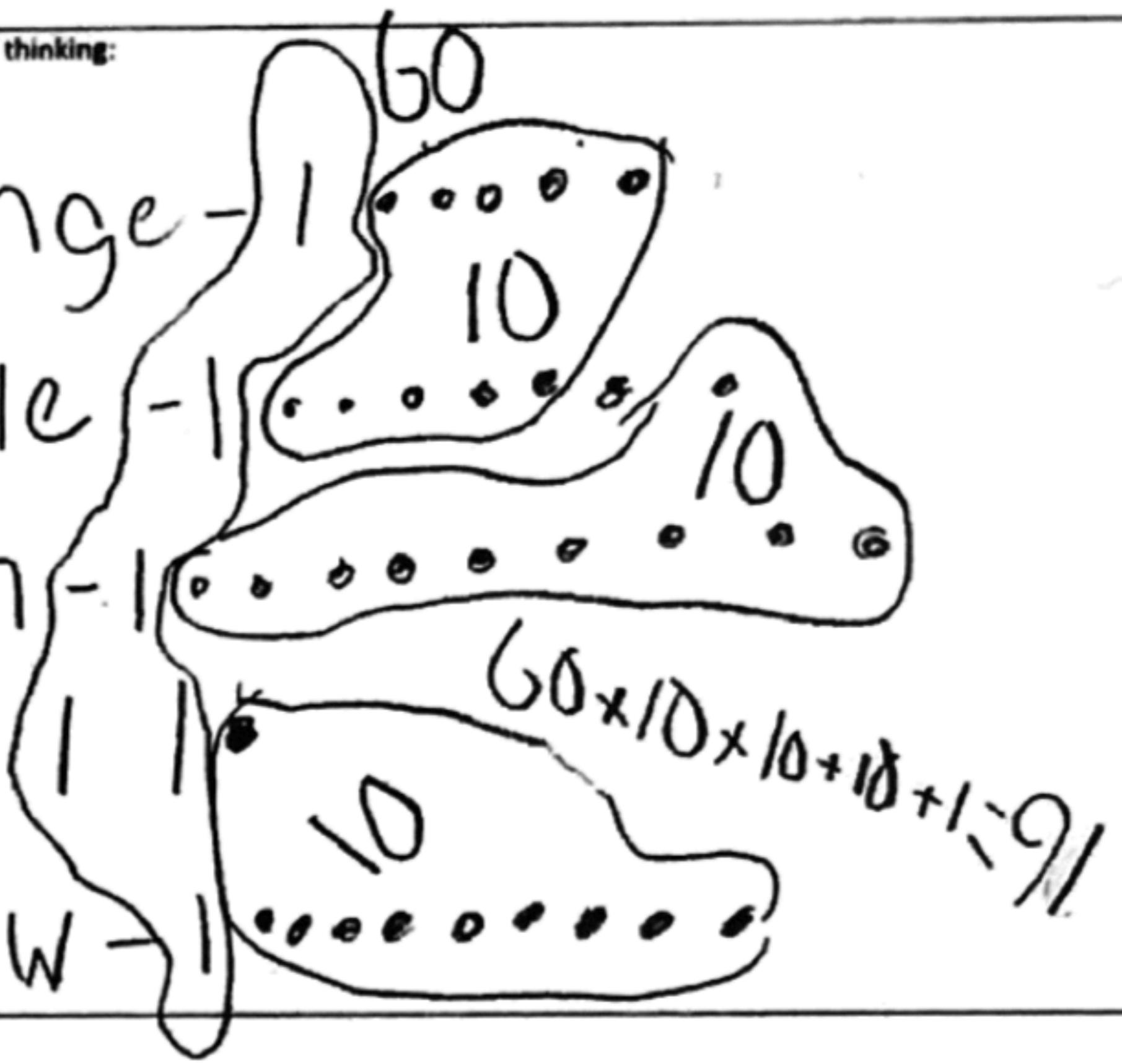
Orange - 1

PURPLE - 1

green - 1

red - 1

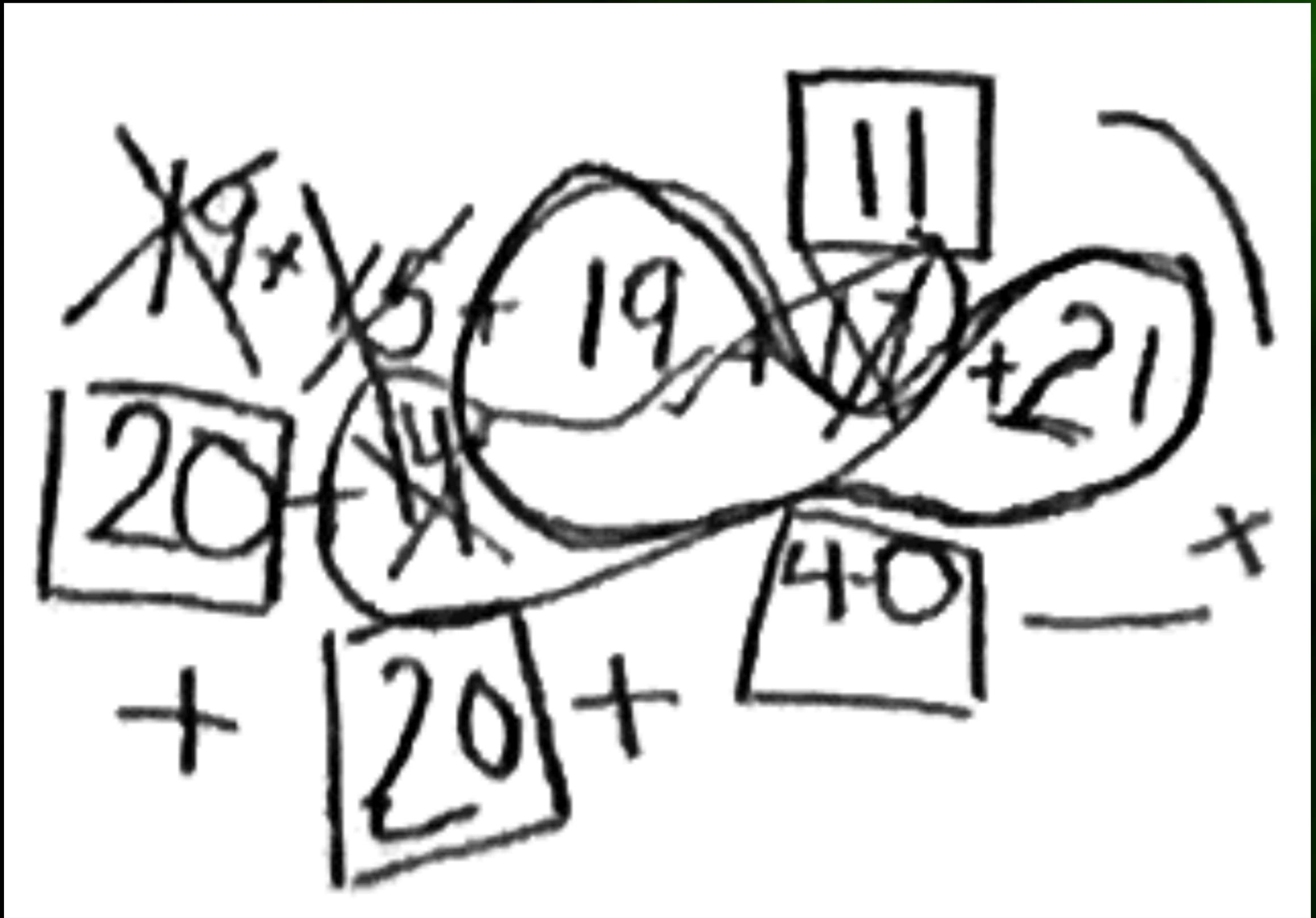
Yellow - 1



Draw a picture to show your thinking:

$$\begin{array}{r} 19 \\ 15 \\ 19 \\ 17 \\ + 21 \\ \hline 91 \\ 60 \\ \hline 91 \end{array}$$

$$19+15+19+17+21$$



Draw a picture or use numbers to show your thinking:



"I counted them all"

Draw a picture or use numbers to show your thinking:



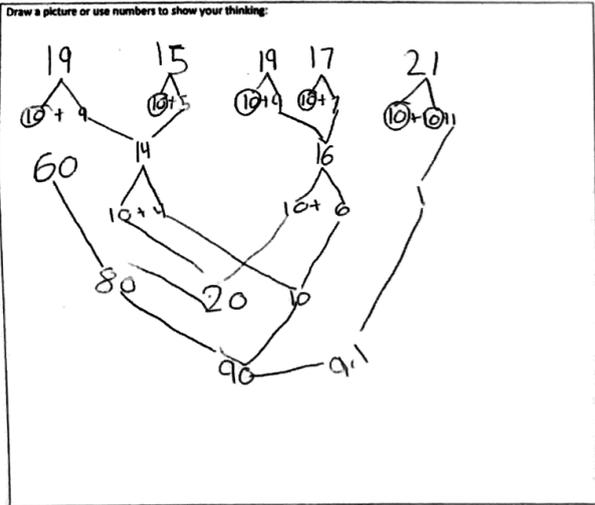
50
61

30

Name: 14

Student #1

Estimate
14

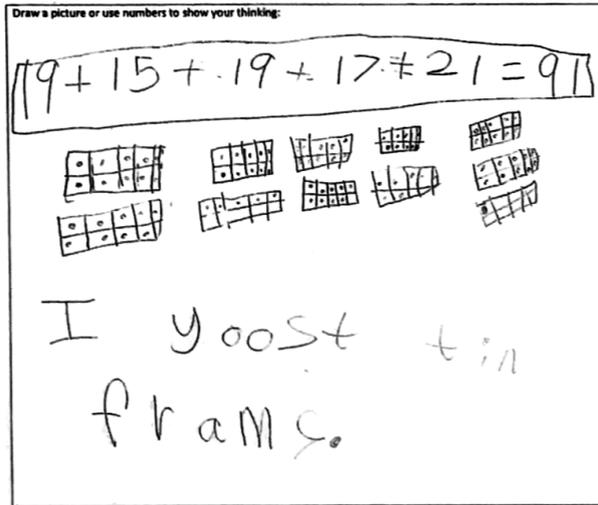


Answer:
91

Name: 11

Student #2

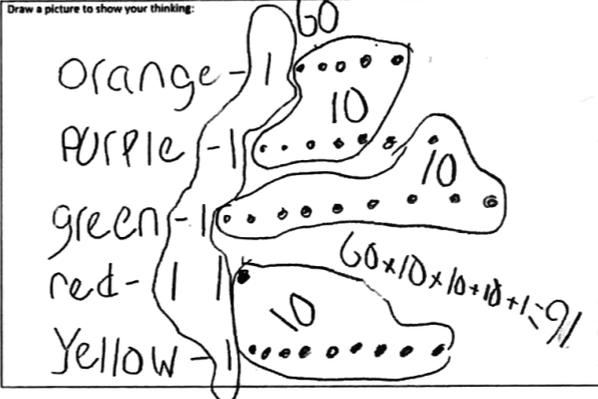
Estimate
13



Answer:
91

Name: Student #3

Estimate
42



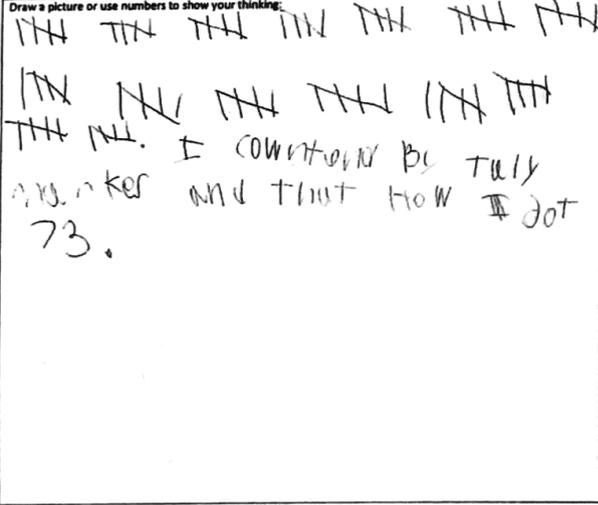
Use numbers to show your thinking:

Answer:
91

Name: 13

Student #4

Estimate
18

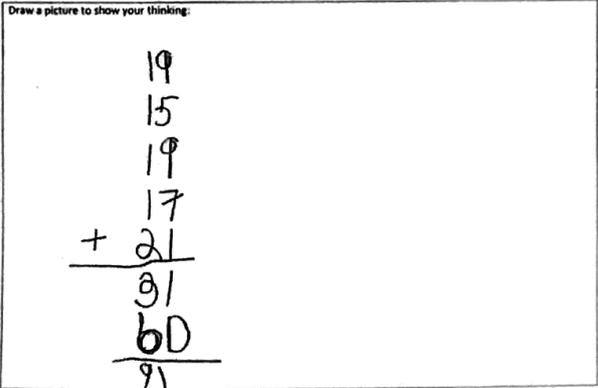


Answer:
73

Name: Student #5

Student #5

Estimate
50



Use numbers to show your thinking:

Answer:

Name: Student #6

Student #6

Estimate
34



Use numbers to show your thinking:

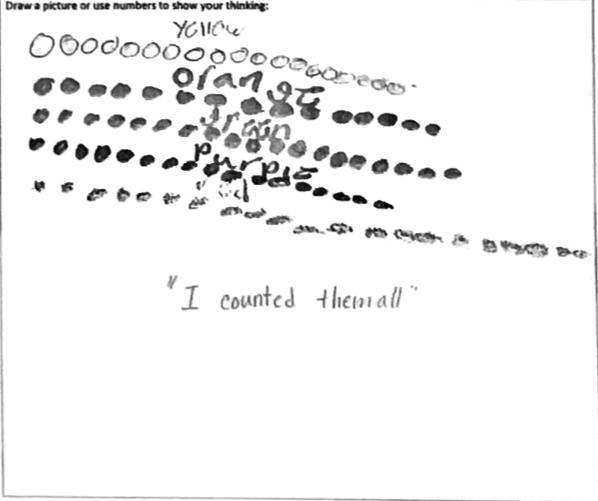
I moved the numbers around

Answer:

Name: Student #7

Student #7

Estimate
81

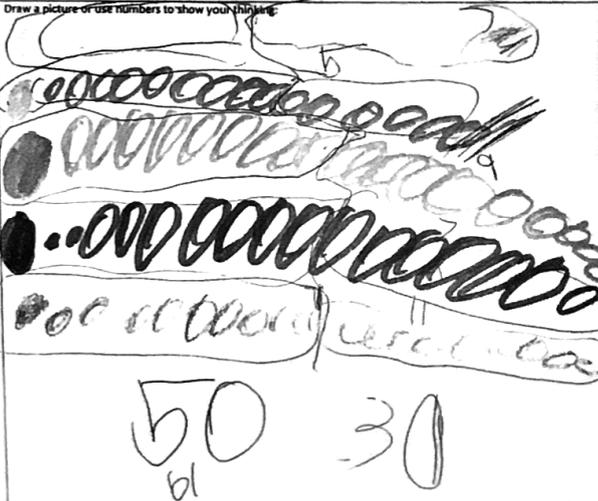


Answer:
91

Name: Student #8

Student #8

Estimate
81

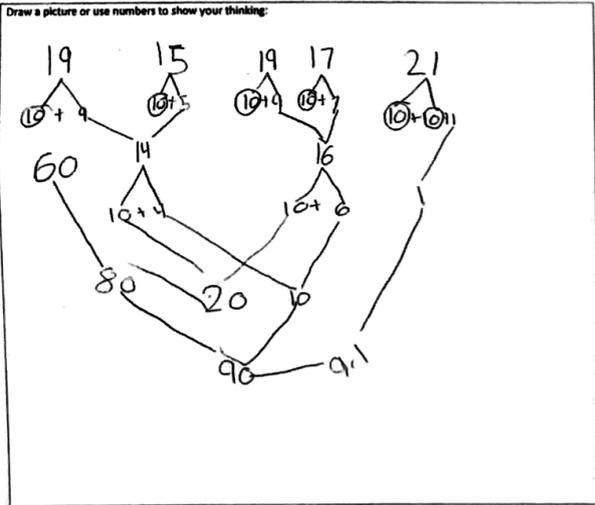


Answer:
91

Name: 14

Student #1

Estimate
14

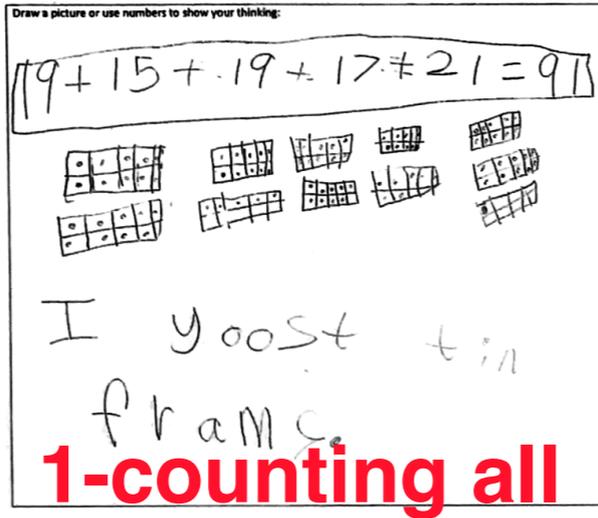


Answer:
91

Name: 11

Student #2

Estimate
13

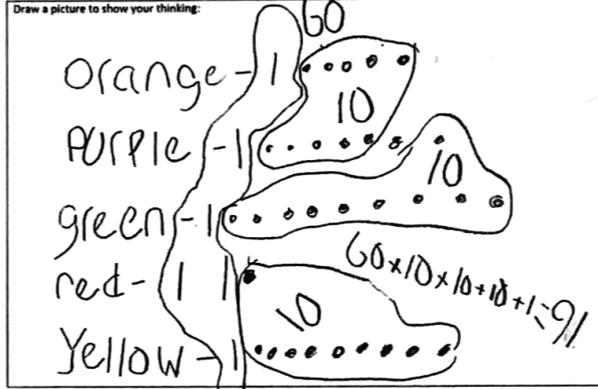


Answer:
91

Name:

Student #3

Estimate
42



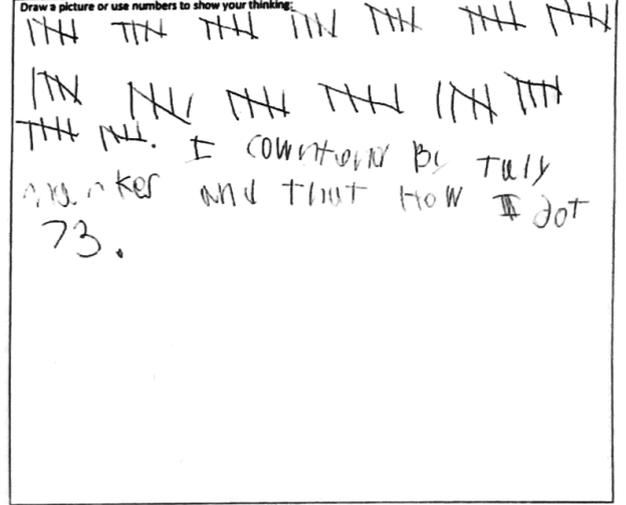
Use numbers to show your thinking:

Answer:
91

Name: 13

Student #4

Estimate
18

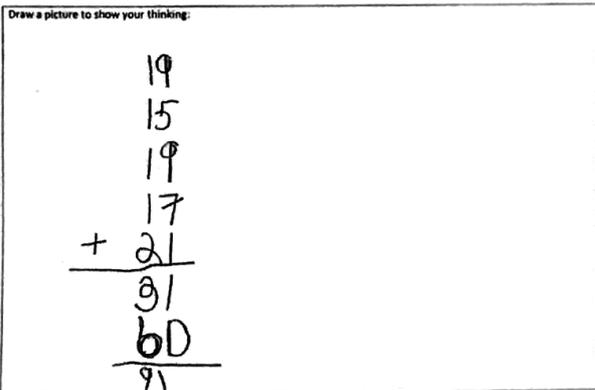


Answer:
73

Name:

Student #5

Estimate
50



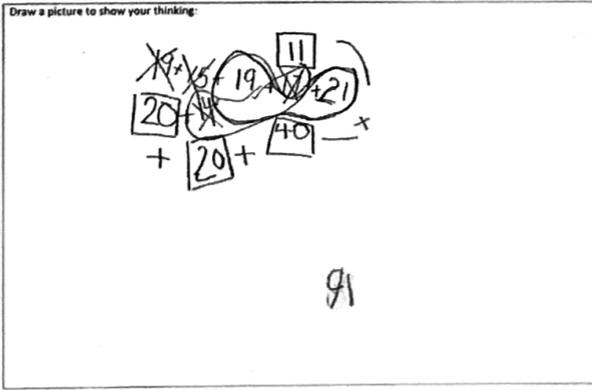
Use numbers to show your thinking:

Answer:

Name:

Student #6

Estimate
34



Use numbers to show your thinking:

I moved the numbers around

Answer:

Name: 203

Student #7

Estimate
81



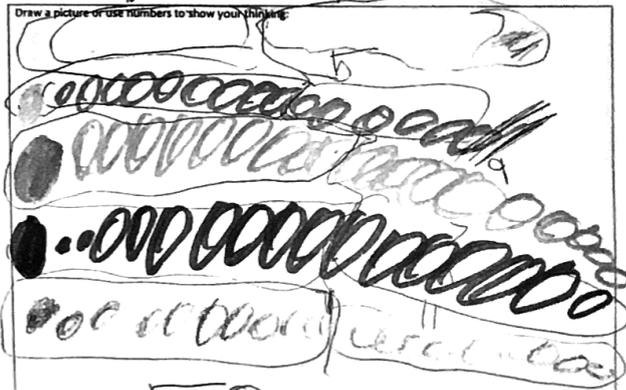
1-counting all

Answer:
91

Name: 4

Student #8

Estimate
81

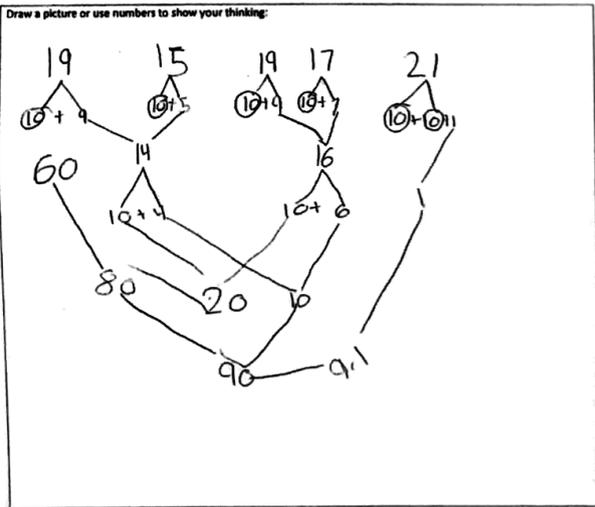


Answer:
71

Name: 14

Student #1

Estimate 14

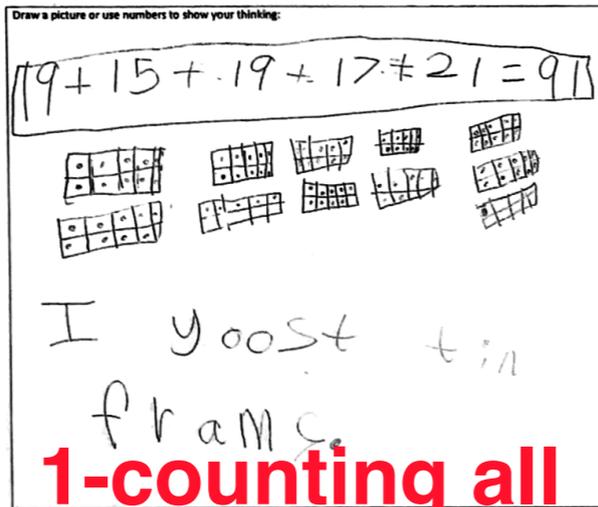


Answer: 91

Name: 11

Student #2

Estimate 13

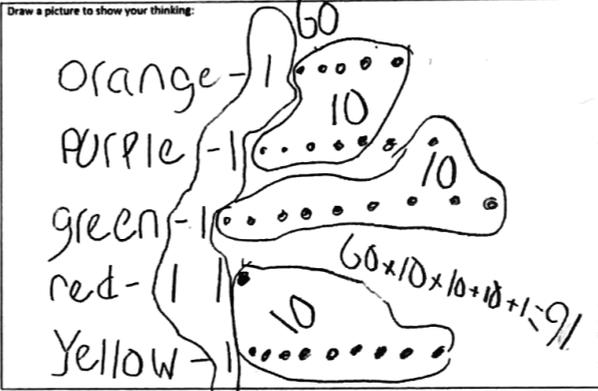


1-counting all

Answer: 91

Name: Student #3

Estimate 42



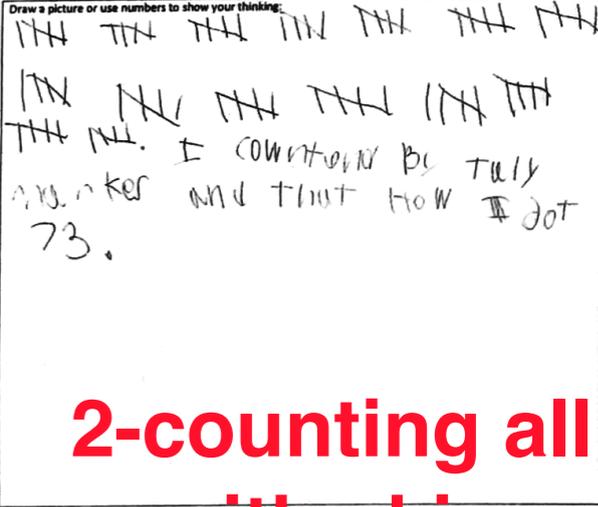
Use numbers to show your thinking:

Answer: 91

Name: 13

Student #4

Estimate 18



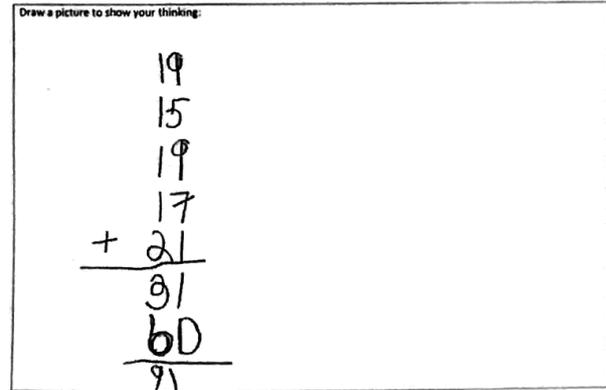
2-counting all with skip counting

Answer: 73

Name: Student #5

Student #5

Estimate 50



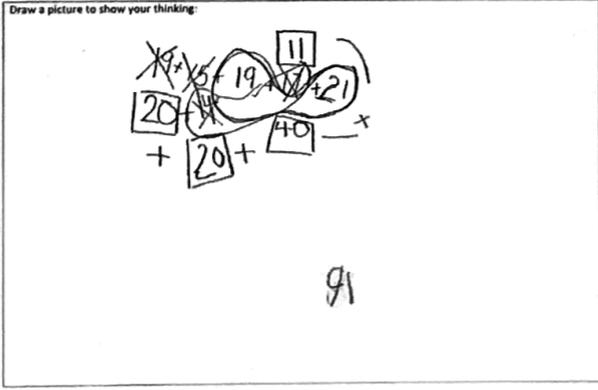
Use numbers to show your thinking:

Answer:

Name: Student #6

Student #6

Estimate 34



Use numbers to show your thinking:

I moved the numbers around

Answer:

Name: Student #7

Student #7

Estimate 81



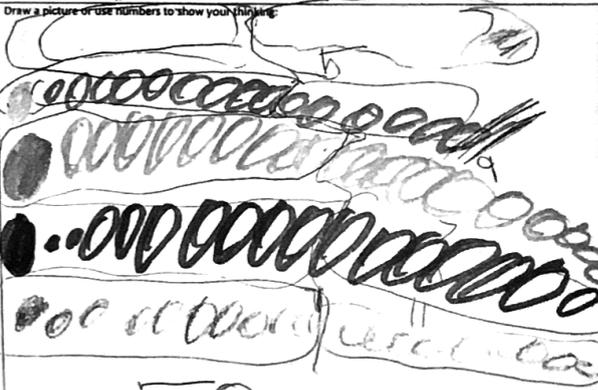
1-counting all

Answer: 91

Name: Student #8

Student #8

Estimate 81



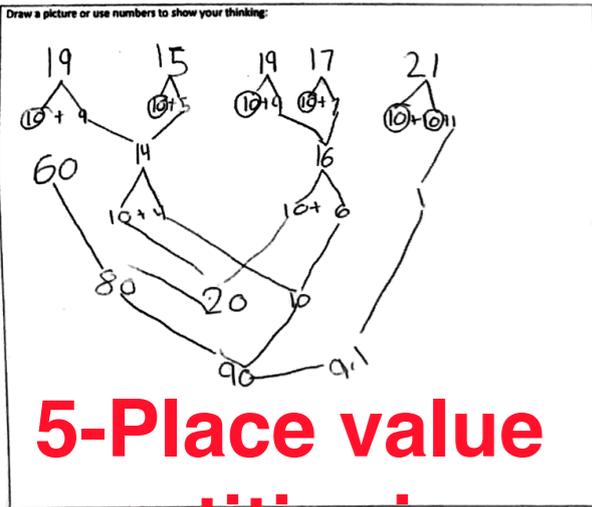
50 30

Answer: 71

Name: 14

Student #1

Estimate
14



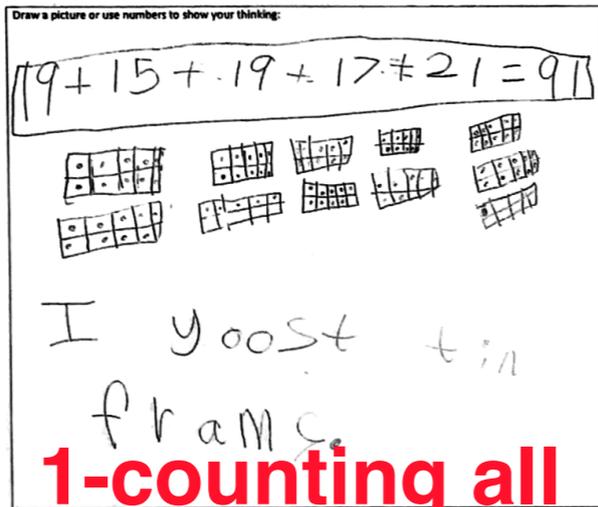
5-Place value partitioning

Answer:
91

Name: 11

Student #2

Estimate
13



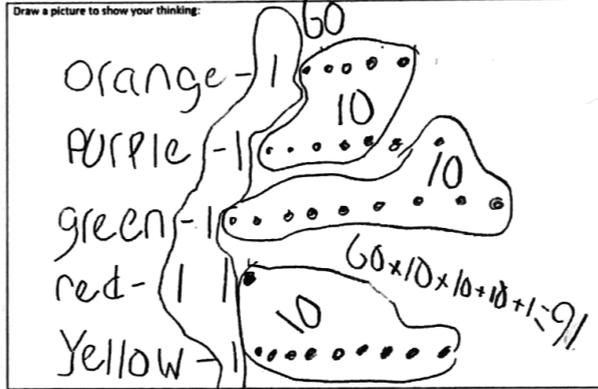
1-counting all

Answer:
91

Name:

Student #3

Estimate
42



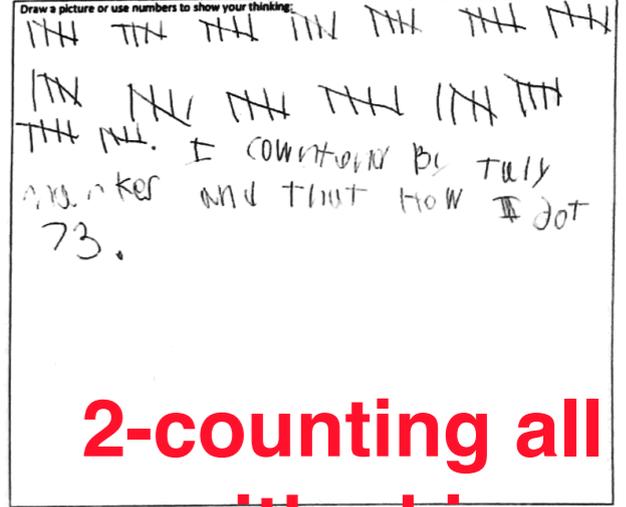
4-making tens from ones

Answer:
91

Name: 13

Student #4

Estimate
18



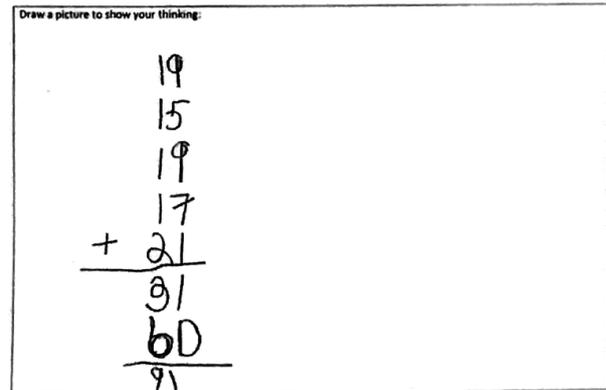
2-counting all with skip counting

Answer:
73

Name:

Student #5

Estimate
50



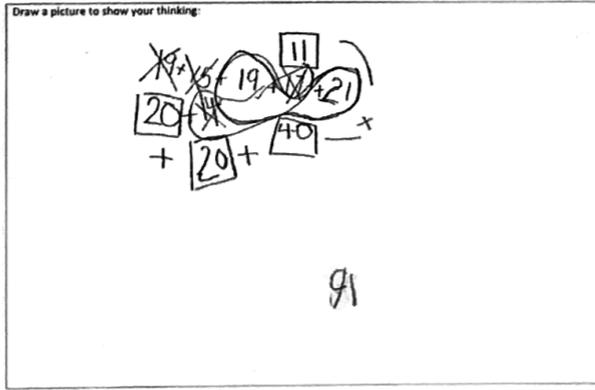
Use numbers to show your thinking:

Answer:

Name:

Student #6

Estimate
34



Use numbers to show your thinking:
I moved the numbers around

Answer:

Name: 203

Student #7

Estimate
81



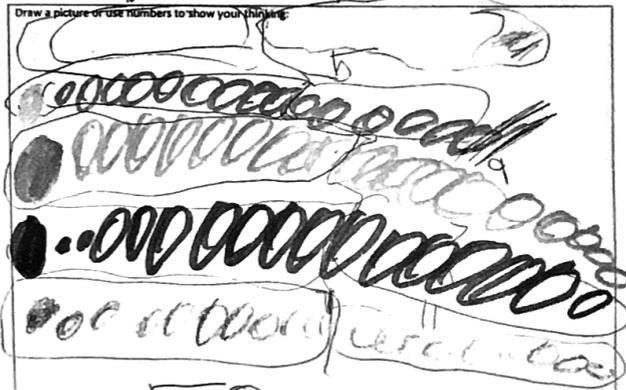
1-counting all

Answer:
91

Name: 4

Student #8

Estimate
81

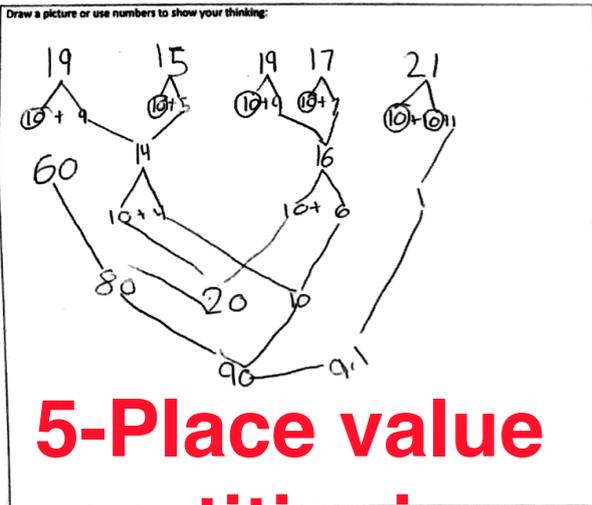


3-making tens from ones

Answer:
91

Name: 14
Student #1

Estimate
14

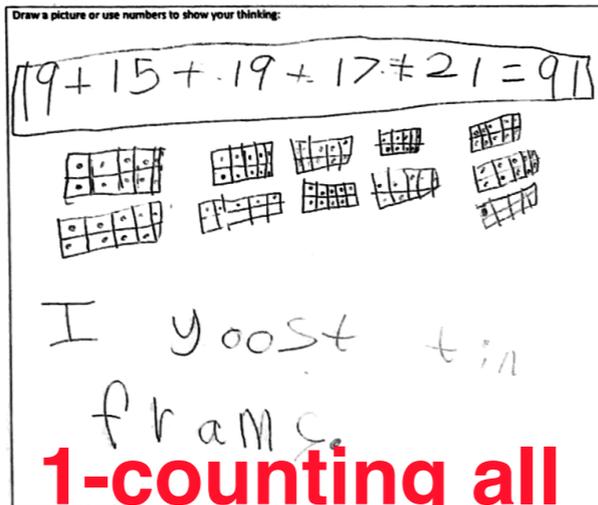


5-Place value partitioning

Answer:
91

Name: 11
Student #2

Estimate
13

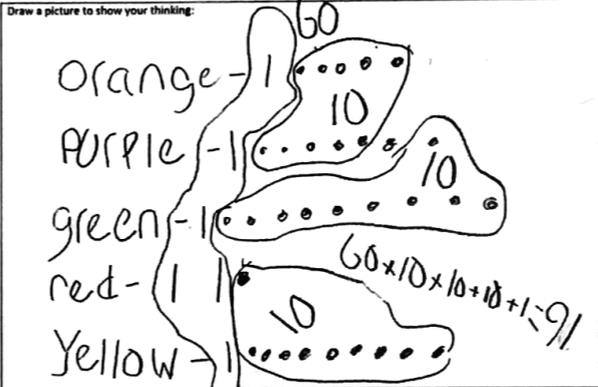


I yooost tin
franc.
1-counting all

Answer:
91

Name: _____
Student #3

Estimate
42

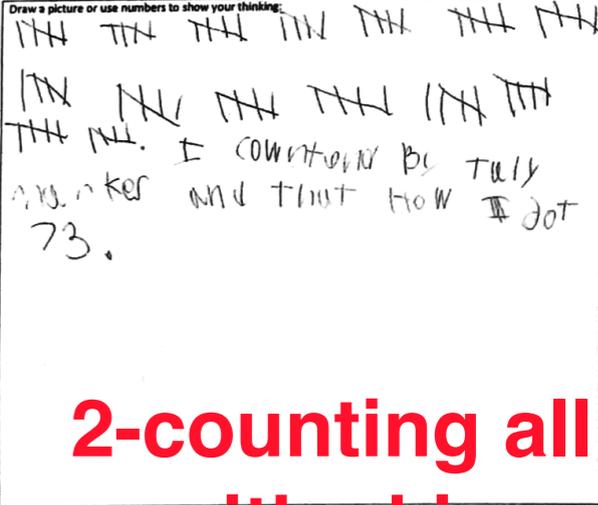


4-making tens from ones

Answer:
91

Name: 13
Student #4

Estimate
18

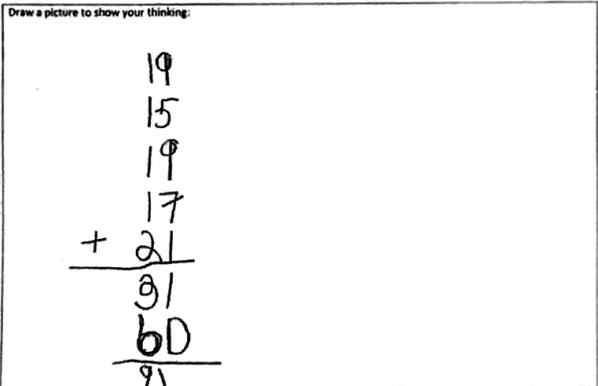


2-counting all with skip counting

Answer:
91

Name: _____
Student #5

Estimate
50

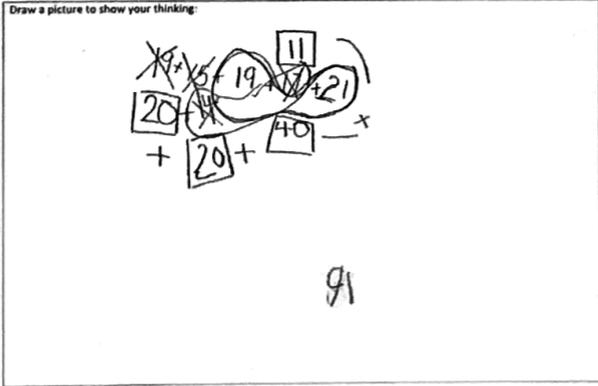


6-Partial Sums

Answer:
91

Name: _____
Student #6

Estimate
34



I moved the numbers around

Answer:
91

Name: 203
Student #7

Estimate
81

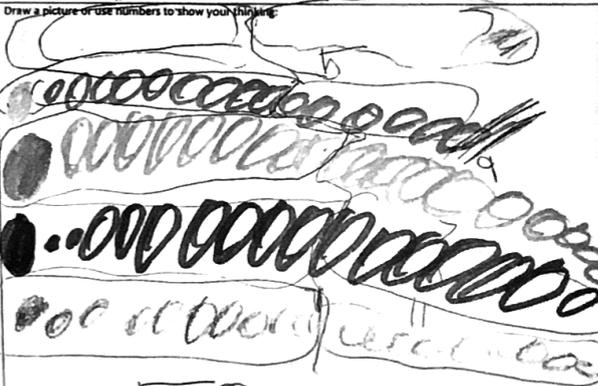


1-counting all

Answer:
91

Name: 4
Student #8

Estimate
81



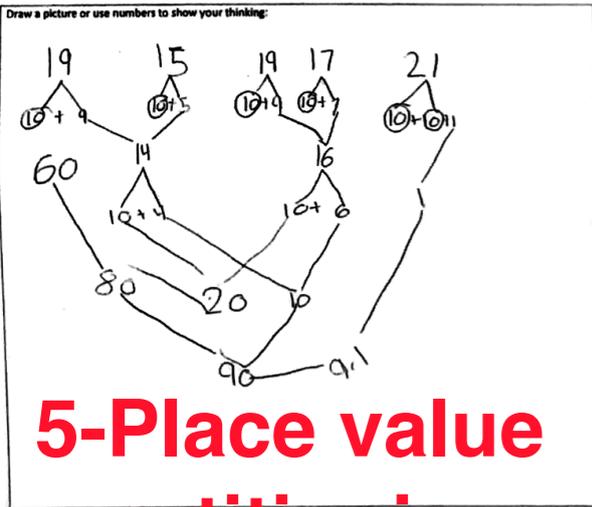
3-making tens from ones

Answer:
91

Name: 14

Student #1

Estimate
14



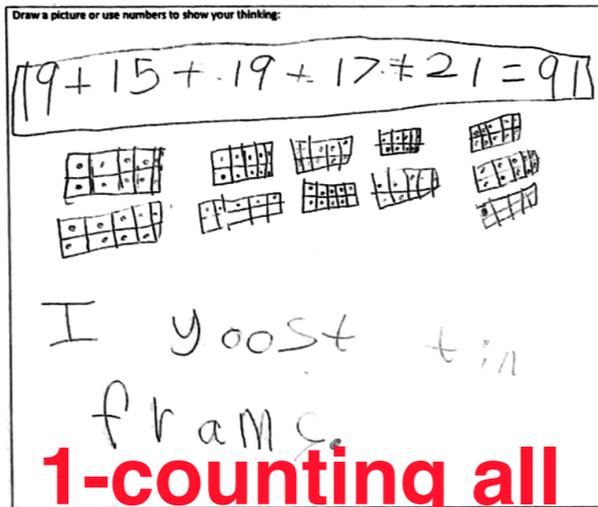
5-Place value partitioning

Answer:
91

Name: 11

Student #2

Estimate
13

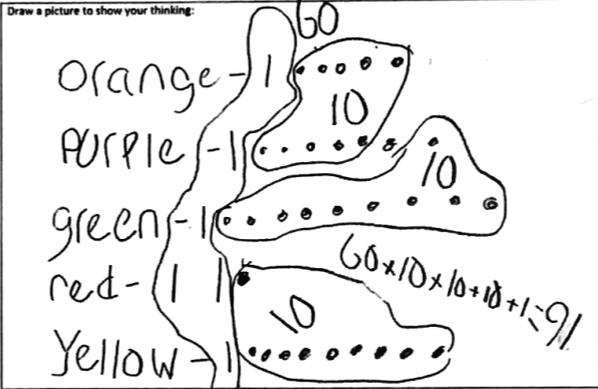


1-counting all

Answer:
91

Name: Student #3

Estimate
42



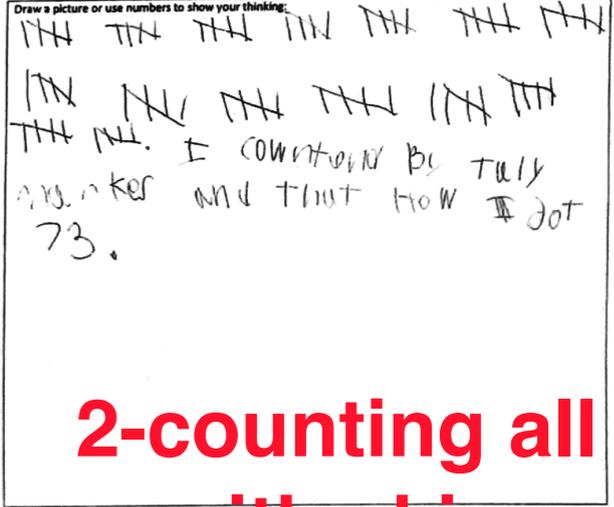
4-making tens from ones

Answer:
91

Name: 13

Student #4

Estimate
18



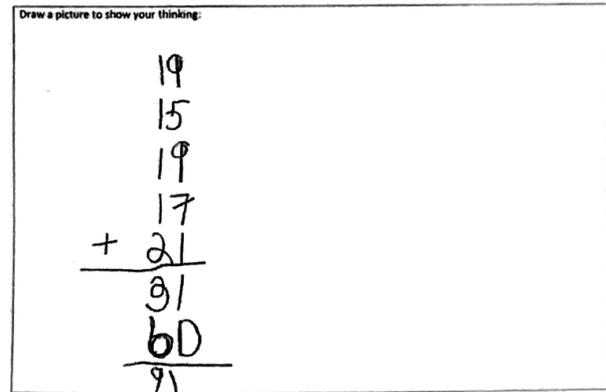
2-counting all with skip counting

Answer:
73

Name: Student #5

Student #5

Estimate
50



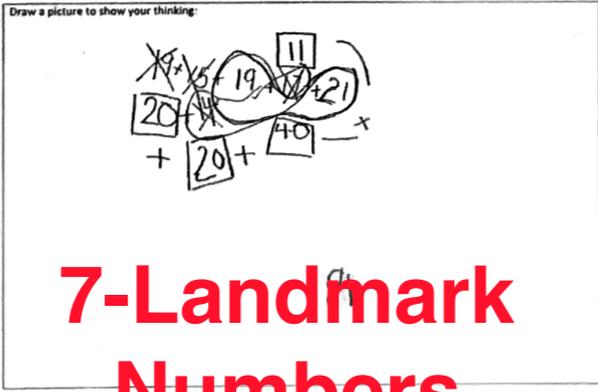
6-Partial Sums

Answer:

Name: Student #6

Student #6

Estimate
34



7-Landmark Numbers

Use numbers to show your thinking:
I moved the numbers around

Answer:

Name: 203

Student #7

Estimate
81



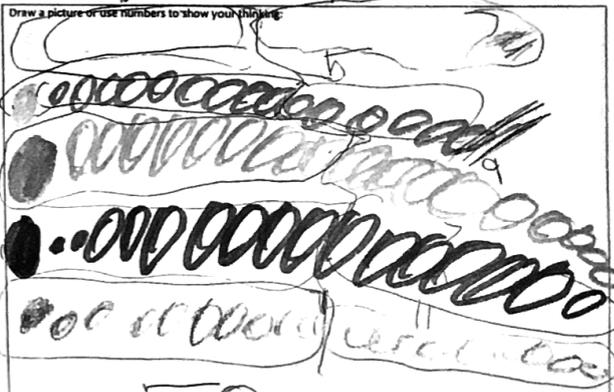
1-counting all

Answer:
91

Name: Student #8

Student #8

Estimate
81



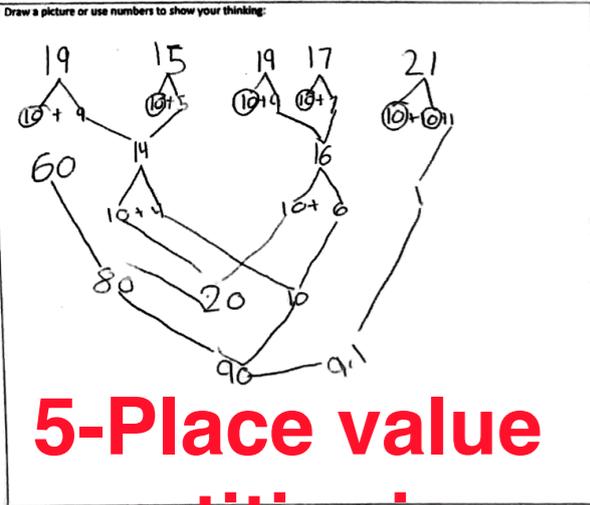
3-making tens from ones

Answer:
91

Name: 14

Student #1

Estimate
14



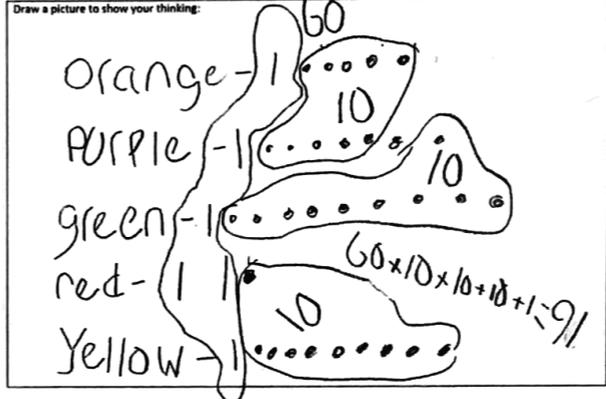
5-Place value partitioning

Answer:
91

Name:

Student #3

Estimate
42



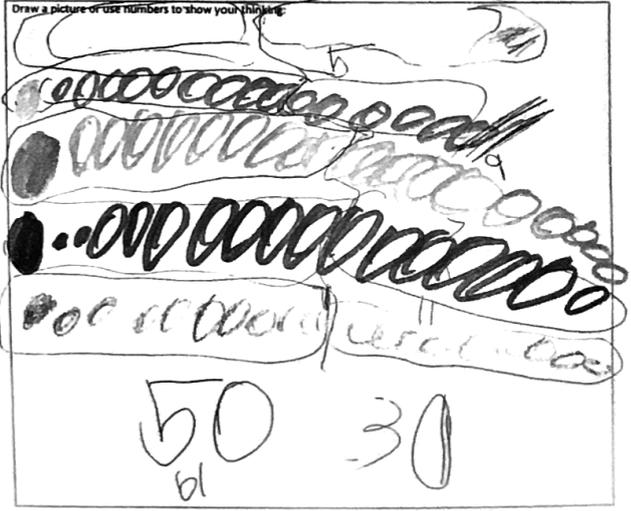
Use numbers to show your thinking:
4-making tens from ones

91

Name: 4

Student #8

Estimate
81

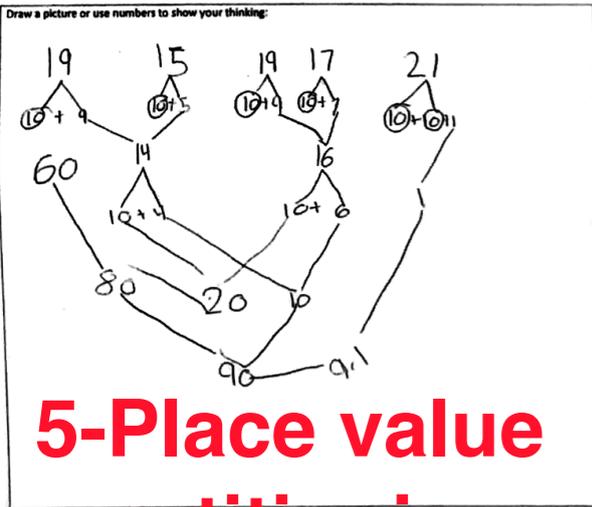


3-making tens from ones

Name: 14

Student #1

Estimate
14



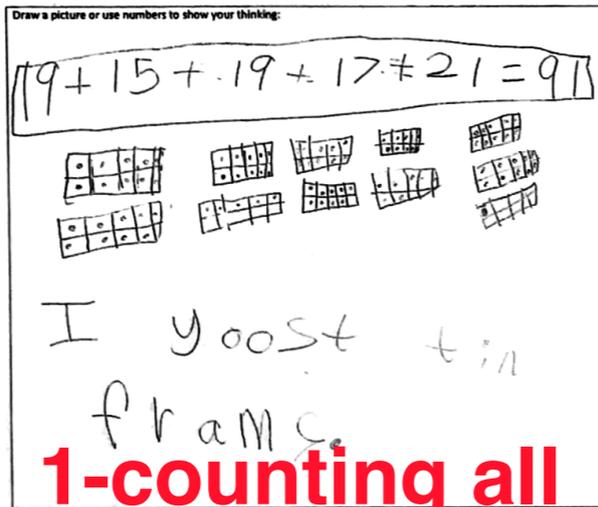
5-Place value partitioning

Answer:
91

Name: 11

Student #2

Estimate
13

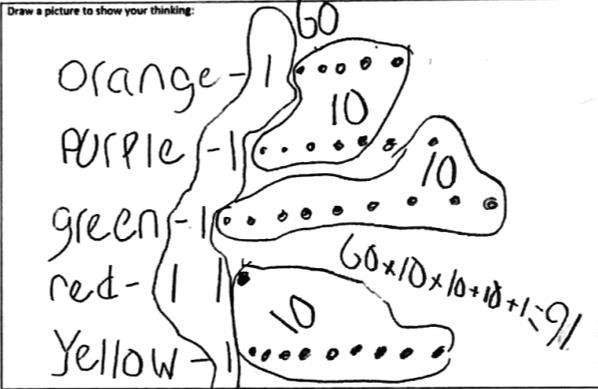


1-counting all

Answer:
91

Name: Student #3

Estimate
42



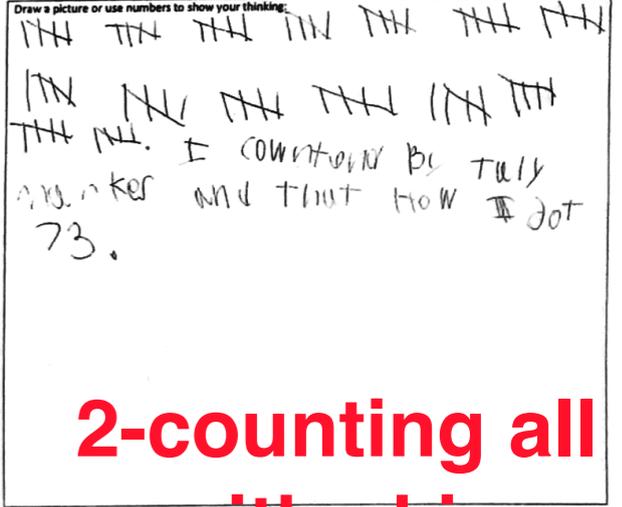
4-making tens from ones

Answer:
91

Name: 13

Student #4

Estimate
18



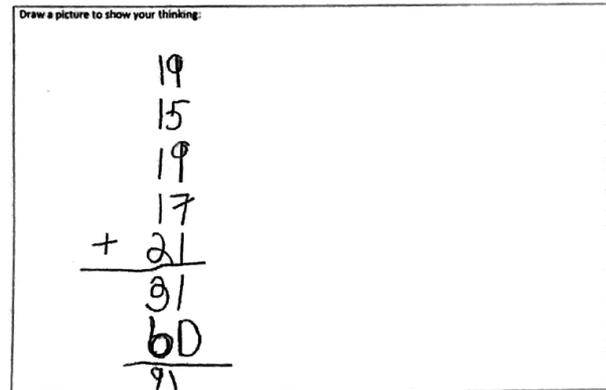
2-counting all with skip counting

Answer:
73

Name: Student #5

Student #5

Estimate
50



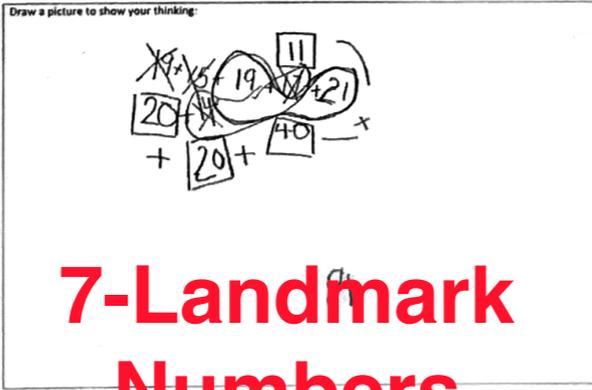
6-Partial Sums

Answer:

Name: Student #6

Student #6

Estimate
34



7-Landmark Numbers

I moved the numbers around

Answer:

Name: Student #7

Student #7

Estimate
81



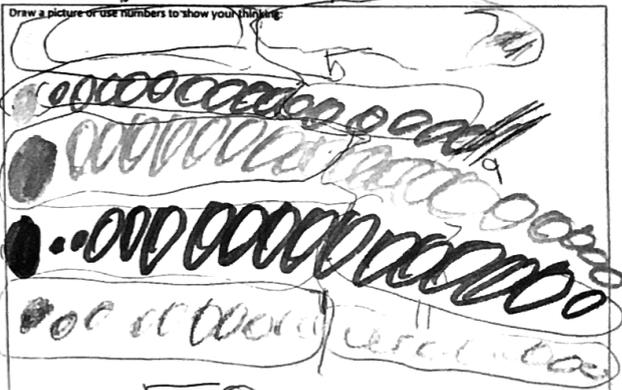
1-counting all

Answer:
91

Name: Student #8

Student #8

Estimate
81



3-making tens from ones

Answer:
91

Making Sense Series

the progression of addition AND subtraction
the standard traditional algorithm

created by Graham Fletcher



@gfletchy

www.gfletchy.com

3-Act Tasks

5 Practices

Progressions

**Using the digits 1-9 at most one time each,
create 4 numbers that have a sum of 91.**

$$\square \square + \square + \square \square + \square \square = 91$$

You can use the 9 & 1 from the cards

Kindergarten?



**CHICLE
TABS**
CHEWING
GUM

25c

 **4-orange**

 **2-red**

 **2-yellow**

 **2-white**

 **0-pink**

Name: _____

Estimate

3

S5

Draw a picture to show your thinking:

$$4 + 2 + 2 + 2 + 0 =$$

10

Use numbers to show your thinking:

Answer:

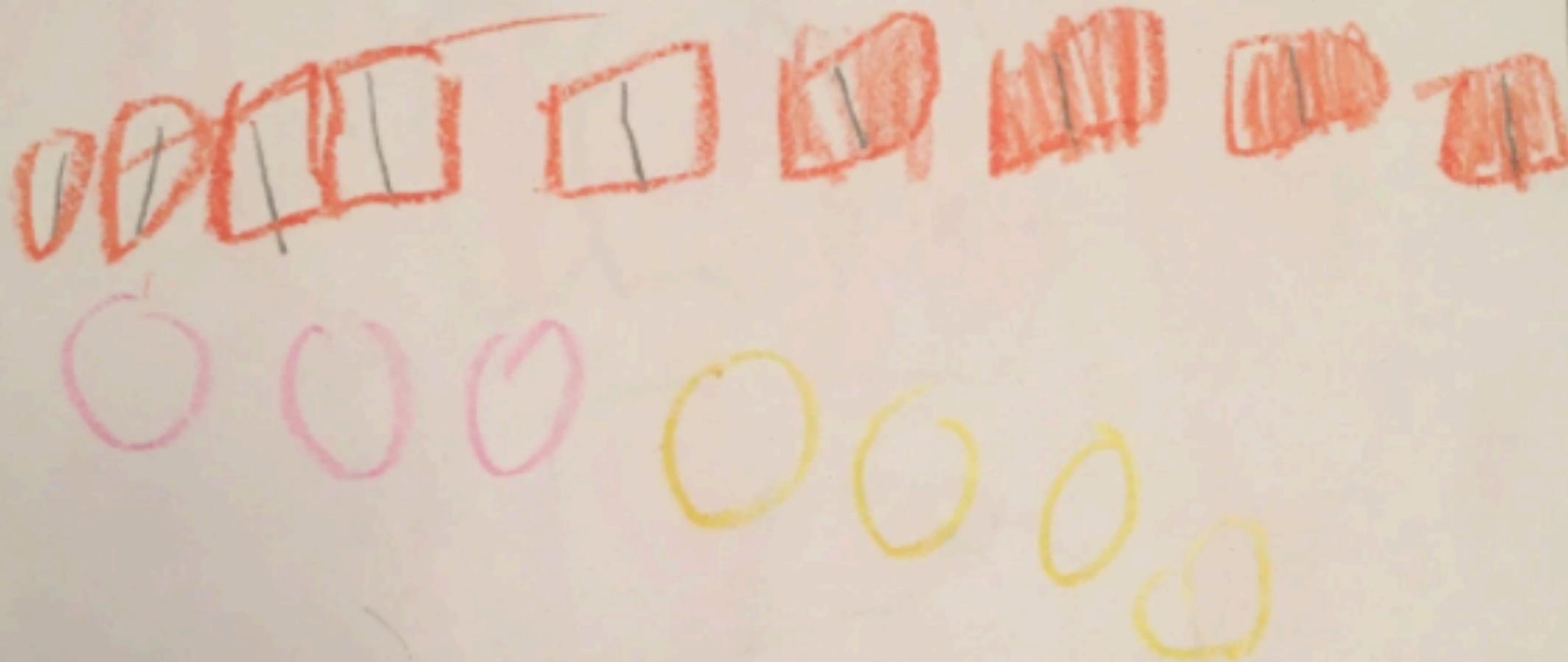
10

Estimate

6

51

Draw a picture to show your thinking:



Estimate

1

86

Draw a picture to show your thinking:

22 2 4
0 2 2 2

Write

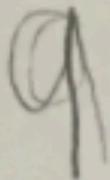
S3

Draw a picture to show your thinking



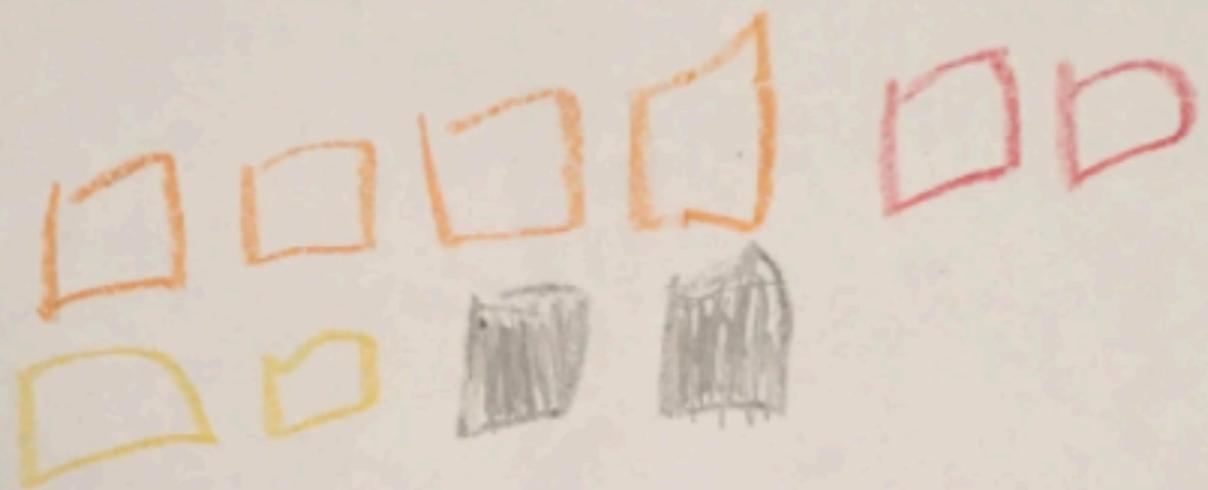
Use numbers to show your thinking

Estimate



S8

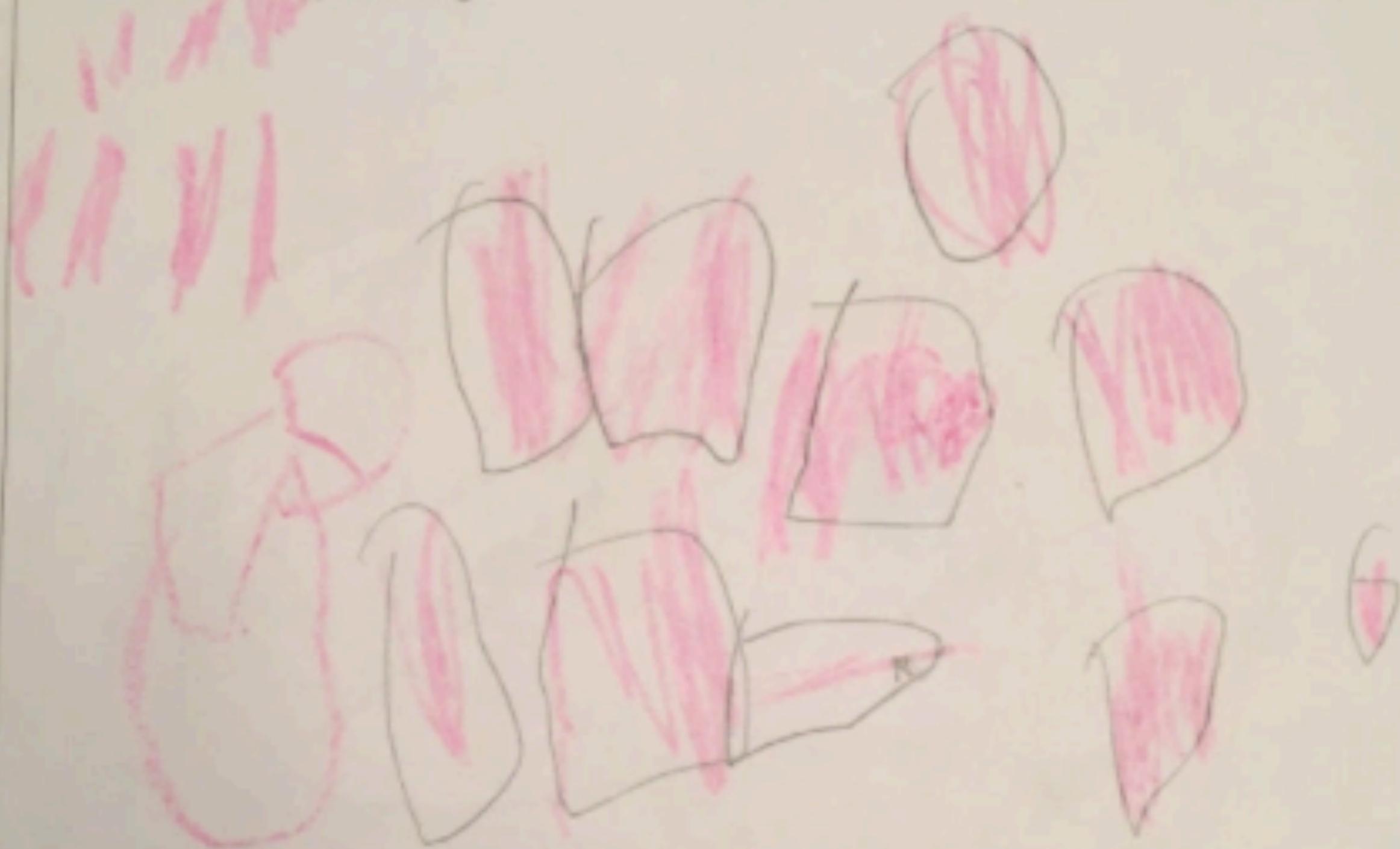
Draw a picture to show your thinking:



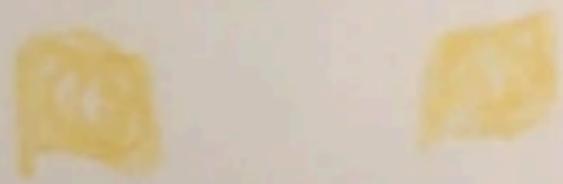
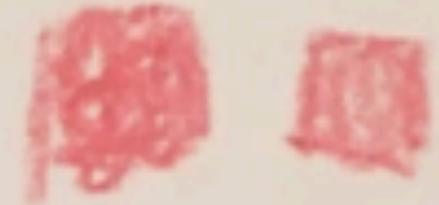
Use numbers to show your thinking:

1 2 3 4 2 6 7 8 9 10

Draw a picture to show your thinking:



Use numbers to show your thinking:



Use numbers to show your thinking:

-  **4-orange**
-  **2-red**
-  **2-yellow**
-  **2-white**
-  **0-pink**

