Harnessing the Power of Modeling Tasks through the Lens of a Math Progression

Graham Fletcher

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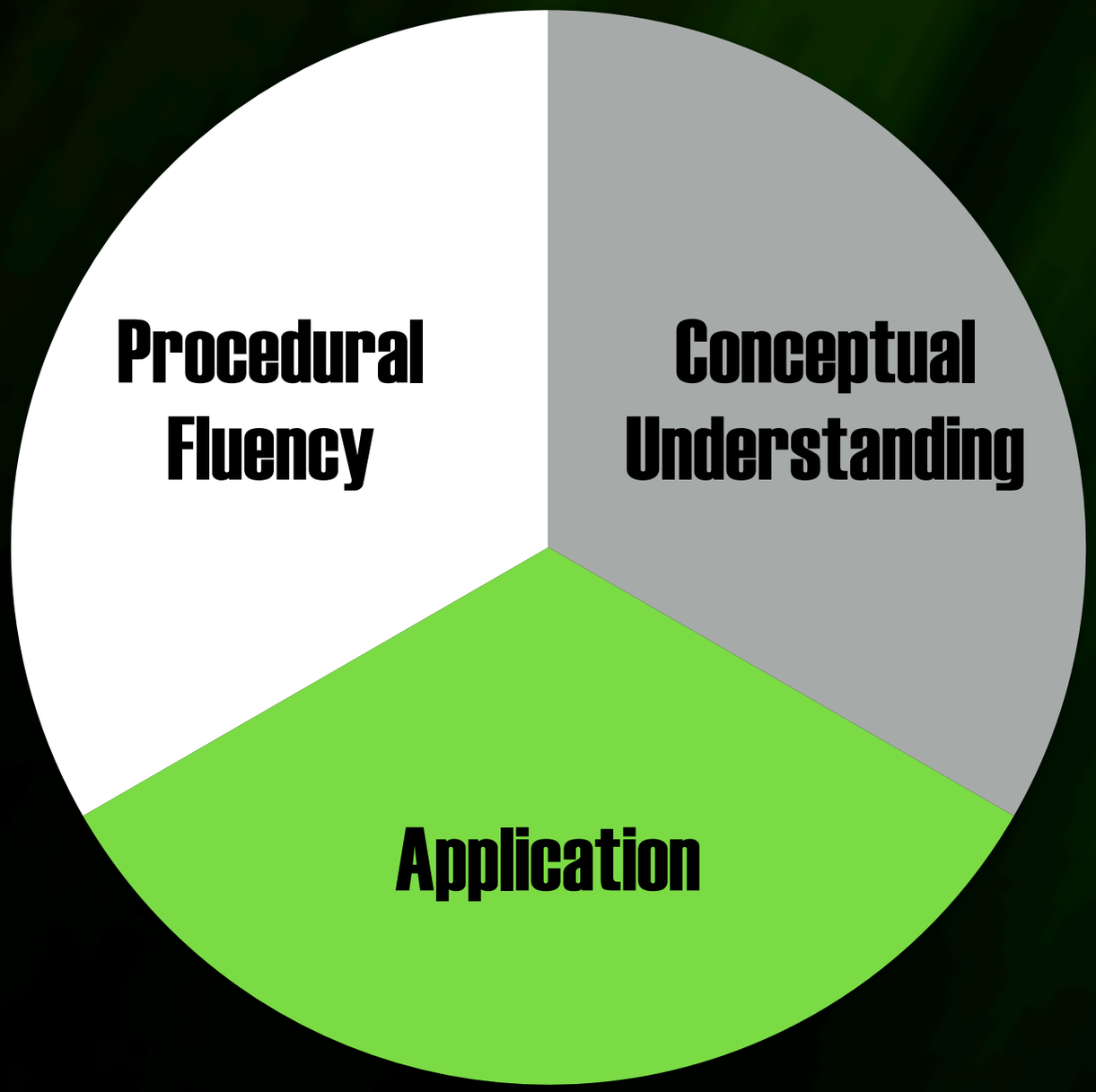
Broken Squares

- Designate a shape keeper
- 6 congruent squares
- No shapes left over
- Everyone is encouraged to OFFER. No one may TAKE. They may ACCEPT a puzzle piece to use if OFFERED.
- NO TALKING



http://www.corestandards.org/other-resources/key-shifts-in-mathematics/

@RobertKaplinsky


NEXT TIME YOU RE AFRAID
TO SHARE IDEAS
REMEMBER SOMEONE
ONCE SAID IN A MEETING
LETS MAKE A FILM WITH A
TORNADO FULL OF SHARKS

The Condominium Problem

In a particular condominium community 2/3 of all of the men are married to 3/5 of all of the women.

What fraction of the entire condominium community are married?

students doing the solving and the sense making

teacher showing a very conceptual approach

top-down, rule oriented approach

3 questions

1 Billion Circles

?

Fletchy

100 circles : minute

144,000 circles : day

1,000,000,000 would take 6944 days

19+ years with no sleep

2nd Question

Mark Chubb @MarkChubb3 · 16m

@gfletchy

Where does 1 billion go?

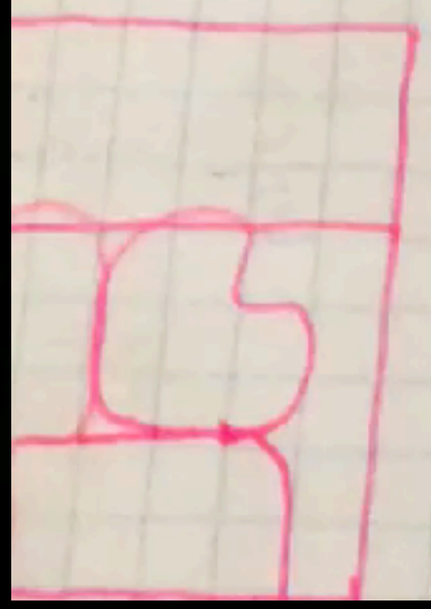
Where does 1 billion go on the number line?

$$6 \times \frac{5}{8}$$

$10 + 3 + 5/8$

9	600	180

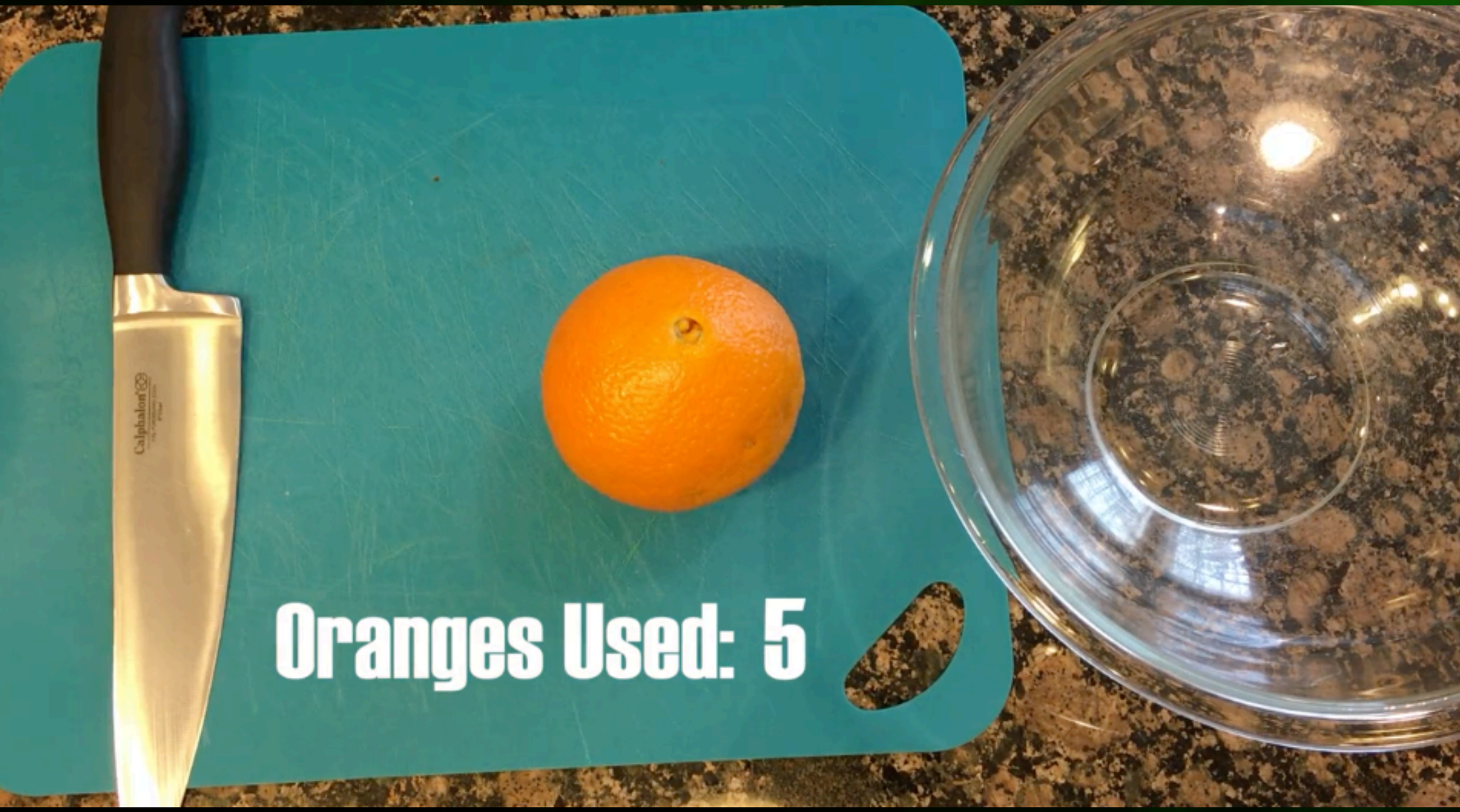
$6 * 5/8 = 3\ 6/8$



$6 \times \frac{5}{8}$

- Understand the structure of 3-act task and see how they fit into the scope and sequence of a unit.
- Explore the importance of progressional understanding and how a good task can be used as formative assessment.
    - Numbers and Operations in Fractions
- Understand the importance of an effective closing and the role it plays in deciding our next move.

Each orange wedges is a quarter.


Calphalon
Oranges Used: 5

Graham had 5 oranges and cut them into quarters.

How many orange wedges did Graham have?

3-Act Tasks

Act 1:

- Real world problem or scenario presented
- What do you notice? What do you wonder?
- Make estimates

Act 2:

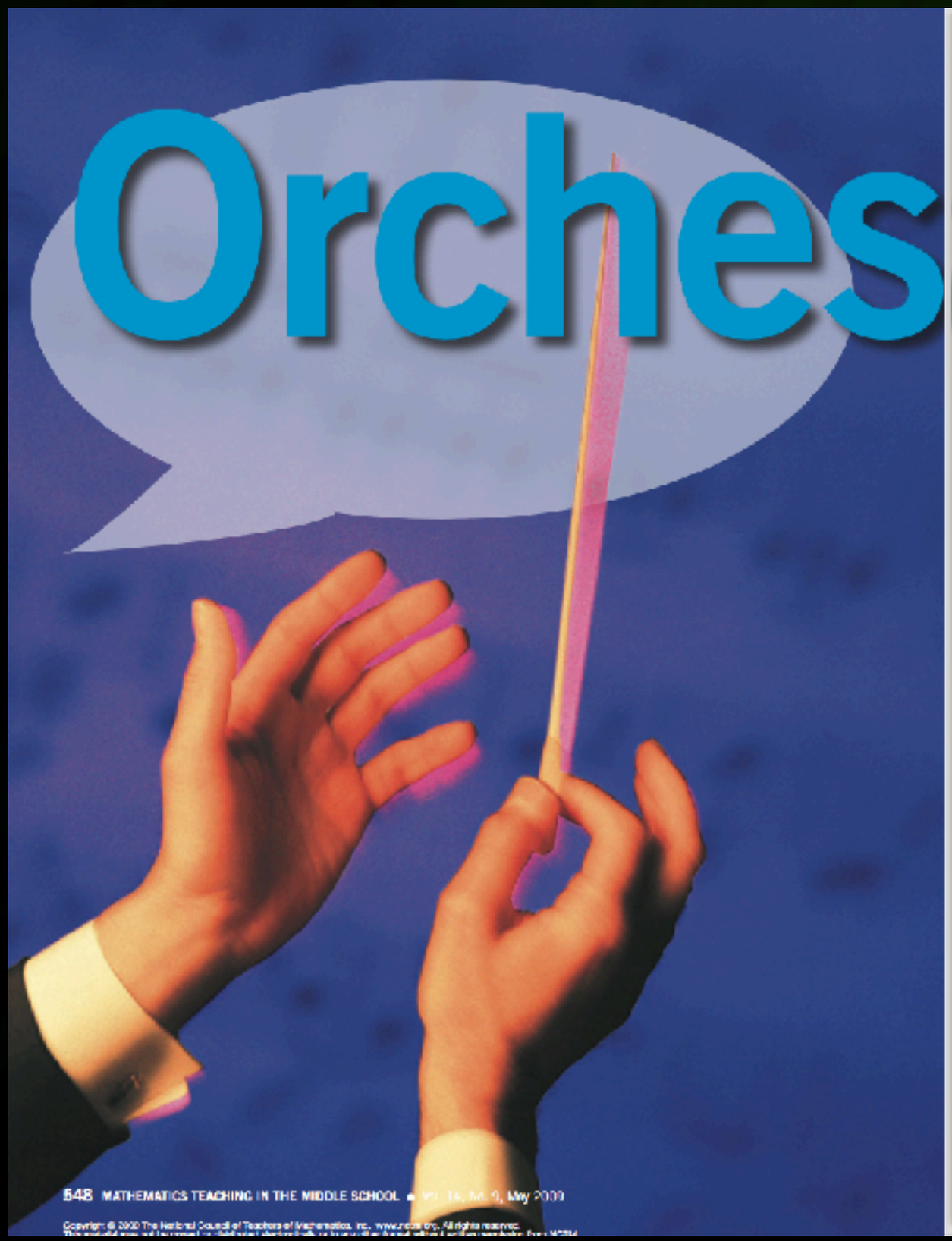
- Identify missing variables and missing variables to solve
- Define solution path using variables

Act 3:

- Solve and interpret results of the solution
- Validate answer

Most asked questions:

- How often should we use 3-Act Tasks? How do they fit into the scope of a unit?
- How long does one task usually take?
- What if we don’t have the time?



Orchestrating Discussions

Five practices constitute a model for effectively using student responses in whole-class discussions that can potentially make teaching with high-level tasks more manageable for teachers.

Margaret S. Smith, Elizabeth K. Hughes, Randi A. Engle, and Mary Kay Stein

Margaret S. Smith, pegsu@pitt.edu, is an associate professor of mathematics education at the University of Pittsburgh. Over the past decade, she has been developing research-based materials for use in the professional development of mathematics teachers and studying what teachers learn from the professional development in which they engage. **Elizabeth K. Hughes**, elizabeth.hughes@uni.edu, recently finished her doctorate in mathematics education at the University of Pittsburgh. Her areas of interest include preservice secondary mathematics teacher education and the use of practice-based materials in developing teachers' understanding of what it means to teach and learn mathematics. **Randi A. Engle**, rengle@berkeley.edu, is an assistant professor of mathematics education and the social context of learning at the University of California, Berkeley. She is interested in developing practical theories for how mathematics teachers can create discussion-based learning environments that promote strong student engagement, learning, and transfer. **Mary Kay Stein**, mkstein@pitt.edu, is a professor of learning sciences and policy and the director of the Learning Policy Center at the University of Pittsburgh. Her research focuses on instructional practice and the organizational and policy conditions that shape it.

Discussions that focus on cognitively challenging mathematical tasks, namely, those that promote thinking, reasoning, and problem solving, are a primary mechanism for promoting conceptual understanding of mathematics (Hiebert and Wearne 1993; Michaels, O'Connor, and Resnick forthcoming). Such discussions give students opportunities to share ideas and clarify understandings, develop convincing arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things from other perspectives (NCTM 2000).

Although discussions about high-level tasks provide important

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The **5** practices are:

1. **Anticipating** student responses to challenging mathematical tasks;
2. **Monitoring** students' work on and engagement with the tasks;
3. **Selecting** particular students to present their mathematical work;
4. **Sequencing** the student responses that will be displayed in a specific order and;
5. C**onnecting** different students' responses and connecting the responses to key mathematical ideas.

MTMS: Vol. 14, No. 9, May 2009-5 Practices for Orchestrating Productive Mathematics Discussions

Task Planning Page

Learning Target:		
Questions and Look-Fors:		
Strategy	Who and What	Order
Notes:		

Anticipating → Monitoring → Selecting → Sequencing → Connecting

5 oranges

Each wedge is a quarter

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[redacted]

S1

Estimate: 20

Draw a picture to show your thinking:

$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1$

$\frac{4}{4} + \frac{8}{4} + \frac{12}{4} + \frac{16}{4} + \frac{20}{4}$

$\frac{20}{4}$ = wedges

Use numbers to show your thinking:

Answer: 5

[redacted]

S2

1. What did you notice? Oranges glass bowl
2. What do you wonder? how many of oranges?
3. Main Question how many wedges?
4. Make an estimate. low estimate — high estimate. Place an "X" to represent your estimate on the number line.
5. What information do you need? How big is the peiss
6. Construct a viable argument or share a reflection. Answer: 20

5 oranges + 5 oranges

4 8 12 16 20

[redacted]

S3

1. What did you notice? Bowl oranges oranges wedges
2. What do you wonder? how many whole oranges?
3. Main Question How many wedges?
4. Make an estimate. low estimate — high estimate. Place an "X" to represent your estimate on the number line.
5. What information do you need? What are the sizes of the wedges?
6. Construct a viable argument or share a reflection. Answer: 20

5 oranges

Oranges cut in 4 wedges

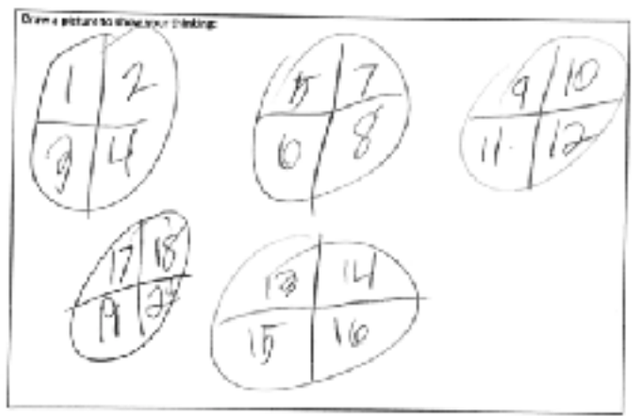
5 × 4 = 20

[redacted]

S4

Estimate: 6

Draw a picture to show your thinking:



Use numbers to show your thinking:

4+4+4+4+4=20

Answer: 20

[redacted]

S5

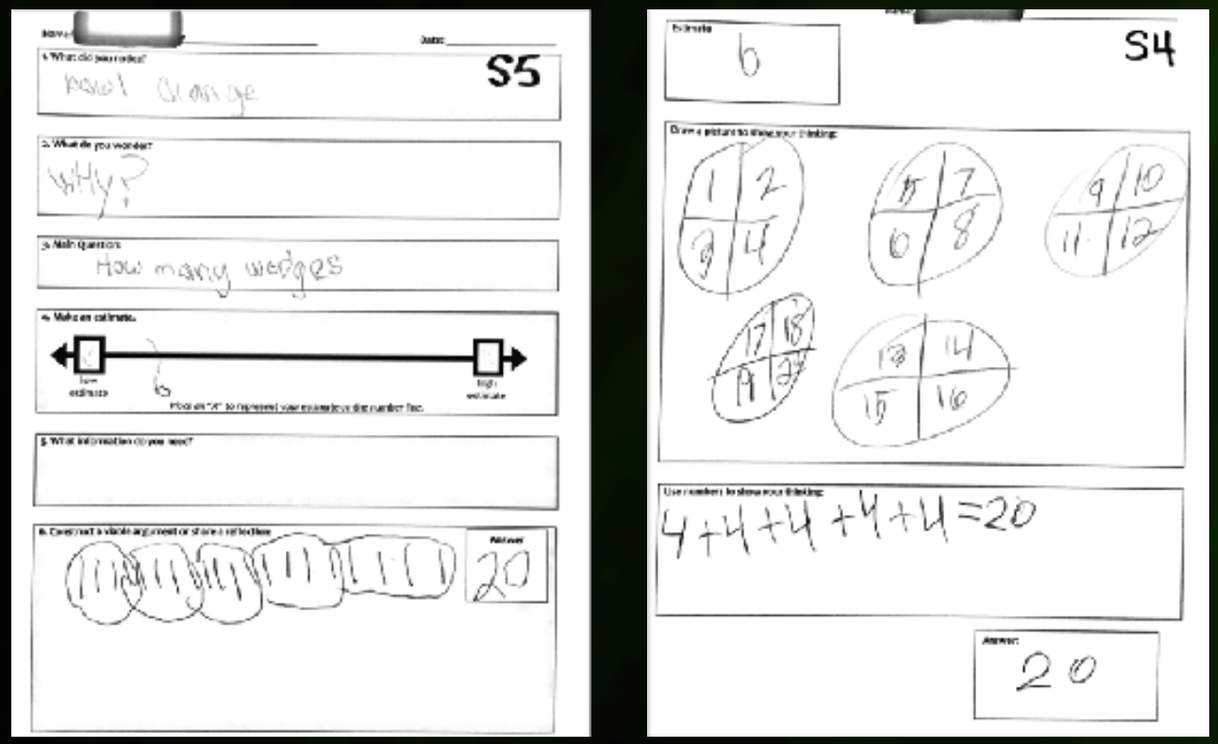
1. What do you notice? bowl orange
2. What do you wonder? why?
3. Main Question How many wedges
4. Make an estimate. low estimate — high estimate. Place an "X" to represent your estimate on the number line. 6
5. What information do you need?
6. Construct a viable argument or share a reflection. Answer: 20

[redacted]

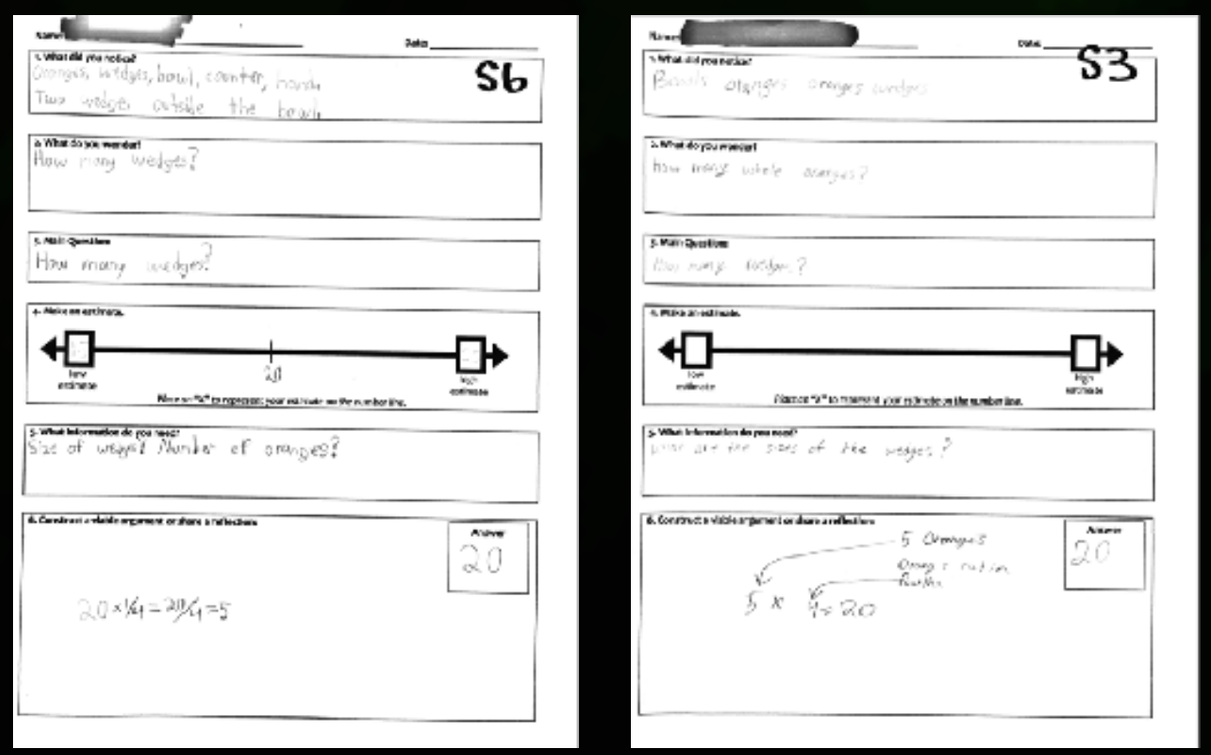
S6

1. What did you notice? Oranges, wedges, bowl, counter, hands. Two wedges outside the bowl.
2. What do you wonder? How many wedges?
3. Main Question How many wedges?
4. Make an estimate. low estimate — high estimate. Place an "X" to represent your estimate on the number line. 20
5. What information do you need? Size of wedges? Number of oranges?
6. Construct a viable argument or share a reflection. Answer: 20

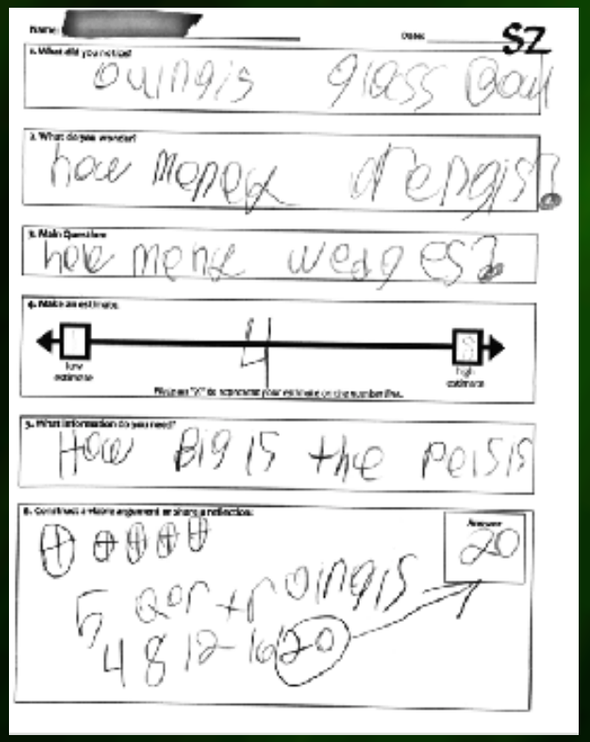
$20 \times \frac{1}{4} = \frac{20}{4} = 5$

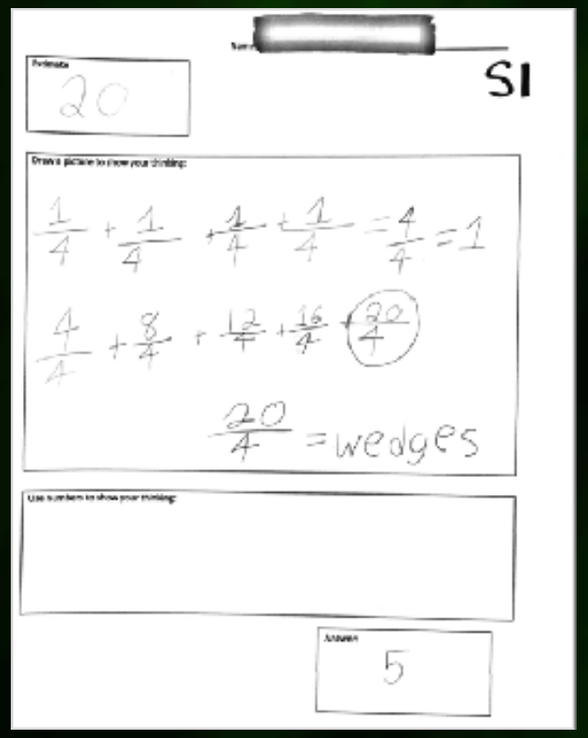


Group 1



Group 2

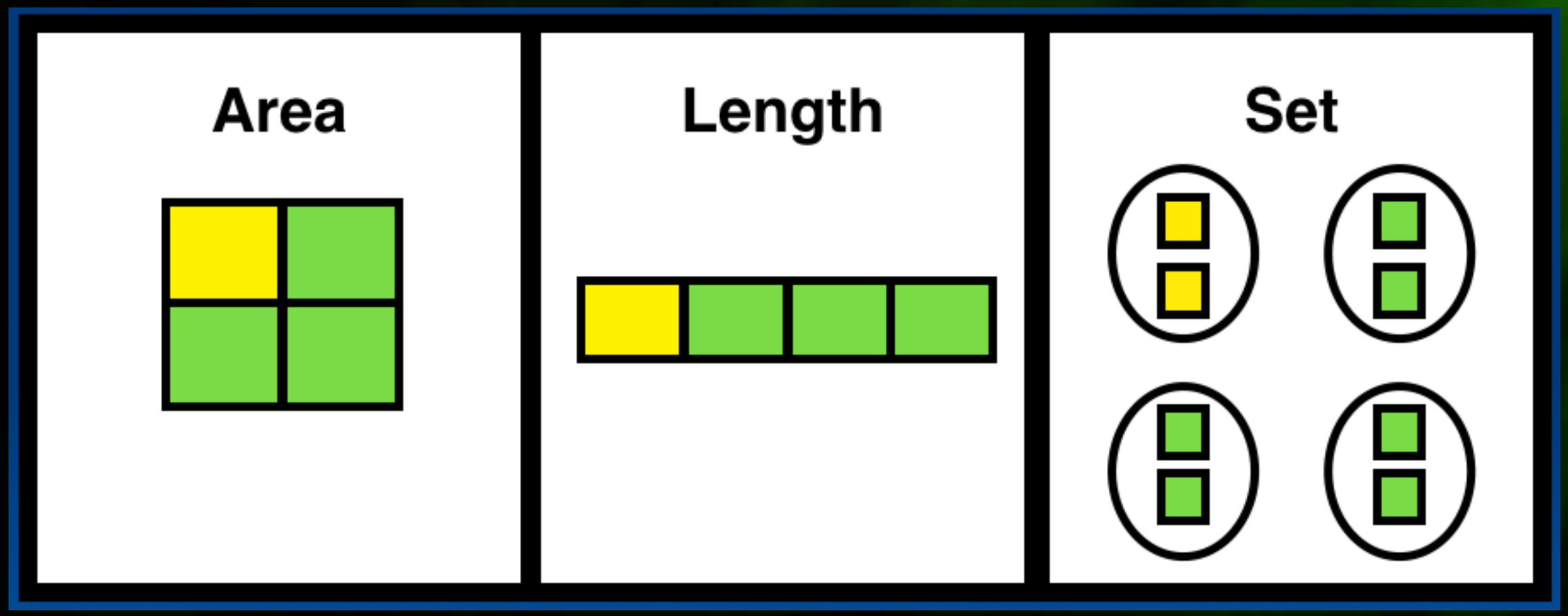




Group 3

Unit Fractions

Representation of a Fraction



unit fraction $\frac{1}{a}$

Say this fraction

$$\frac{3}{4}$$

Say this fraction

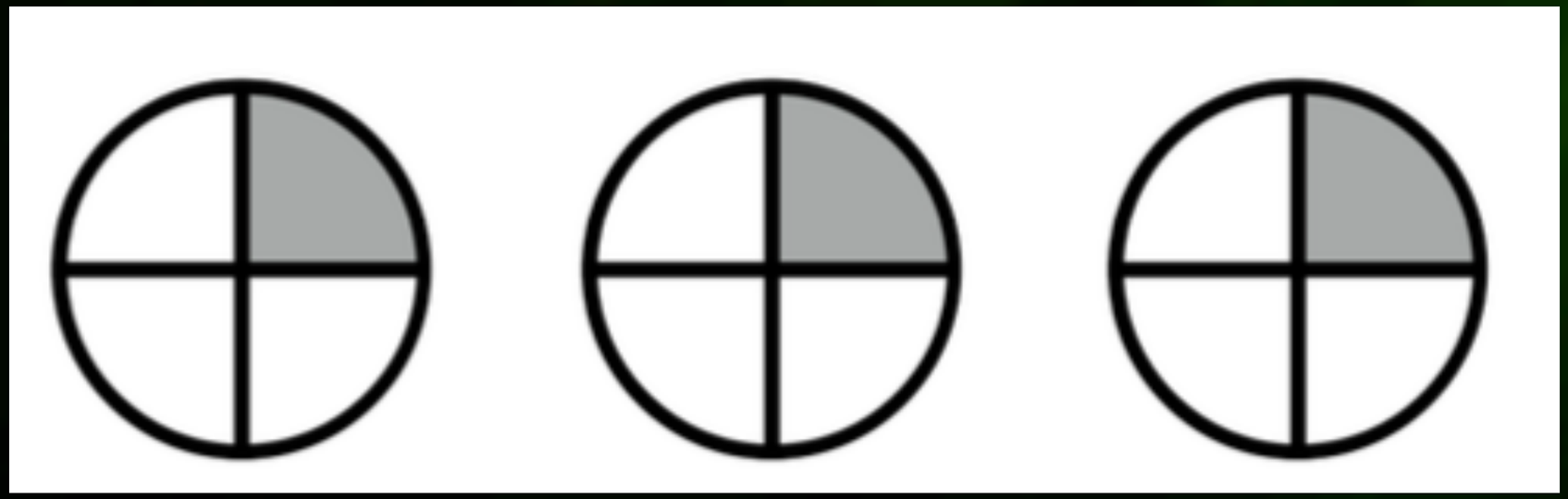
$$\frac{3}{4}$$

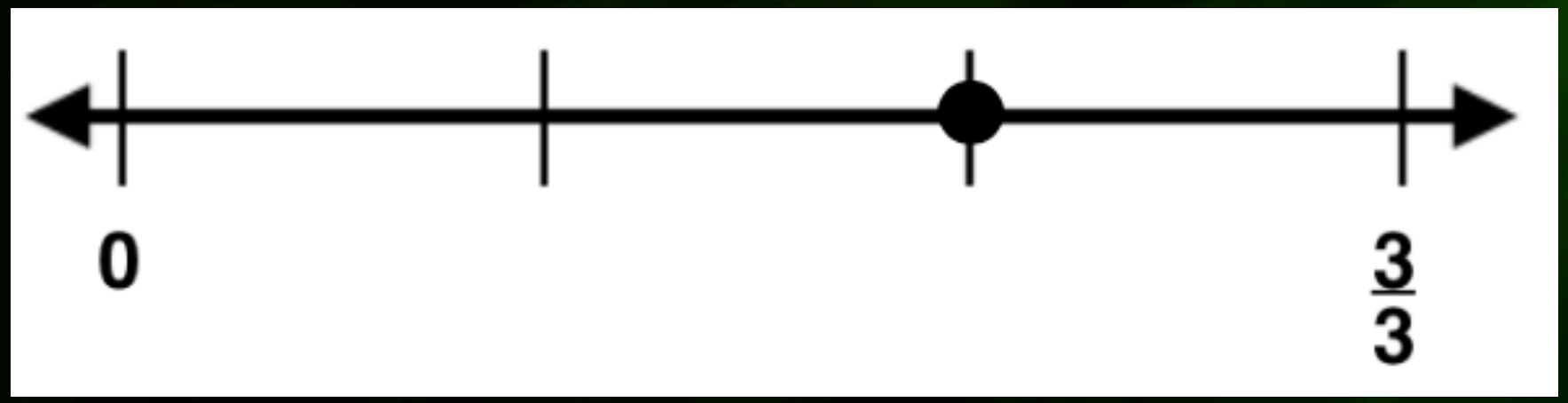
three one-fourths

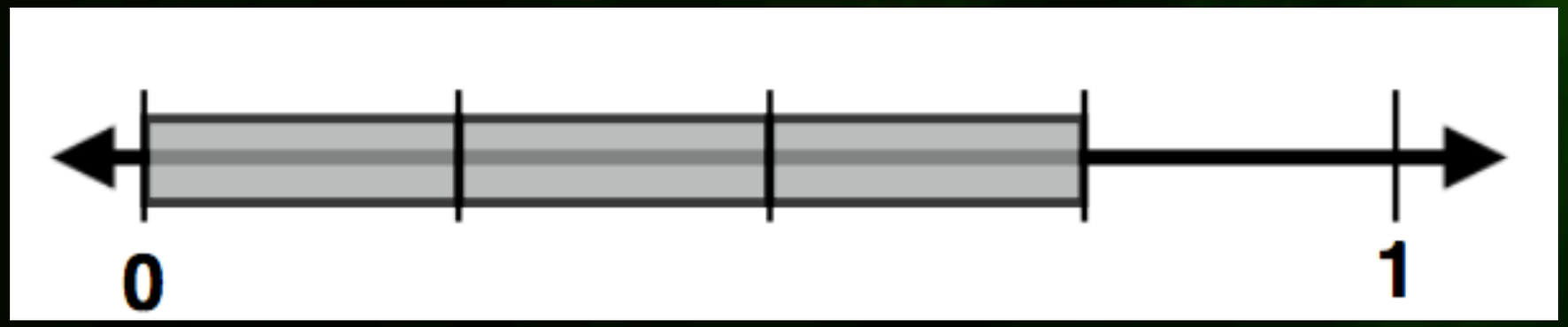
$$3 = 1 + 1 + 1$$

$$3 = 1 + 1 + 1$$

$$\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$




0
3/3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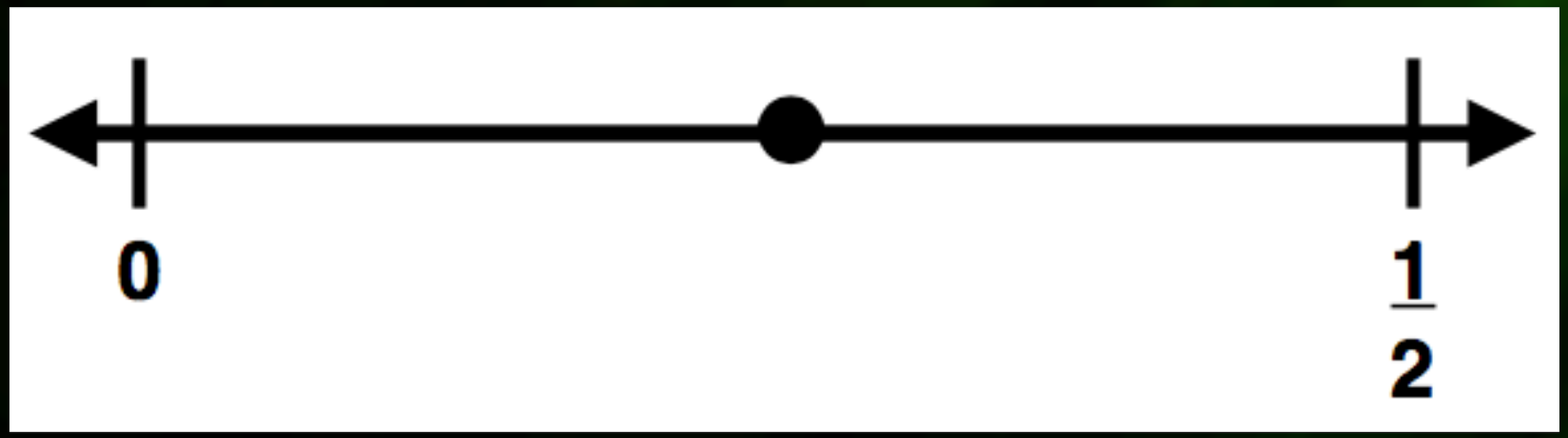
1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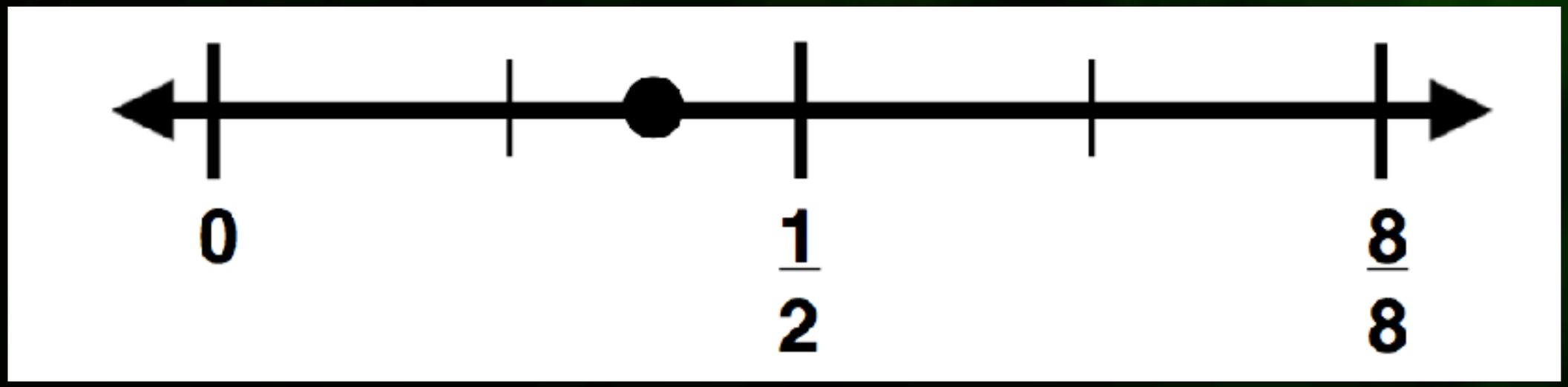
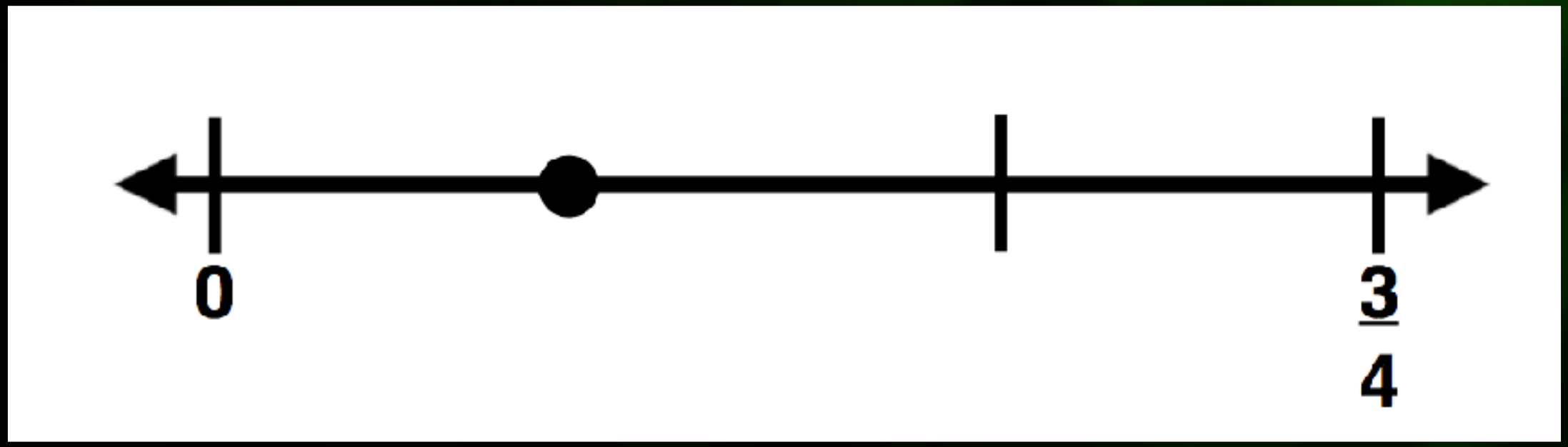
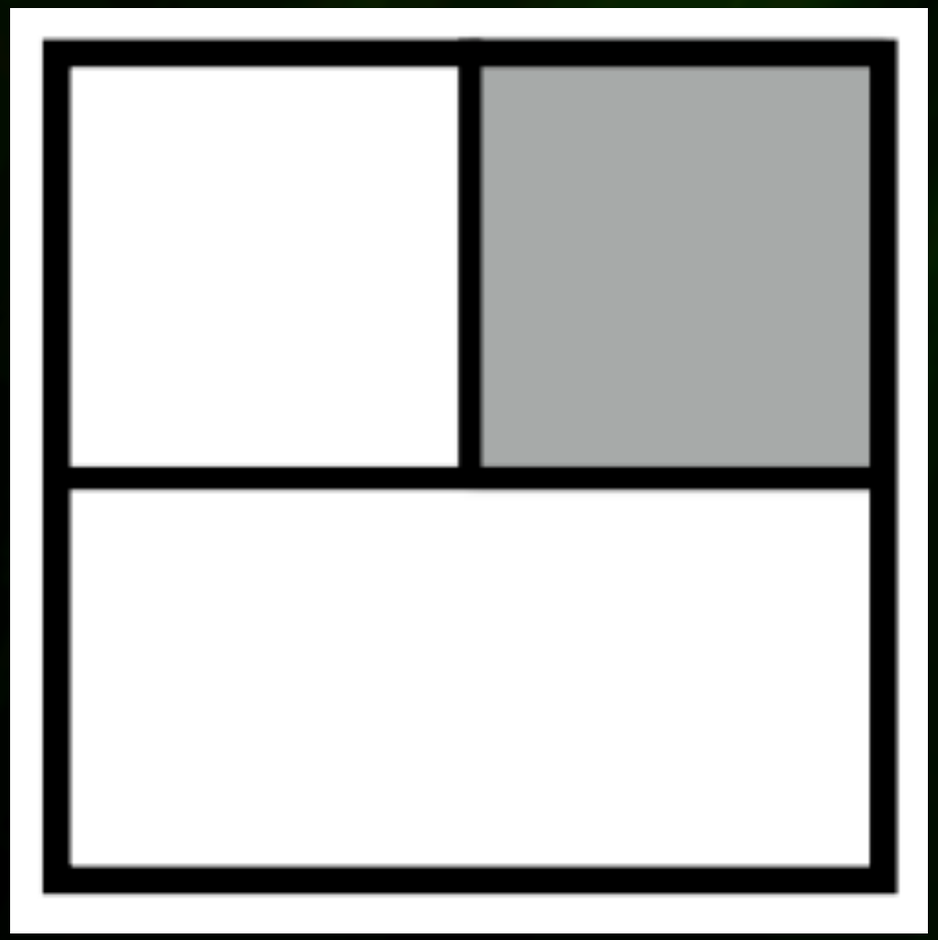
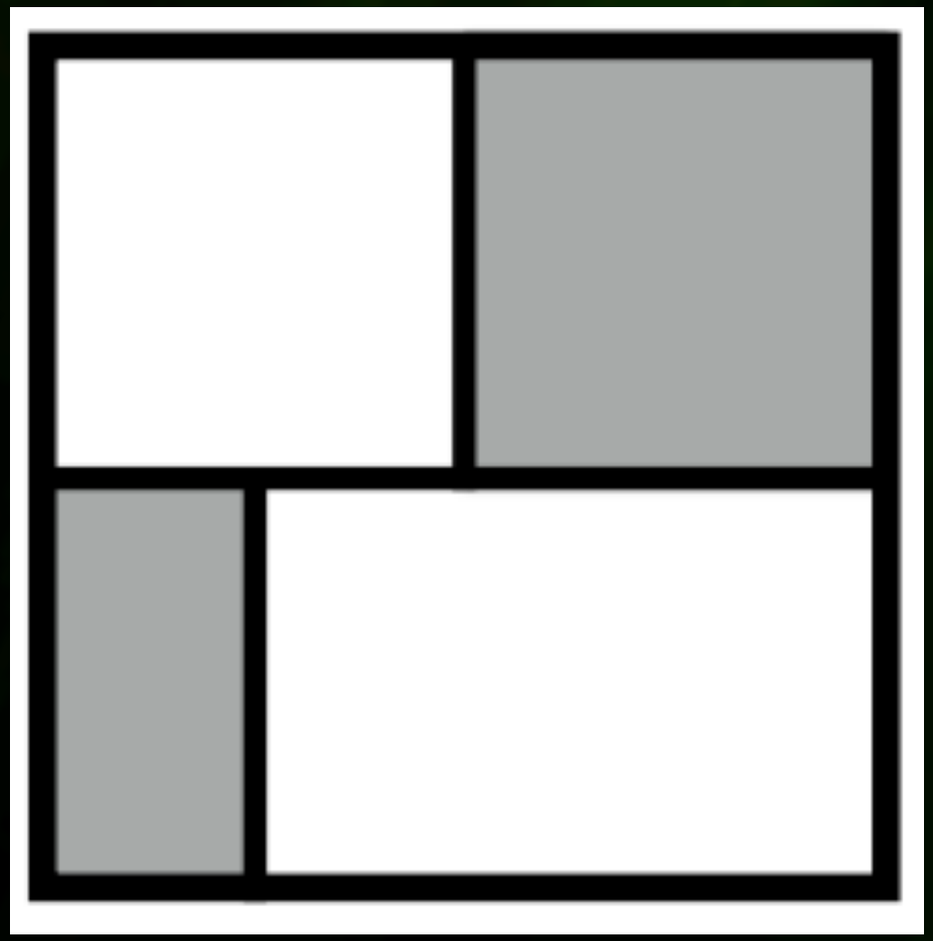
1/2


0
1/2
8/8

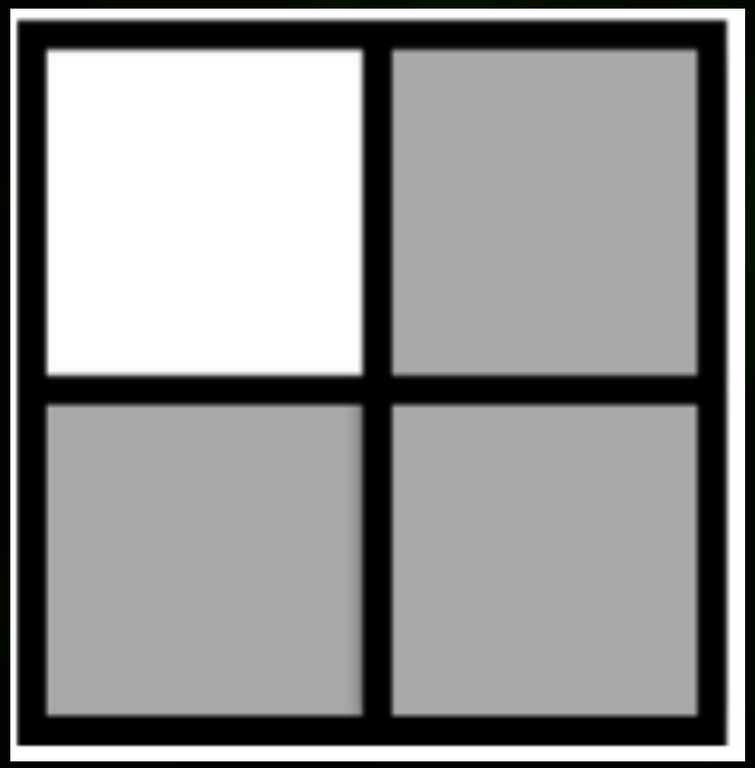

0
3
4





What's the Sum?

What's the Sum?



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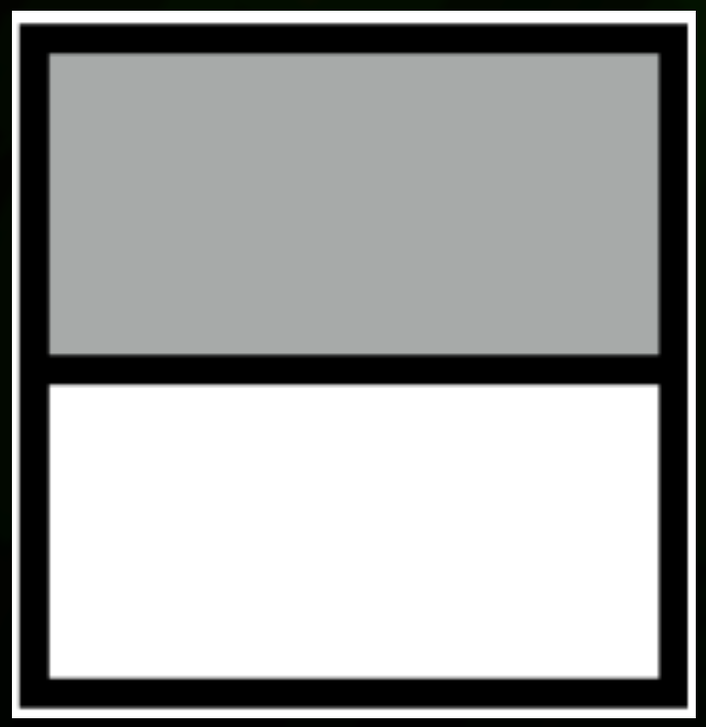
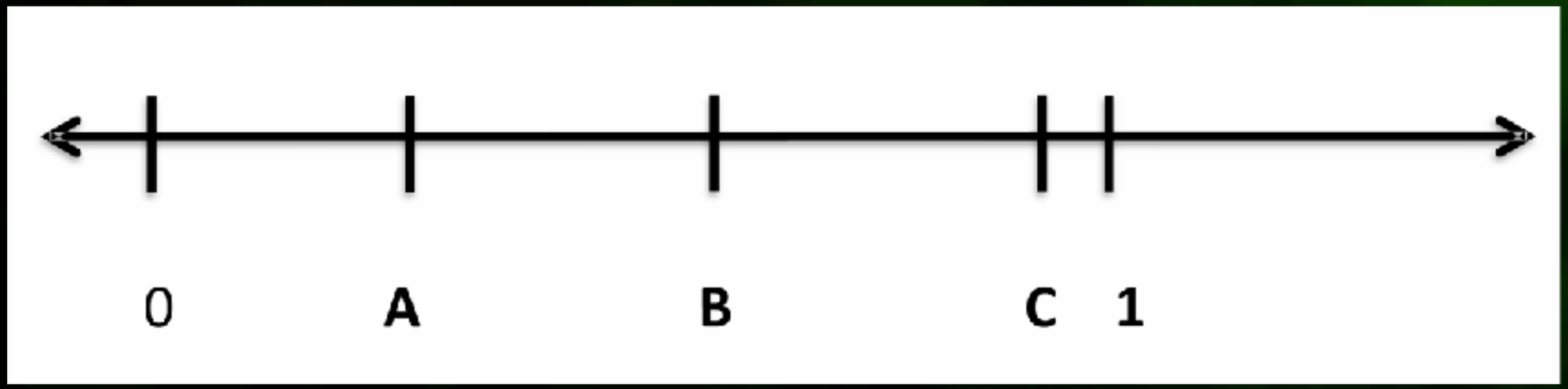



0
A
B
C
1


½
A
2
B

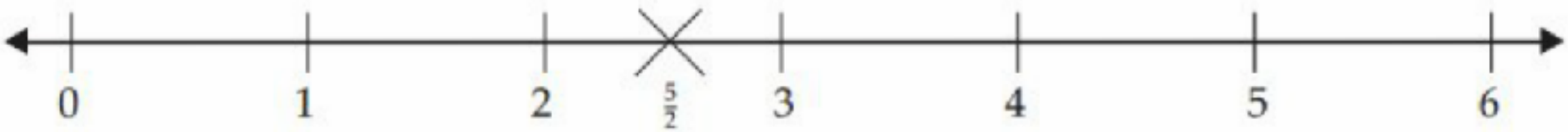
?

$\frac{1}{2}$ $2\frac{1}{4}$

Dotty Pairs

Dotty Pairs Game

The students play in pairs. One student takes dots, the other takes crosses. Place the cards (cards 1–6, two lots, see Material Master 4-1) face down in a pile. The players take turns turning over two cards. The numbers are used to form a fraction, e.g., 2 and 5 are turned over, so $\frac{5}{2}$ or $\frac{2}{5}$ can be made. One fraction is chosen, made with the fraction pieces, if necessary, and marked on a 0–6 number line with the player's identifying mark (dot or cross).



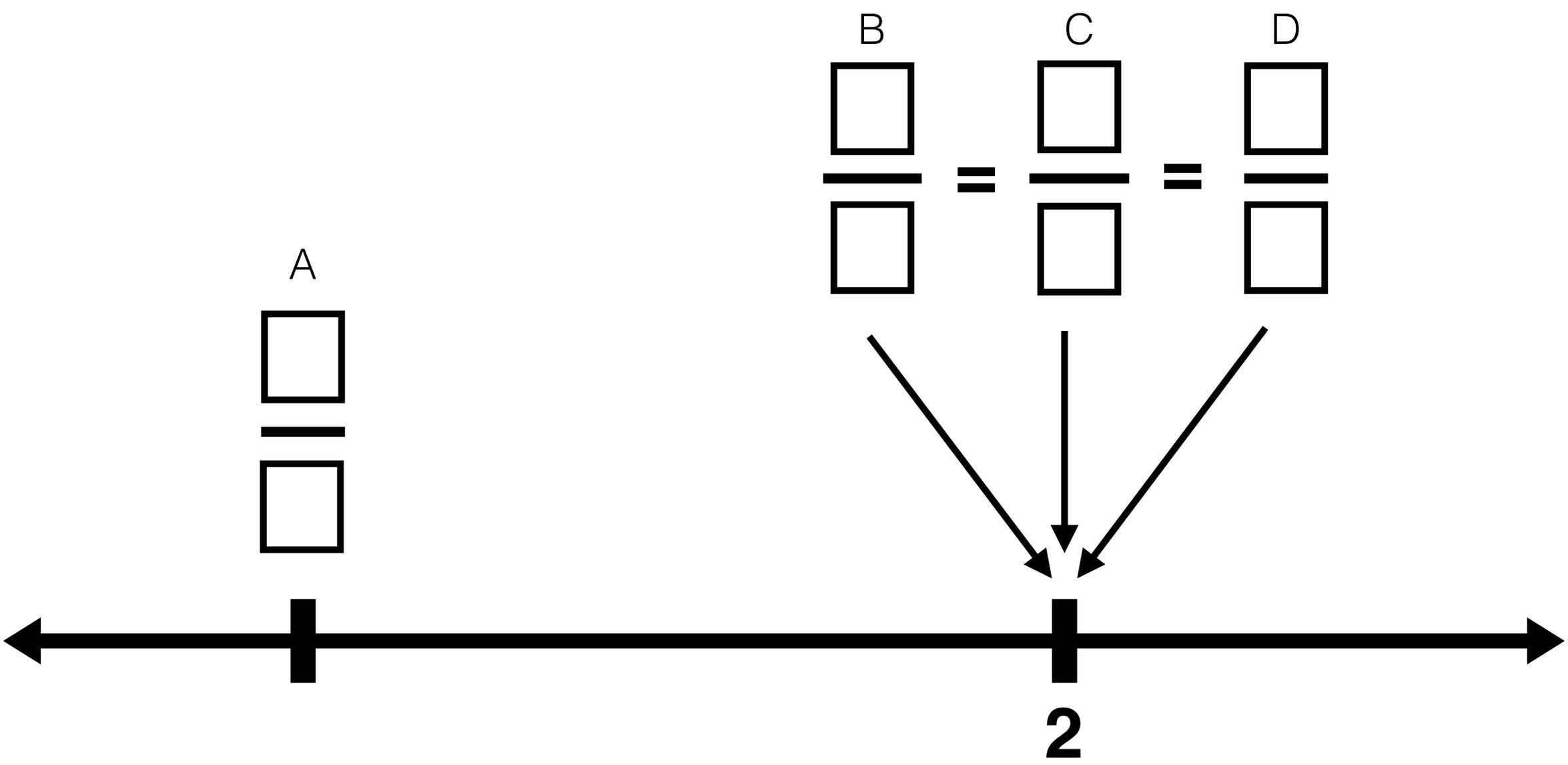
Players take turns. The aim of the game is to get three of their marks uninterrupted by their opponent's marks on the number line. If a player chooses a fraction that is equivalent to a mark that is already there, they miss that turn.

NB: A fraction such as $\frac{4}{1}$ can be made using the cards. Students may not be familiar with fractions in this form and the meaning of the numerator and denominator will need to be explored with the fraction circles.

G → random dice roller

Open Middle

Directions: Using the whole numbers 1-9 no more than one time each, create and place 4 fractions on the number line in the correct order. A is less than 2. Fractions B, C, and D equal 2.

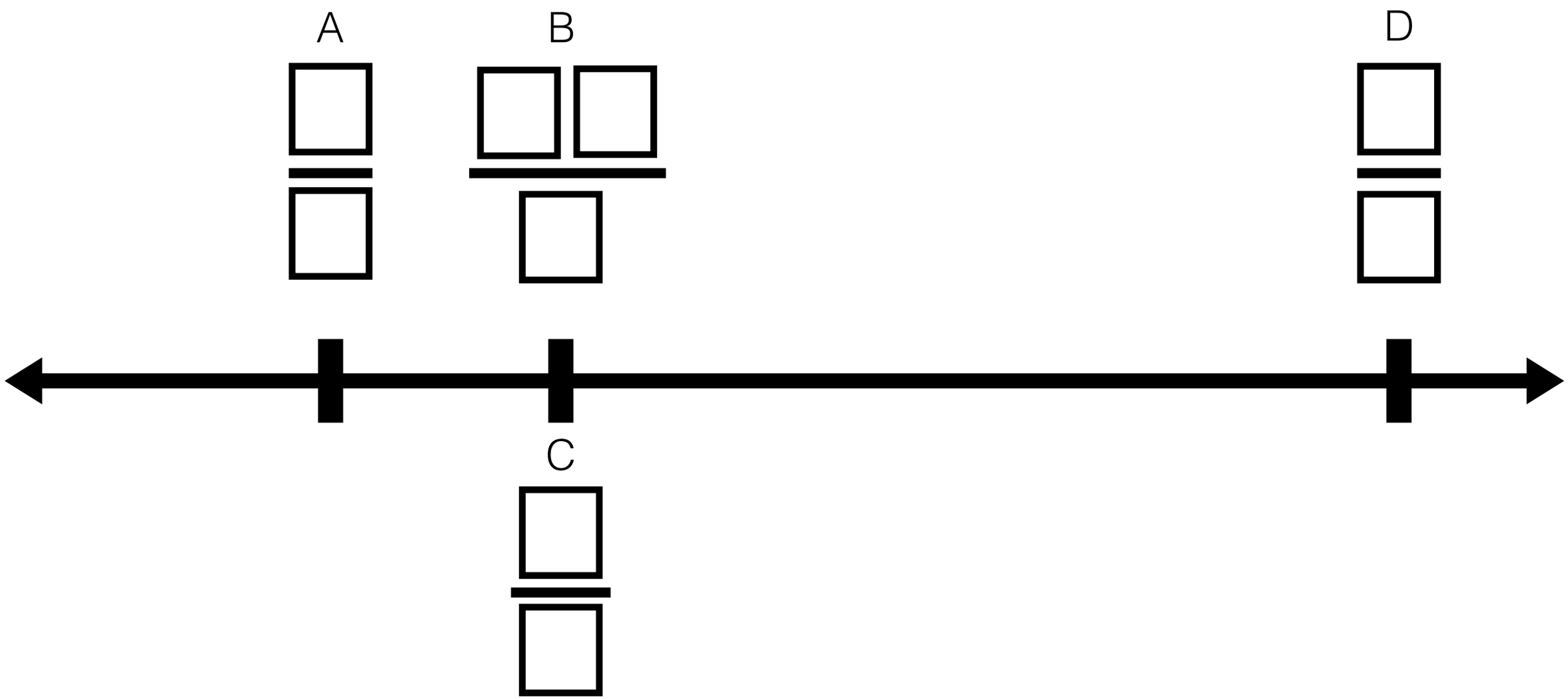


@gfletchy

Open Middle

CCSS.MATH.CONTENT.4.NF.A.2

Directions: Using the whole numbers 1-9 once each, create and place 4 fractions *greater than 1* on the number line in the correct order. *(fractions B & C are equal)*



@gfletchy

$\frac{1}{20}$	$\frac{20}{25}$
$\frac{2}{3}$	$\frac{5}{4}$

http://wodb.ca/numbers.html

Equivalent Fractions

Equal Fraction

$$\frac{2}{3} = \frac{\blacksquare}{\blacksquare} \qquad \frac{3}{4} = \frac{\blacksquare}{\blacksquare}$$

$$\frac{2}{6} = \frac{\blacksquare}{\blacksquare}$$

Equal Fraction

$$\frac{2}{3} = \frac{5}{6} \qquad \frac{3}{4} = \frac{7}{8}$$

$$\frac{2}{6} = \frac{5}{9}$$

Name _____ Date _____

1. Draw a rectangular fraction model to find the sum. Simplify your answer, if possible.

a. $\frac{1}{4}+\frac{1}{3}=$

b. $\frac{1}{4}+\frac{1}{5}=$

It is possible to over-emphasize the importance of simplifying fractions in this way. There is no mathematical reason why fractions must be written in simplified form, although it may be convenient to do so in some cases.

What about “the test”

$\frac{3}{6} + \frac{1}{6}$ is equal to which of the following?

a. $\frac{4}{12}$

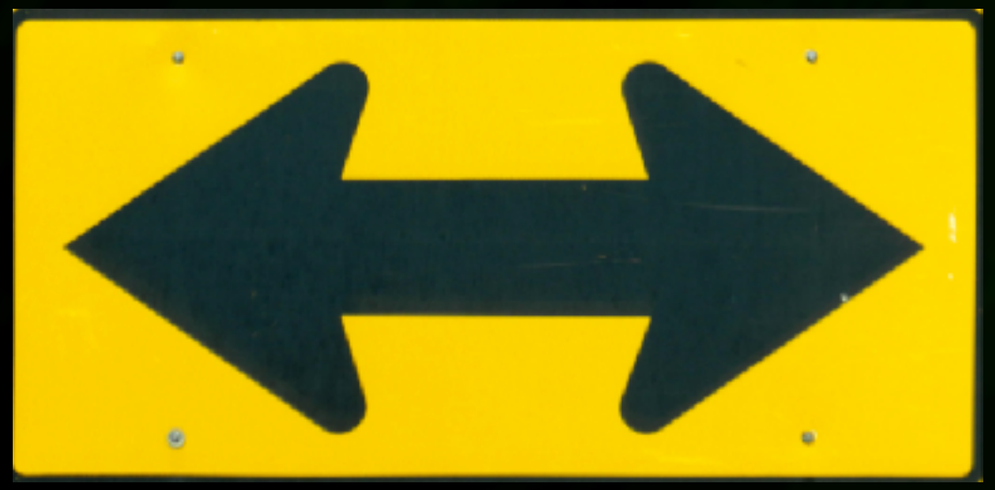
b. $\frac{8}{12}$

c. $\frac{3}{6}$

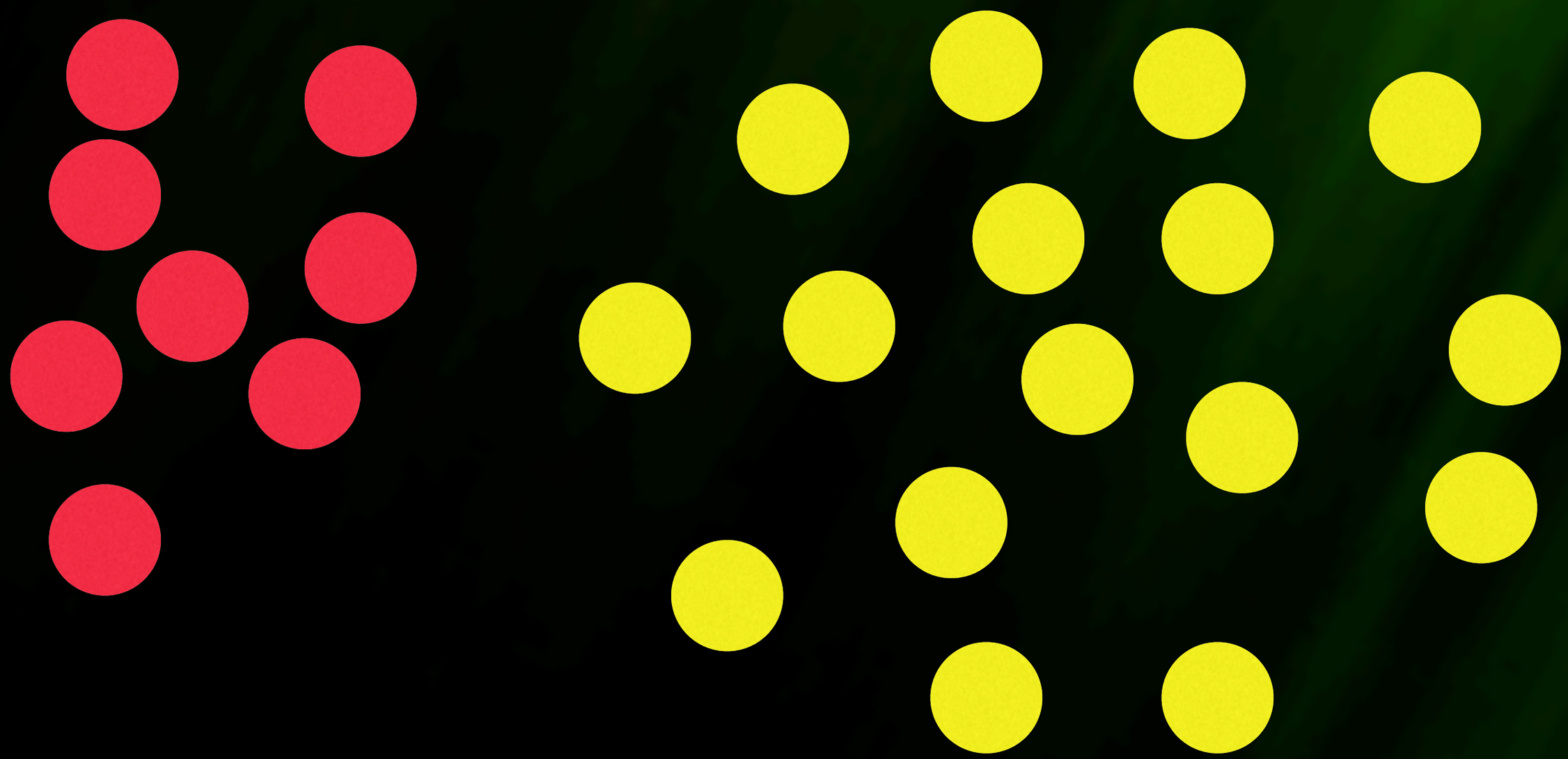
d. None of the above

Simplifying

Equivalence

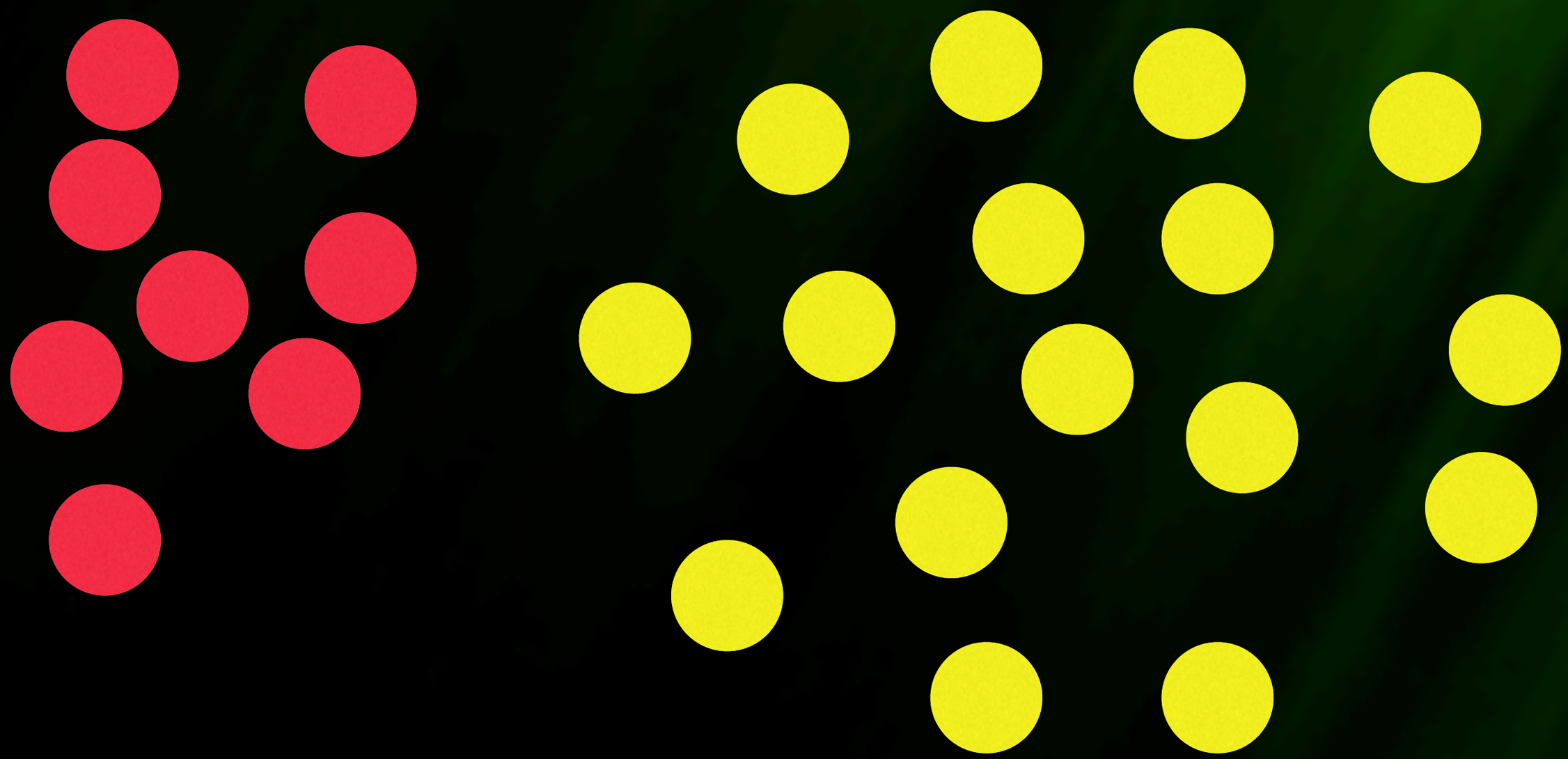


Get 24 counters.
8 red and 16 yellow
Using all the counters, how many different fractions can you represent?



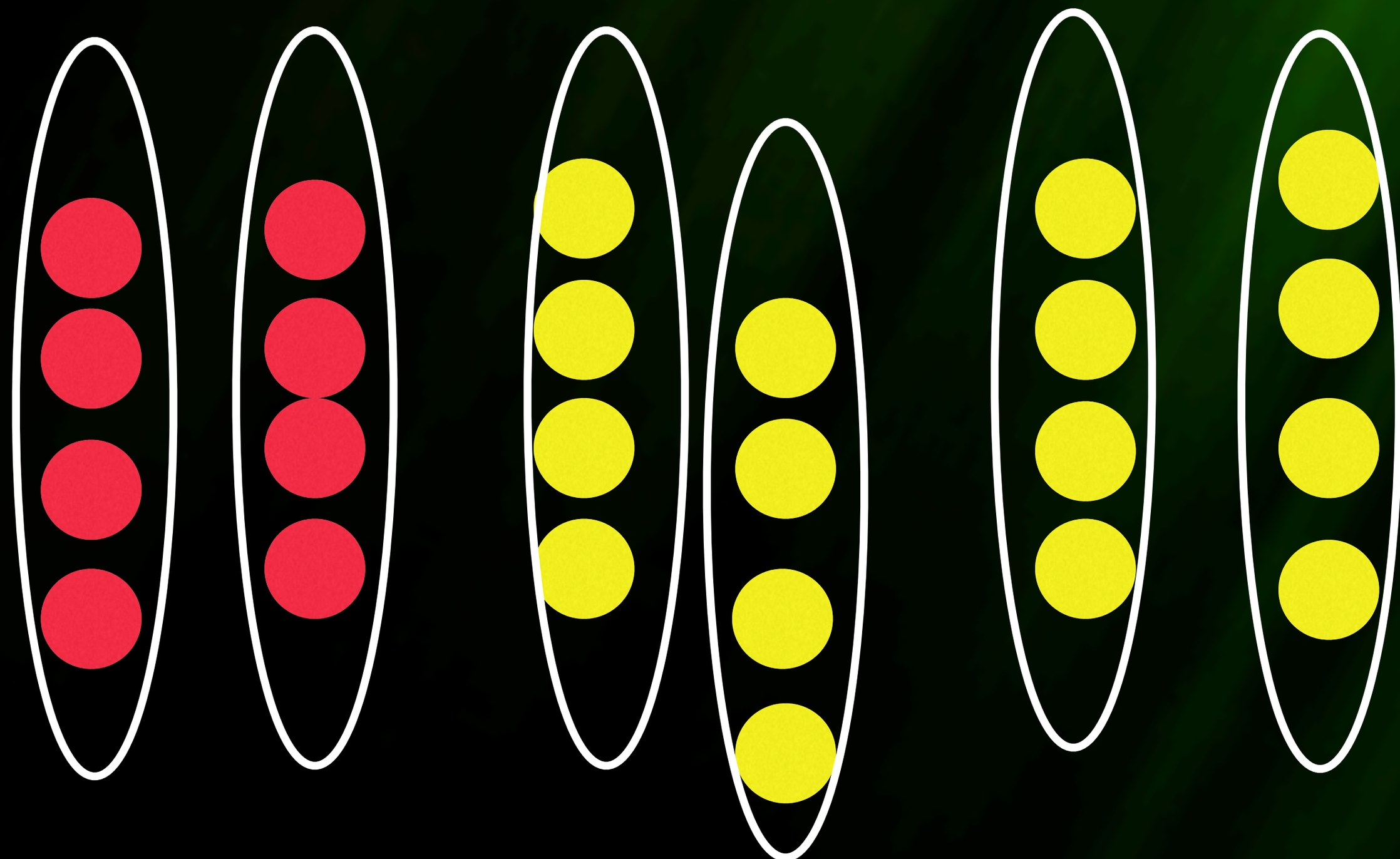
16/24 are yellow

8/24 are red



4/6 are yellow

2/6 are red



2/3 are yellow

1/3 are red

8/12 are yellow

4/12 are red

Comparing Fractions

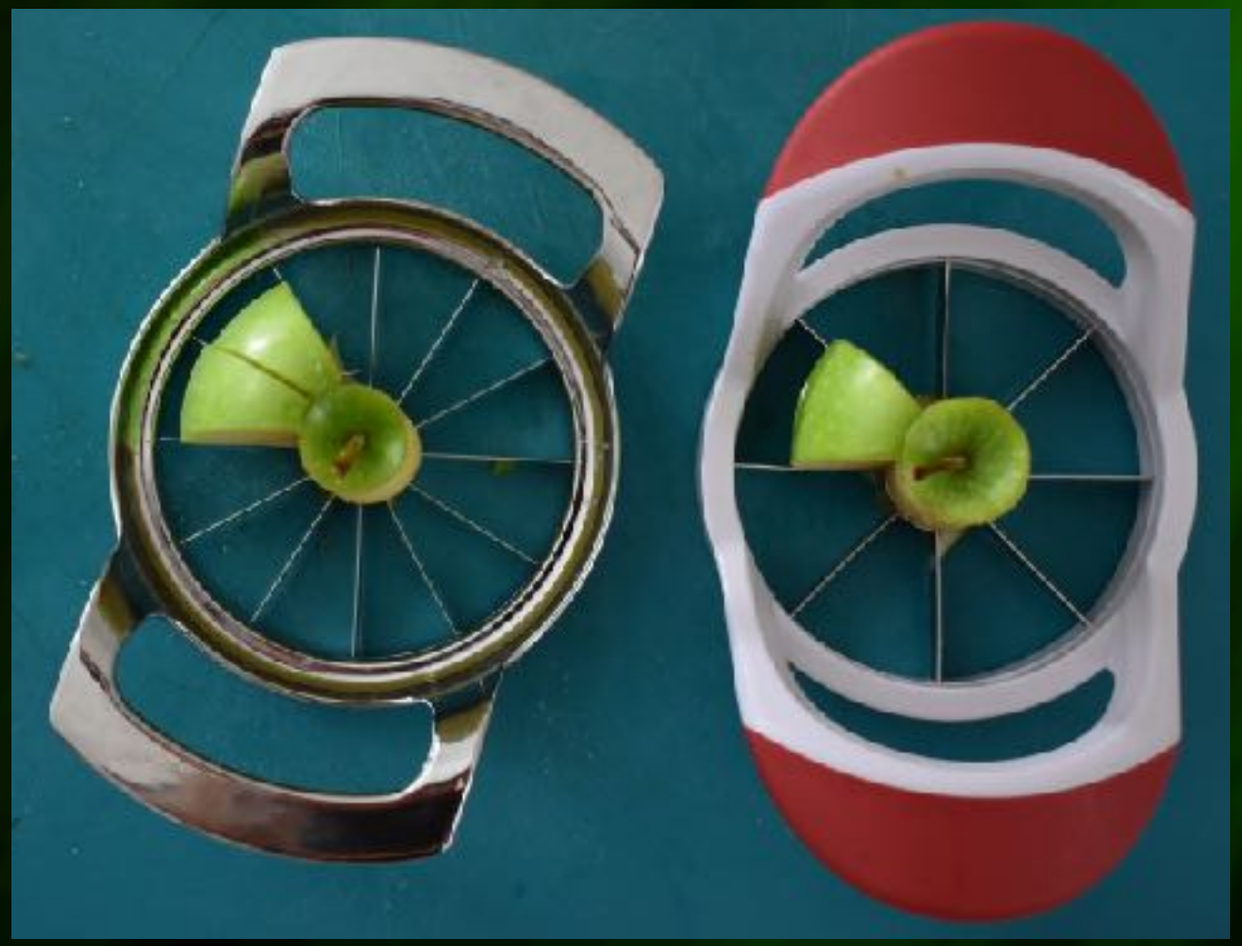
twelfths

Pause

||

eighths

Apple Eat Off Act-3



Big sister ate $\frac{10}{12}$ of an apple and little sister ate $\frac{7}{8}$ of an apple. Which sister ate more apple?

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S1

6. Construct a viable argument or share a reflection:

$\frac{7}{8}$ $\frac{10}{12}$

$\frac{1}{8}$ is a smaller pice left over

$\frac{1}{8}$

$\frac{2}{12} = \frac{1}{6}$ $\frac{7}{8} > \frac{10}{12}$

S2

6. Construct a viable argument or share a reflection:

Answer

$\frac{10}{12}$ 2 pieces left over

$\frac{1}{8}$ 1 piece leftover more

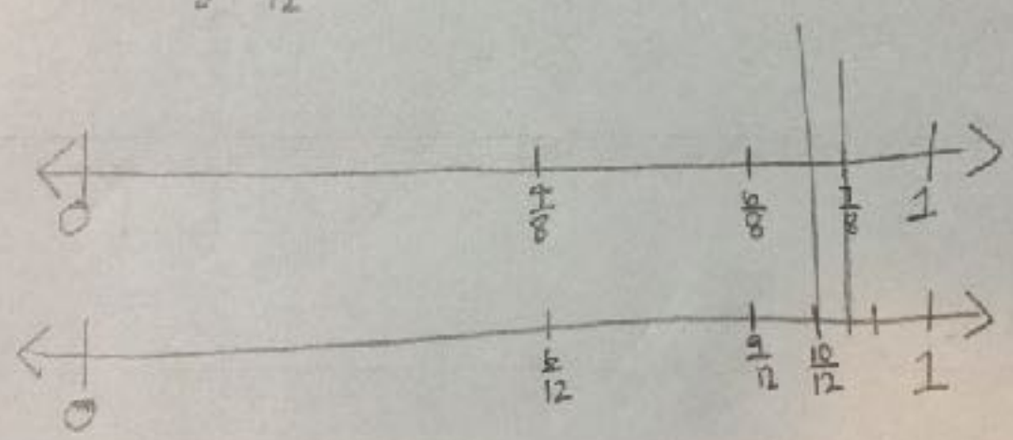
$\frac{10}{12} > \frac{1}{8}$

Pause

S3

6. Construct a viable argument or share a reflection:

$\frac{7}{8} > \frac{10}{12}$



S4

6. Construct a viable argument or share a reflection:

$\frac{10}{12}$ ×2 ×2 $\frac{20}{24}$

$\frac{7}{8}$ ×3 ×3 $\frac{21}{24}$

$\frac{20}{24} < \frac{21}{24}$

$\frac{10}{12} < \frac{7}{8}$

S5

6. Construct a viable argument or share a reflection:

Answer

$\frac{10}{12} > \frac{7}{10}$ ten is more than seven

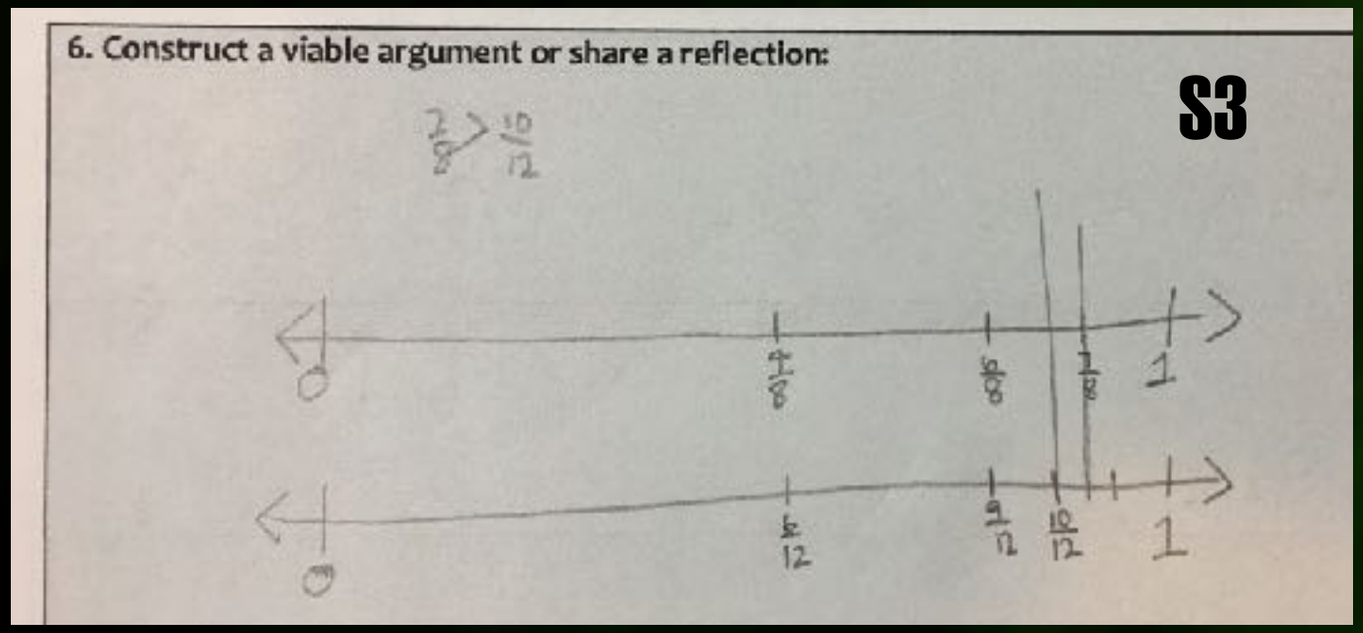
S6

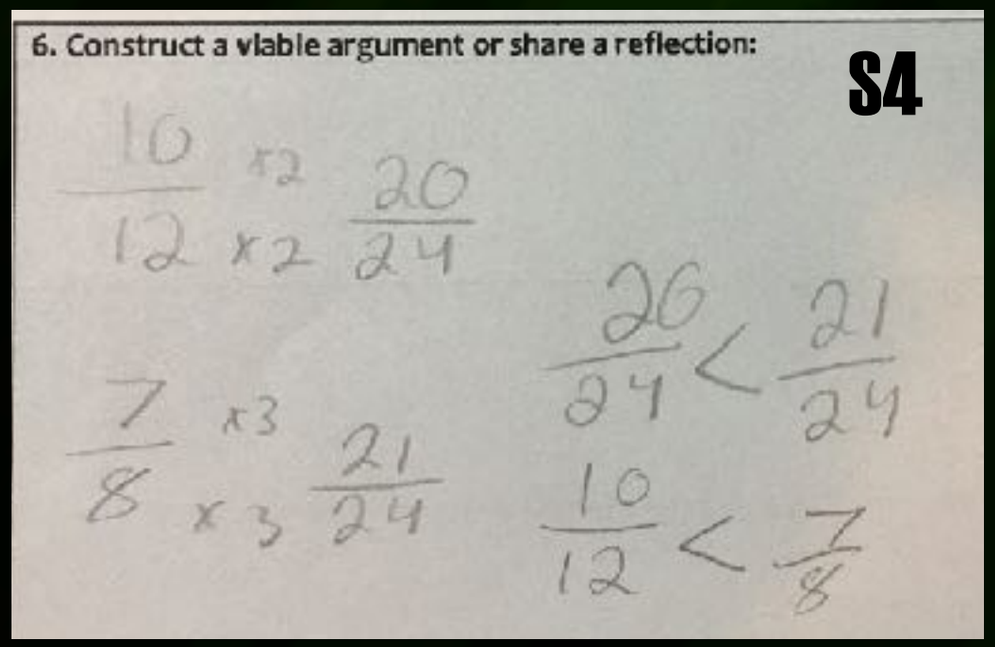
6. Construct a viable argument or share a reflection:

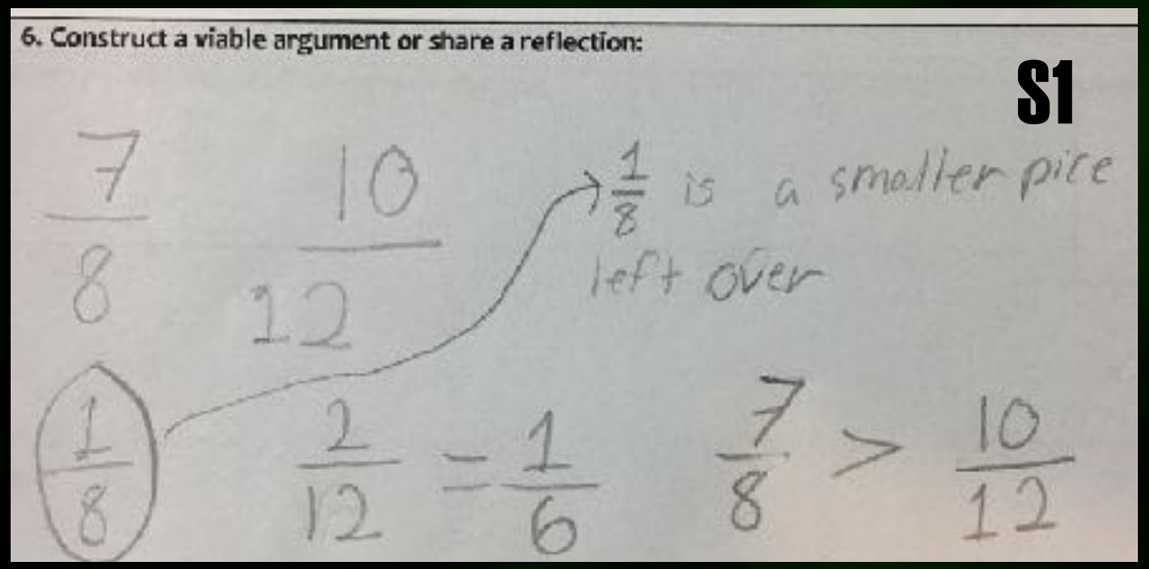
80 < 84

10 7

12 8


6. Construct a viable argument or share a reflection:
S3
7/8 > 10/12
0
4/8
6/8
7/8
1
0
5/12
9/12
10/12
1


6. Construct a viable argument or share a reflection:
S4
10/12 x2 20/24
7/8 x3 21/24
20/24 < 21/24
10/12 < 7/8


6. Construct a viable argument or share a reflection:
S1
7/8
10/12
1/8 is a smaller pice left over
1/8
2/12 = 1/6
7/8 > 10/12

It Takes 3 to Prove it to Me

$\frac{1}{4}$ $\frac{3}{4}$

Common Denominator

Unit Fraction Understanding

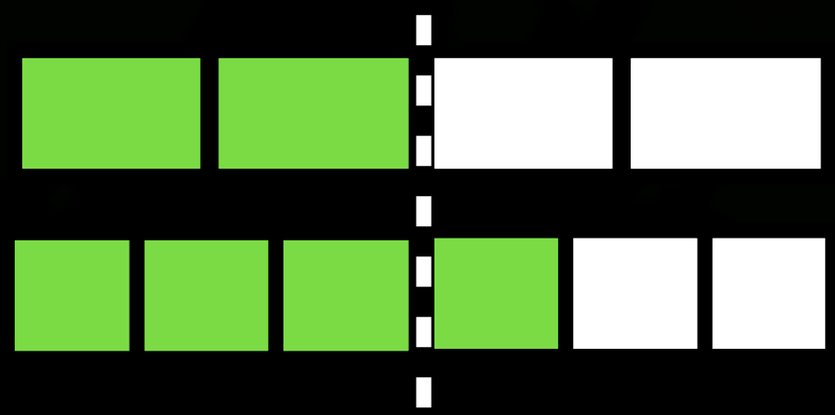
$\frac{5}{6}$ $\frac{3}{4}$

Missing Parts

Unit Fraction Understanding

$\frac{2}{4}$ $\frac{4}{6}$

Benchmark



$\frac{2}{3}$ $\frac{2}{5}$

Common Numerator

Unit Fraction Understanding

$$\frac{8}{11} \qquad \frac{4}{7}$$

$$\frac{8}{11} \qquad \frac{8}{14}$$

Comparing Fractions

CCSS.MATH.CONTENT.3.NF.A.3.D
Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

CCSS.MATH.CONTENT.4.NF.A.2
Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Making Sense Series

The Progression of Fractions
Meaning, Equivalence, & Comparison

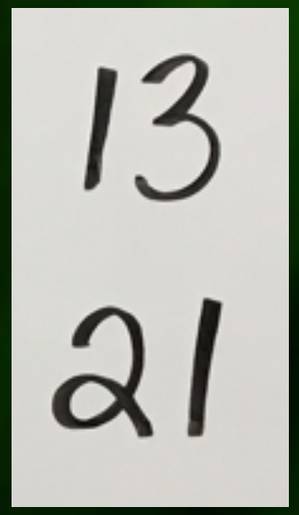
created by Graham Fletcher
@gfletchy
www.gfletchy.com

Joe had some playing cards in his bag. Ashley gave him 13 more cards. Joe now has 21 cards. How many cards did Joe have in his bag?

13

21

You little plucker!



John SanGovanni, NCTM '16

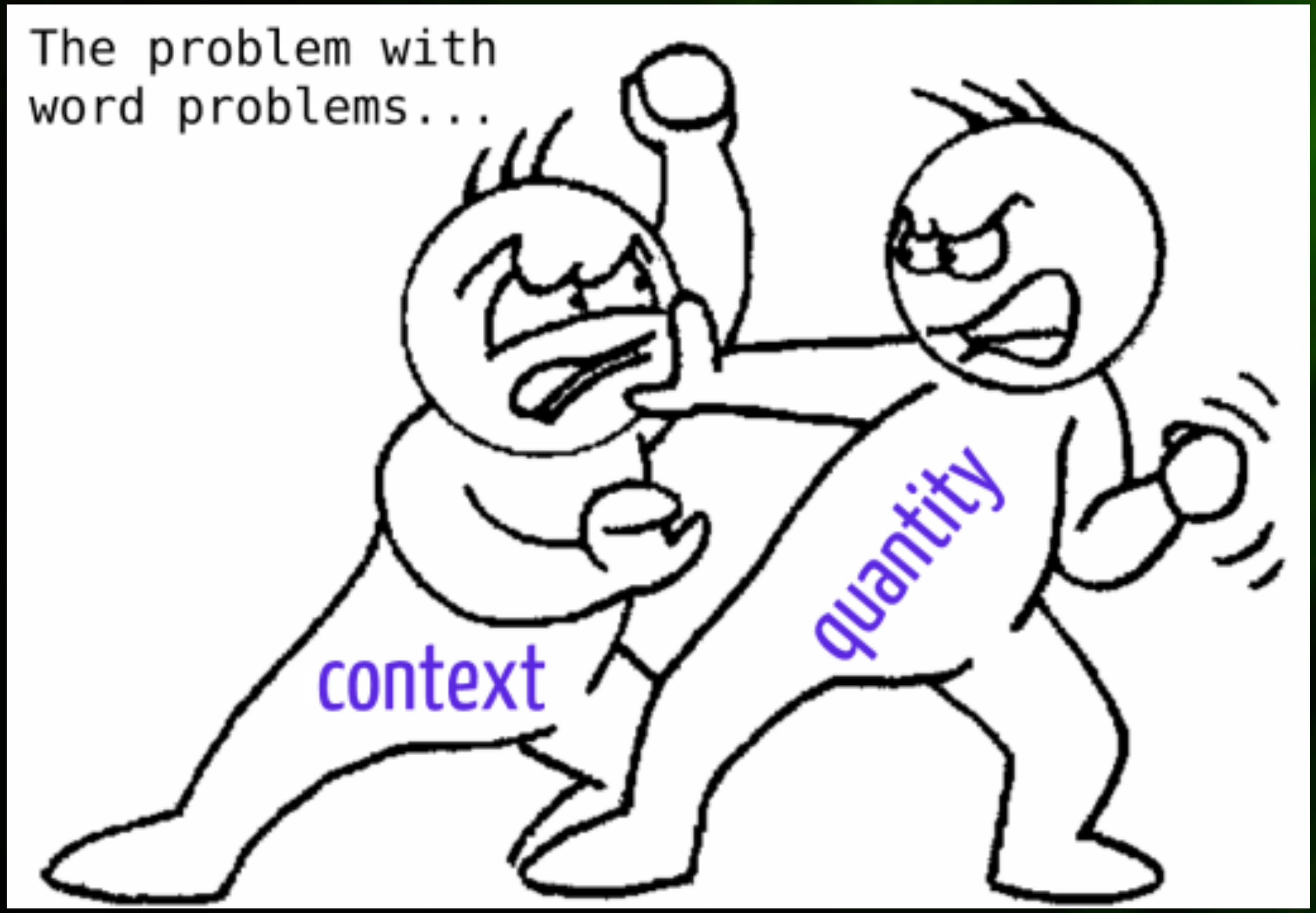
number

You little plucker!

13

21

John SanGovanni, NCTM '16



@gfletchy

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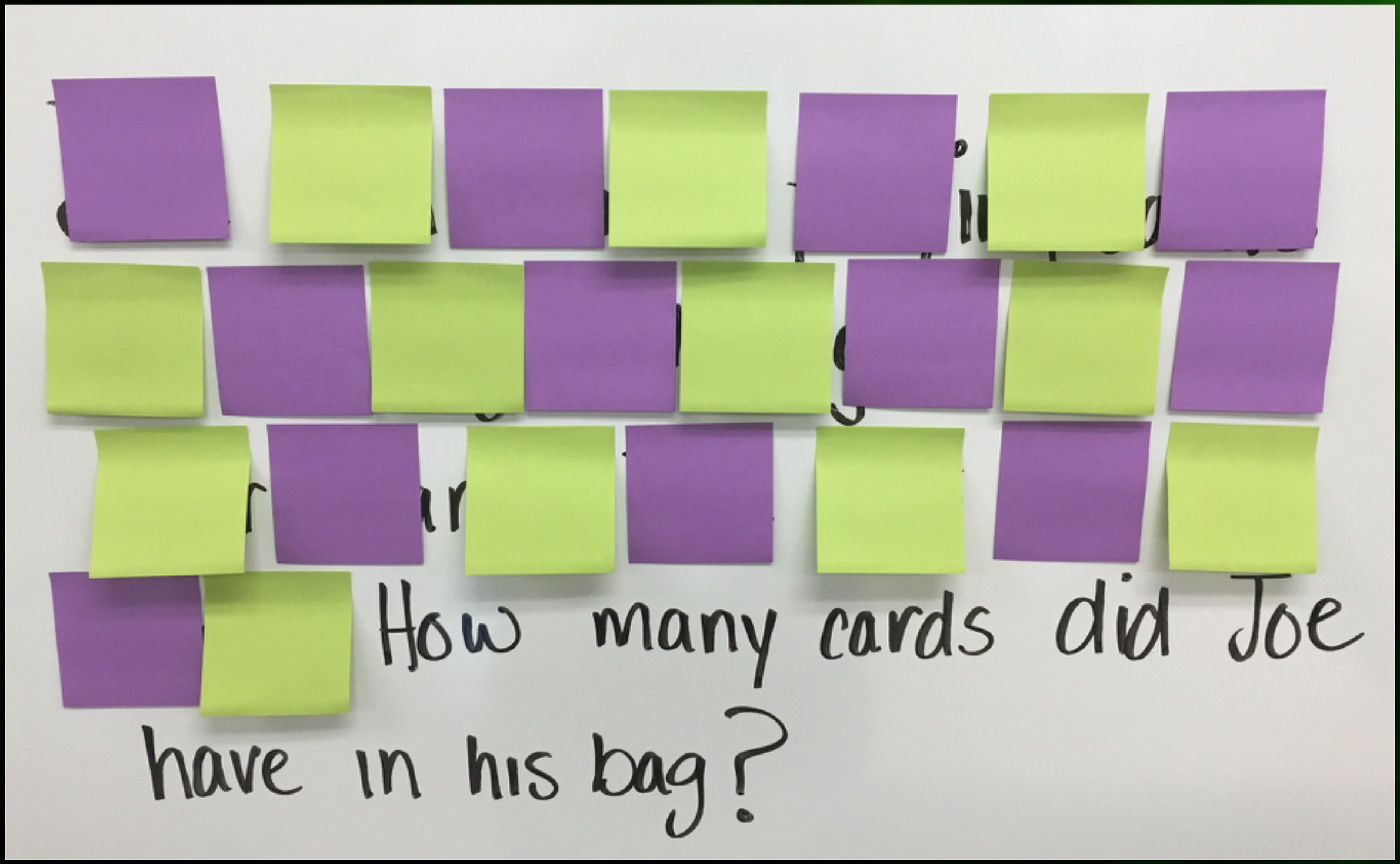

How many cards did Joe
have in his bag?

There are ■ yards of ribbon for ■ people to share. How many yards of ribbon can each person get if they share the ribbon equally?

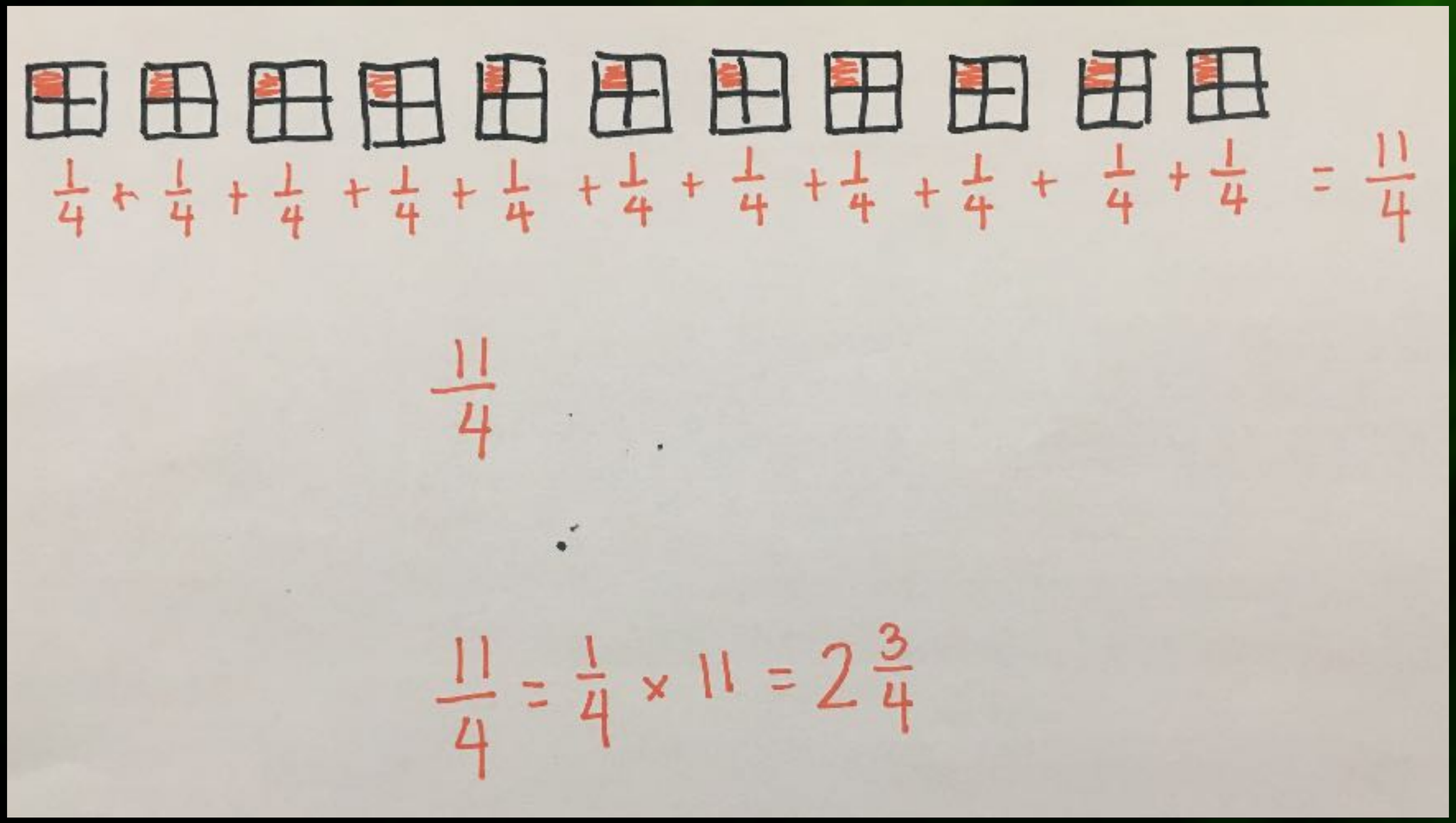
There are 11 yards of ribbon for 4 people to share. How many yards of ribbon can each person get if they share the ribbon equally?

2. There are 11 yards of ribbon for 4 people to share. How many yards of ribbon can each person get if they share the ribbon equally?

$$2 + \frac{1}{2} + \frac{1}{4} = 2\frac{3}{4}$$

Additive Coordination: Sharing one item at a time

2. There are 11 yards of ribbon for 4 people to share. How many yards of ribbon can each person get if they share the ribbon equally?



Additive Coordination: Sharing one item at a time

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