

Harnessing the Power of Modeling Tasks through the Lens of a Math Progression

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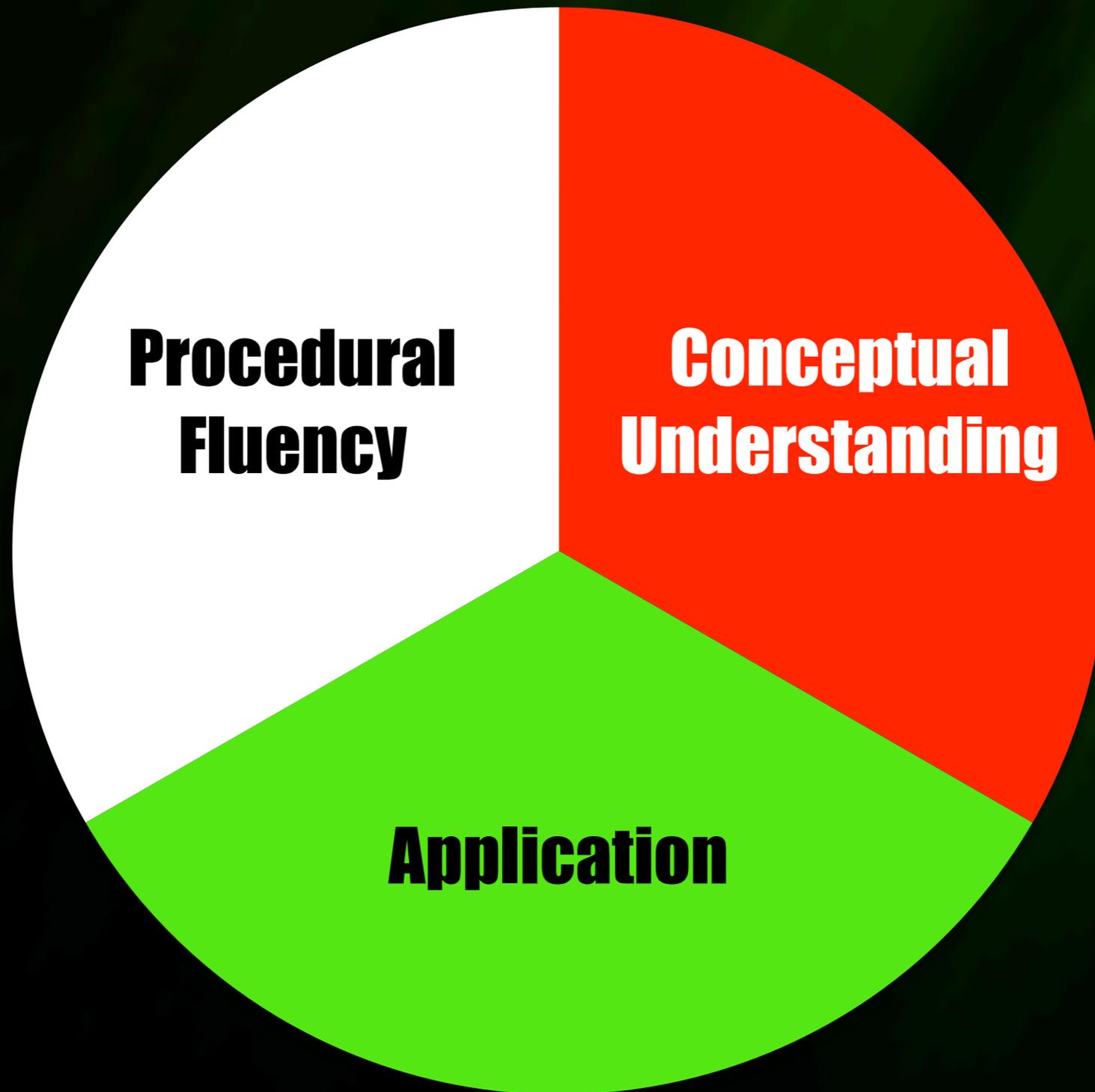


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Broken Squares

- Designate a shape keeper
- 6 congruent squares
- No shapes left over
- Everyone is encouraged to OFFER. No one may TAKE. They may ACCEPT a puzzle piece to use if OFFERED.
- NO TALKING



**Procedural
Fluency**

**Conceptual
Understanding**

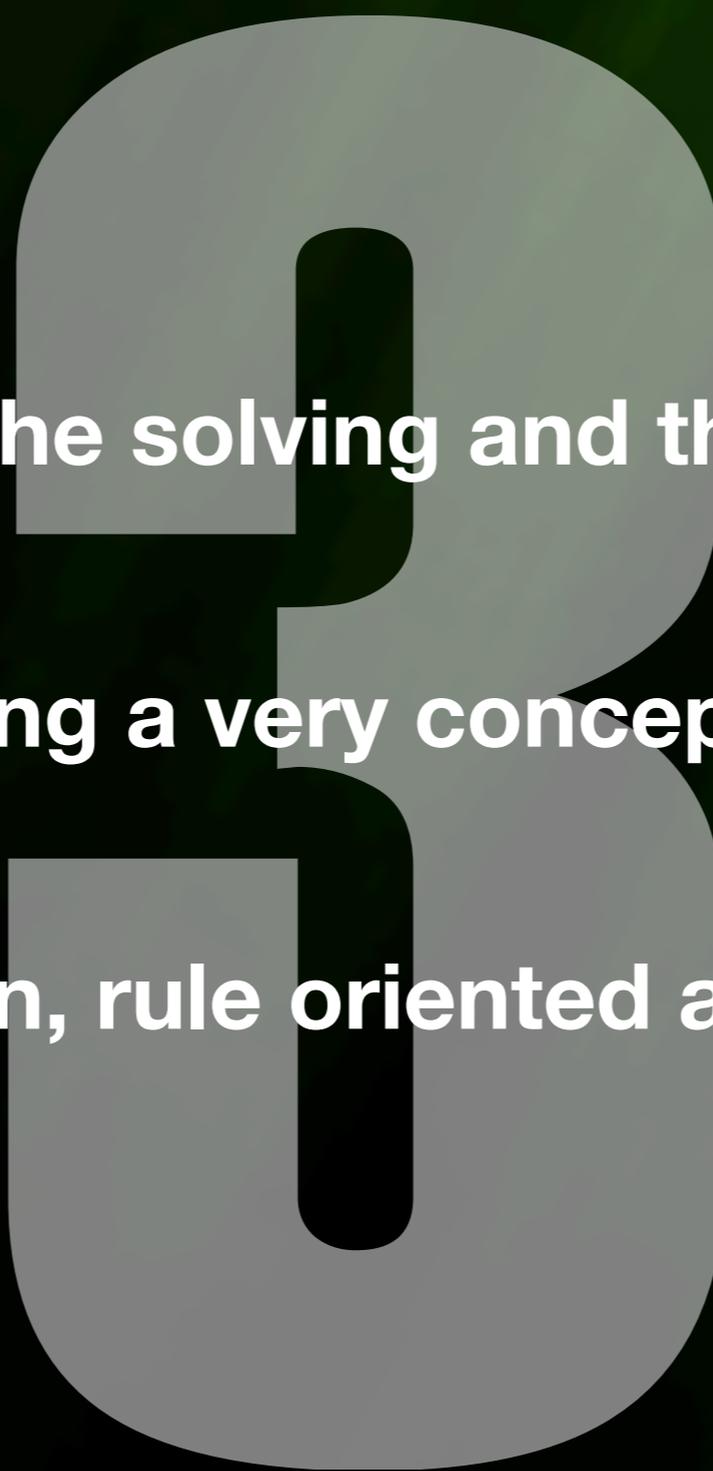
Application

**NEXT TIME YOU'RE AFRAID
TO SHARE IDEAS
REMEMBER SOMEONE
ONCE SAID IN A MEETING
LET'S MAKE A FILM WITH A
TORNADO FULL OF SHARKS**

The Condominium Problem

In a particular condominium community $\frac{2}{3}$ of all of the men are married to $\frac{3}{5}$ of all of the women.

What fraction of the entire condominium community are married?



students doing the solving and the sense making

teacher showing a very conceptual approach

top-down, rule oriented approach



3 questions

1
Billion Circles

How long to draw 1 billion circles:

- 100 circles : minute
- 144,000 circles : day
- 1,000,000,000 would take 6944 days
- 19+ years with no sleep





Mark Chubb @MarkChubb3 · 16m

@gfletchy

Where does 1 billion go?



Where does 1 billion go on the number line?



Where does 1 billion go on the number line?



$$6 \times \frac{5}{8}$$

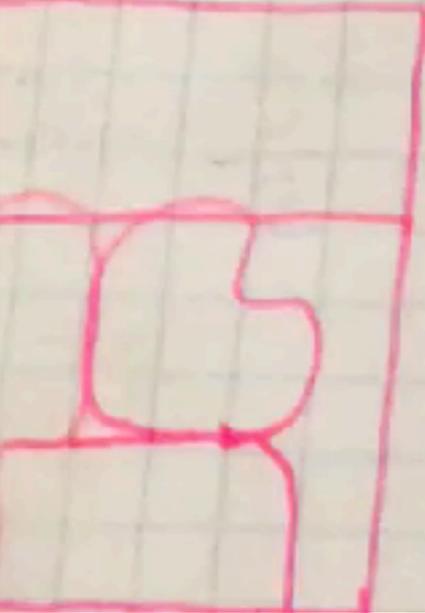
$$6 \times \frac{5}{8}$$

$$6 \text{ @ } \frac{10 + 3 + 5/8}{600 | 180}$$

$$6 \times 5/8 = 3 \frac{6}{8}$$

$$\begin{array}{r} 10 + 3 + 5/8 \\ \hline 600 \mid 180 \end{array}$$

$$6 \times 5/8 = 3 \frac{6}{8}$$


$$6 \times \frac{5}{8}$$

8

$$6 \times \frac{2}{5}$$



How many orange wedges are in the bowl?



Estimate

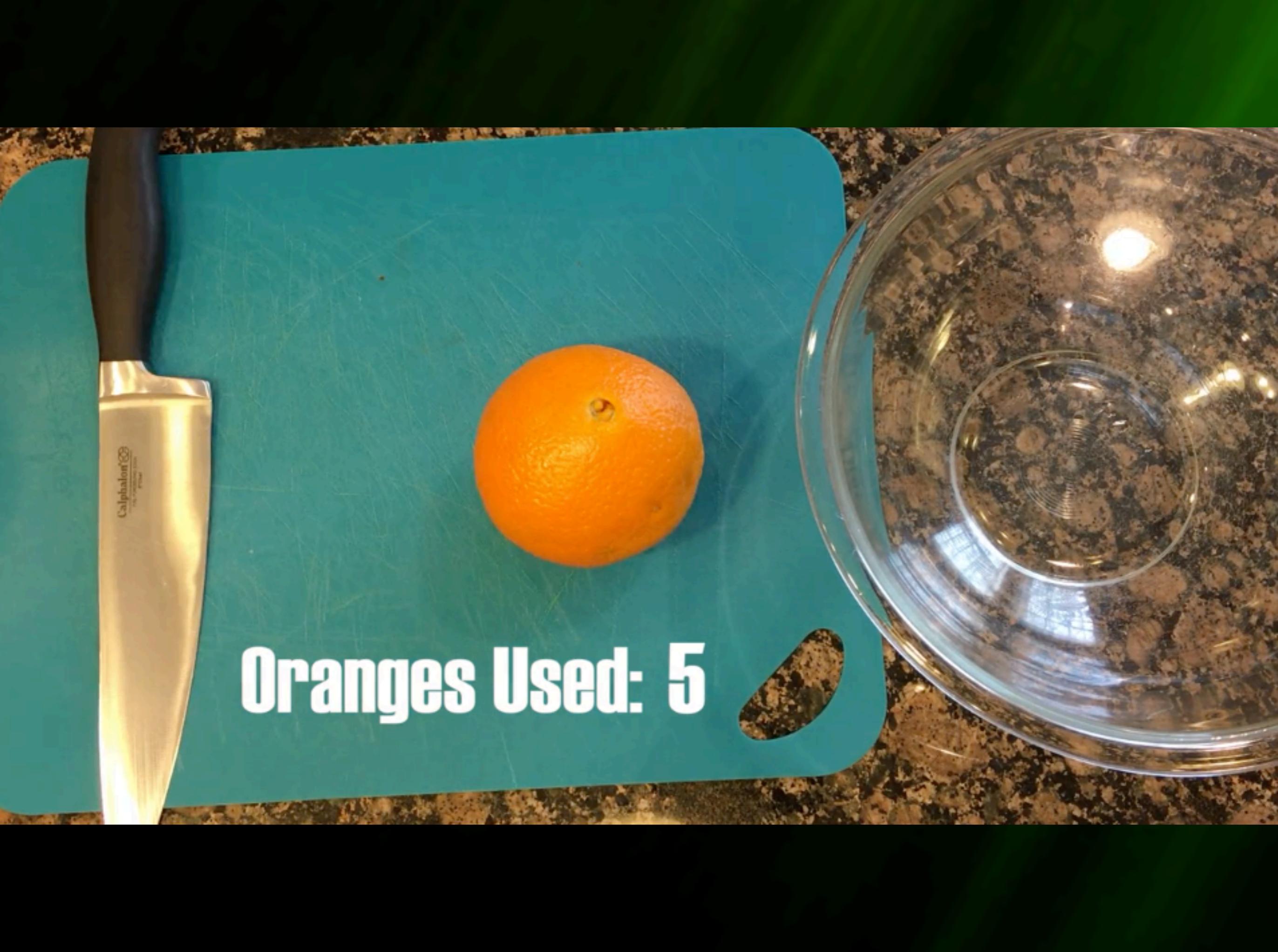


How many orange wedges are in the bowl?

What information do you need to know?

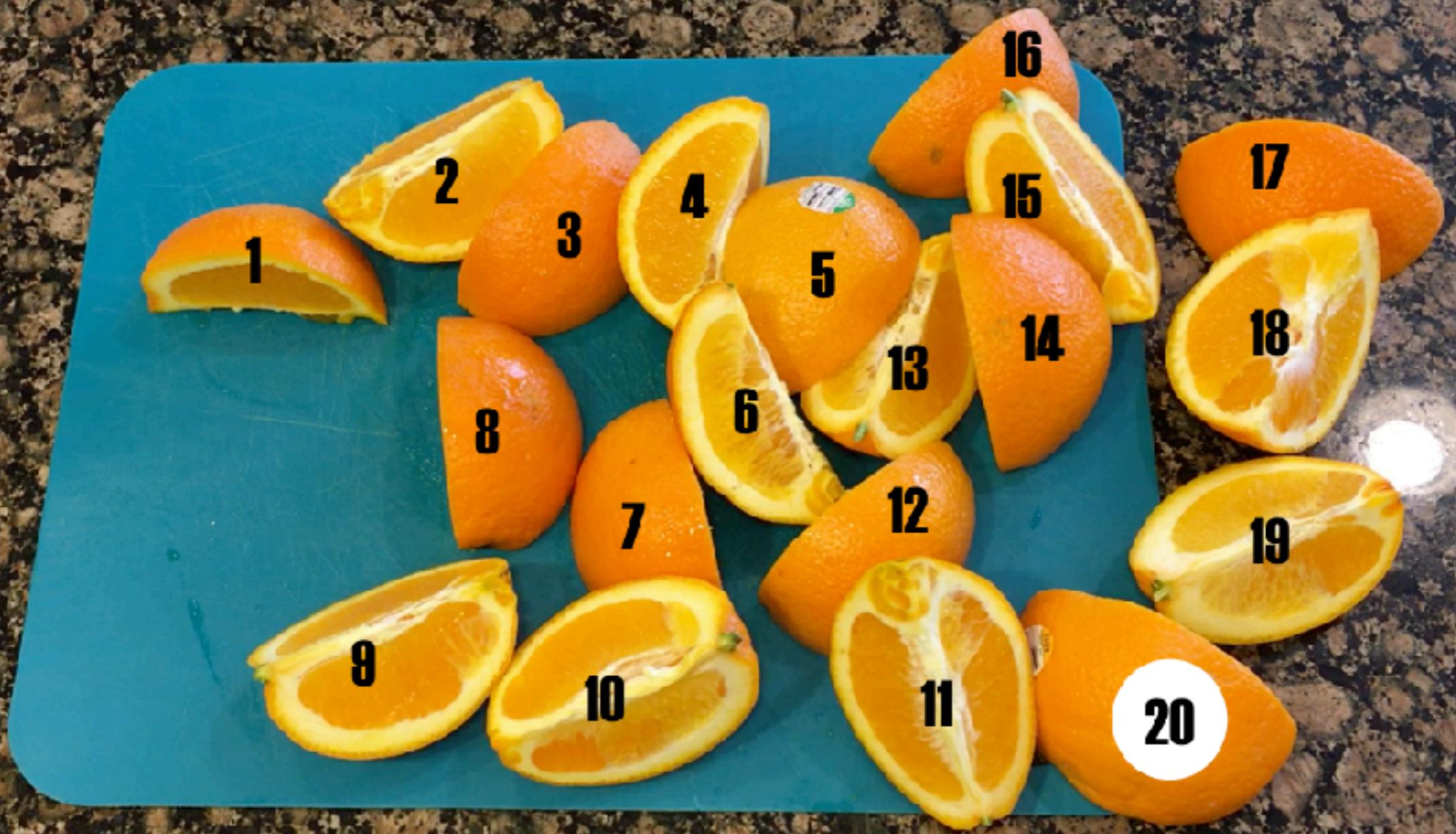


Each orange wedges is a quarter.

A kitchen scene featuring a teal cutting board on a granite countertop. A Calphalon knife is positioned on the left side of the board. In the center of the board lies a single, whole orange. To the right of the cutting board is a clear glass bowl. The background is a dark green wall.

Oranges Used: 5





1

2

3

4

5

16

15

17

18

14

13

8

6

7

12

19

9

10

11

20

Graham had 5 oranges and cut them into quarters.

How many orange wedges did Graham have?

3-Act Tasks



Act 1:

- Real world problem or scenario presented
- What do you notice? What do you wonder?
- Make estimates

Act 2:

- Identify missing variables and missing variables to solve
- Define solution path using variables

Act 3:

- Solve and interpret results of the solution
- Validate answer

Most asked questions:

- How often should we use 3-Act Tasks?
How do they fit into the scope of a unit?
- How long does one task usually take?
- What if we don't have the time?

Orchestrating Discussions

Five practices constitute a model for effectively using student responses in whole-class discussions that can potentially make teaching with high-level tasks more manageable for teachers.

Margaret S. Smith, Elizabeth K. Hughes, Randi A. Engle, and Mary Kay Stein



Margaret S. Smith, pegso@pitt.edu, is an associate professor of mathematics education at the University of Pittsburgh. Over the past decade, she has been developing research-based materials for use in the professional development of mathematics teachers and studying what teachers learn from the professional development in which they engage. Elizabeth K. Hughes, elizabeth.hughes@pitt.edu, recently finished her doctorate in mathematics education at the University of Pittsburgh. Her areas of interest include preservice secondary mathematics teacher education and the use of practice-based materials in developing teachers' understanding of what it means to teach and learn mathematics. Randi A. Engle, raengle@berkeley.edu, is an assistant professor of mathematics education and the social context of learning at the University of California Berkeley. She is interested in developing practical theories for how mathematics teachers can create discussion-based learning environments that promote strong student engagement, learning, and transfer. Mary Kay Stein, mkslein@pitt.edu, is a professor of learning sciences and policy and the director of the Learning Policy Center at the University of Pittsburgh. Her research focuses on instructional practice and the organizational and policy conditions that shape it.

Discussions that focus on cognitively challenging mathematical tasks, namely, those that promote thinking, reasoning, and problem solving, are a primary mechanism for promoting conceptual understanding of mathematics (Blömeke and Inagaki 1991; Michaels, O'Connor, and Resnick forthcoming). Such discussions give students opportunities to share ideas and clarify understandings, develop convincing arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things from other perspectives (NCTM 2000).

Although discussions about high-level tasks provide important

The **5** practices are:

1. **Anticipating** student responses to challenging mathematical tasks;
2. **Monitoring** students' work on and engagement with the tasks;
3. **Selecting** particular students to present their mathematical work;
4. **Sequencing** the student responses that will be displayed in a specific order and;
5. **Connecting** different students' responses and connecting the responses to key mathematical ideas.



5 oranges

Each wedge is a quarter

The **5** practices are:

1. **Anticipating** student responses to challenging mathematical tasks;
2. **Monitoring** students' work on and engagement with the tasks;
3. **Selecting** particular students to present their mathematical work;
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The **5** practices are:

1. **Anticipating** student responses to challenging mathematical tasks;
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3. **Selecting** particular students to present their mathematical work;
4. **Sequencing** the student responses that will be displayed in a specific order and;
5. **Connecting** different students' responses and connecting the responses to key mathematical ideas.

Name: _____ Date: _____ **S1**

1. What do you need?
20

2. What do you wonder?
Draw a picture to show your thinking:

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

$$\frac{4}{4} + \frac{8}{4} + \frac{12}{4} + \frac{16}{4} = \frac{20}{4}$$

$\frac{20}{4} = \text{wedges}$

3. Main Question
Use numbers to show your thinking:

Answer: 5

Name: _____ Date: _____ **S2**

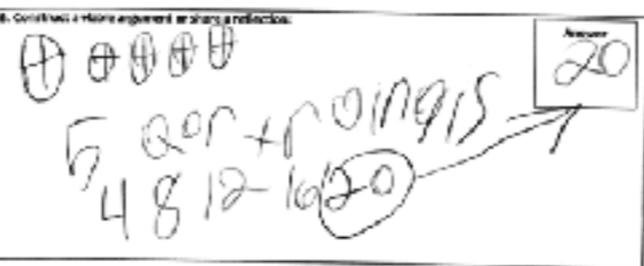
1. What do you need?
Owning's glass bowl

2. What do you wonder?
how many wedges?

3. Main Question
how many wedges?

4. Make an estimate.


5. What information do you need?
How big is the peisis

6. Construct a viable argument or share a reflection.


Name: _____ Date: _____ **S3**

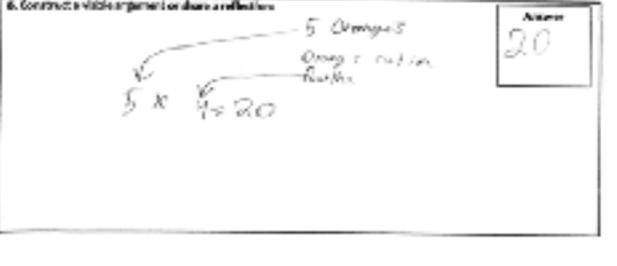
1. What do you need?
Bowl, oranges, oranges wedges

2. What do you wonder?
how many whole oranges?

3. Main Question
How many wedges?

4. Make an estimate.


5. What information do you need?
What are the size of the wedges?

6. Construct a viable argument or share a reflection.


Name: _____ Date: _____ **S4**

1. What do you need?
6

2. What do you wonder?
Draw a picture to show your thinking:



3. Main Question
Use numbers to show your thinking:

$$4 + 4 + 4 + 4 + 4 = 20$$

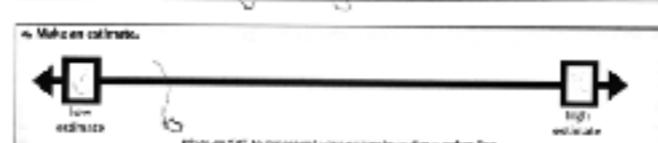
Answer: 20

Name: _____ Date: _____ **S5**

1. What do you need?
Bowl Orange

2. What do you wonder?
why?

3. Main question
How many wedges

4. Make an estimate.


5. What information do you need?

6. Construct a viable argument or share a reflection.


Name: _____ Date: _____ **S6**

1. What do you need?
Oranges, wedges, bowl, counter, hands
Two wedges outside the bowl

2. What do you wonder?
How many wedges?

3. Main Question
How many wedges?

4. Make an estimate.


5. What information do you need?
Size of wedges? Number of oranges?

6. Construct a viable argument or share a reflection.

$$20 \div 4 = 5$$

Answer: 20

Name: _____

SI

Estimate

20

Draw a picture to show your thinking:

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

$$\frac{4}{4} + \frac{8}{4} + \frac{12}{4} + \frac{16}{4} + \frac{20}{4}$$

$$\frac{20}{4} = \text{wedges}$$

Use numbers to show your thinking:

Answer:

5

Name: _____

Date: _____

SZ

1. What did you notice?

owning a glass bowl

2. What do you wonder?

how much it weighs?

3. Main Question:

how much weight is?

4. Make an estimate.



Place an "X" to represent your estimate on the number line.

5. What information do you need?

How big is the peisis

6. Construct a viable argument or share a reflection:

⊕ ⊕ ⊕ ⊕ ⊕

5 20 + 10 = 30

4 8 12 16 20

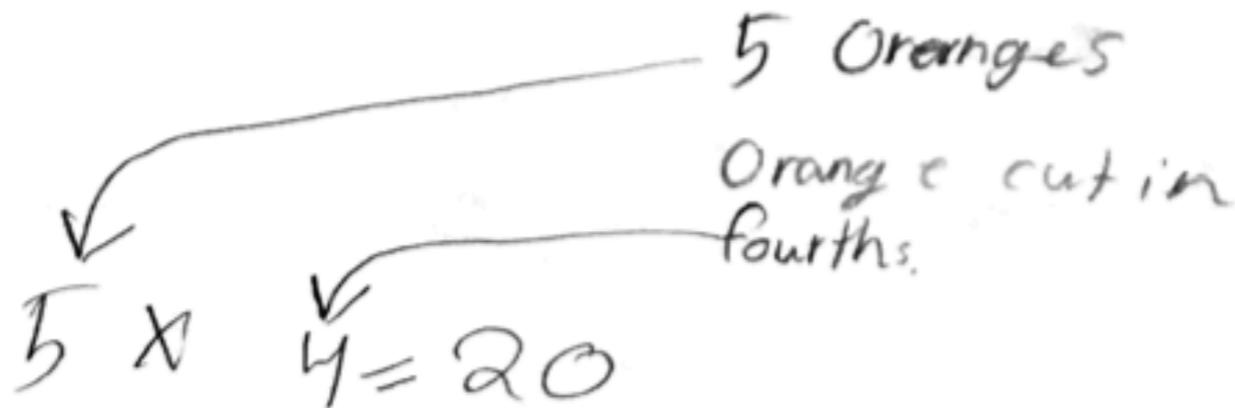
Answer
20

estimate estimate
Place an "X" to represent your estimate on the number line.

5. What information do you need?

What are the sizes of the wedges?

6. Construct a viable argument or share a reflection:



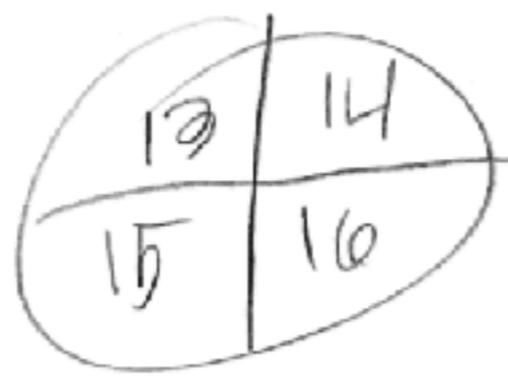
Answer

20

Estimate
6

S4

Draw a picture to show your thinking:



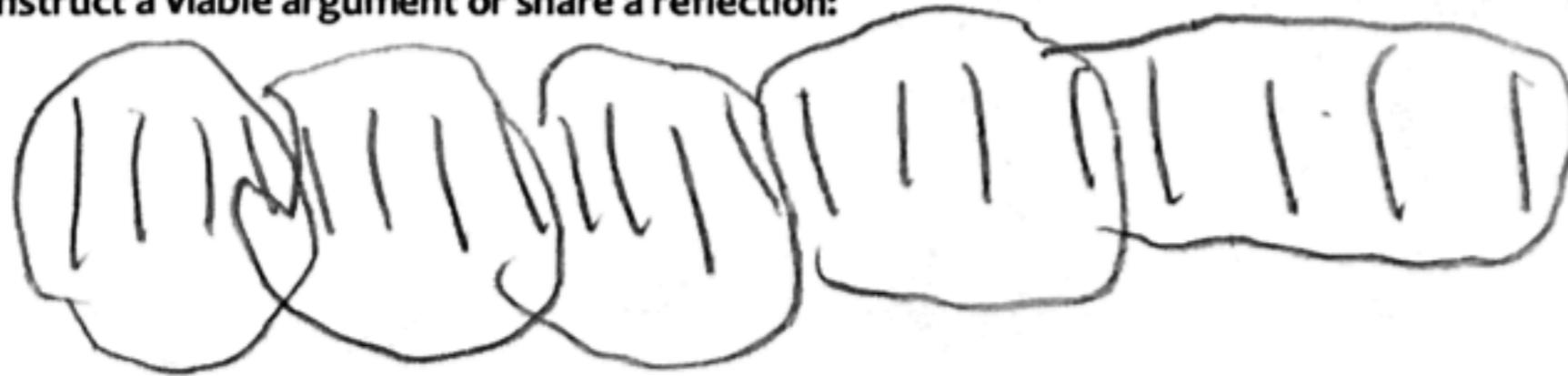
Use numbers to show your thinking:

$$4 + 4 + 4 + 4 + 4 = 20$$

Answer:

20

6. Construct a viable argument or share a reflection:



Answer

20

5. What information do you need?

Size of wedges? Number of oranges?

6. Construct a viable argument or share a reflection:

Answer

20

$$20 \times \frac{1}{4} = \frac{20}{4} = 5$$

Name: _____ Date: _____ **S1**

Estimate: 20

Draw a picture to show your thinking:

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

$$\frac{4}{4} + \frac{8}{4} + \frac{12}{4} + \frac{16}{4} = \frac{20}{4}$$

$\frac{20}{4} = \text{wedges}$

Use numbers to show your thinking:

Answer: 5

Name: _____ Date: _____ **S2**

1. What did you notice?
Owning's glass bowl

2. What do you wonder?
how many oranges?

3. Main Question:
how many wedges?

4. Make an estimate.

Place an "X" to represent your estimate on the number line.

5. What information do you need?
How big is the peels

6. Construct a viable argument or share a reflection:

⊕ ⊕ ⊕ ⊕ ⊕
5 oranges + 10 wedges
4 8 12 16 20

Answer: 20

Name: _____ Date: _____ **S3**

1. What did you notice?
Bowl oranges oranges wedges

2. What do you wonder?
How many whole oranges?

3. Main Question:
How many wedges?

4. Make an estimate.

Place an "X" to represent your estimate on the number line.

5. What information do you need?
What are the sizes of the wedges?

6. Construct a viable argument or share a reflection:

5 Oranges
Orange cut in fourths
 $5 \times 4 = 20$

Answer: 20

Name: _____ Date: _____ **S4**

Estimate: 6

Draw a picture to show your thinking:

Use numbers to show your thinking:

$$4 + 4 + 4 + 4 + 4 = 20$$

Answer: 20

Name: _____ Date: _____ **S5**

1. What did you notice?
bowl orange

2. What do you wonder?
why?

3. Main Question:
How many wedges

4. Make an estimate.

Place an "X" to represent your estimate on the number line.

5. What information do you need?

6. Construct a viable argument or share a reflection:

Answer: 20

Name: _____ Date: _____ **S6**

1. What did you notice?
Oranges, wedges, bowl, counter, hands.
Two wedges outside the bowl.

2. What do you wonder?
How many wedges?

3. Main Question:
How many wedges?

4. Make an estimate.

Place an "X" to represent your estimate on the number line.

5. What information do you need?
Size of wedges! Number of oranges?

6. Construct a viable argument or share a reflection:

$$20 \times \frac{1}{4} = \frac{20}{4} = 5$$

Answer: 20

Name: _____ Date: _____ **S1**

Estimate: 20

Draw a picture to show your thinking:

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

$$\frac{4}{4} + \frac{8}{4} + \frac{12}{4} + \frac{16}{4} = \frac{20}{4}$$

$\frac{20}{4} = \text{wedges}$

Use numbers to show your thinking:

Answer: 5

Name: _____ Date: _____ **S2**

1. What did you notice?
Owning's glass bowl

2. What do you wonder?
how many oranges?

3. Main Question:
how many wedges?

4. Make an estimate.

Place an "X" to represent your estimate on the number line.

5. What information do you need?
How big is the peels

6. Construct a viable argument or share a reflection:

⊕ ⊕ ⊕ ⊕ ⊕
5 oranges + 10 wedges
4 8 12 16 20

Answer: 20

Name: _____ Date: _____ **S3**

1. What did you notice?
Bowl oranges oranges wedges

2. What do you wonder?
How many whole oranges?

3. Main Question:
How many wedges?

4. Make an estimate.

Place an "X" to represent your estimate on the number line.

5. What information do you need?
What are the sizes of the wedges?

6. Construct a viable argument or share a reflection:

5 Oranges
Orange cut in fourths
 $5 \times 4 = 20$

Answer: 20

Name: _____ Date: _____ **S4**

Estimate: 6

Draw a picture to show your thinking:

Use numbers to show your thinking:

$$4 + 4 + 4 + 4 + 4 = 20$$

Answer: 20

Name: _____ Date: _____ **S5**

1. What did you notice?
bowl orange

2. What do you wonder?
why?

3. Main Question:
How many wedges

4. Make an estimate.

Place an "X" to represent your estimate on the number line.

5. What information do you need?

6. Construct a viable argument or share a reflection:

Answer: 20

1-Counting Up

Name: _____ Date: _____ **S6**

1. What did you notice?
Oranges, wedges, bowl, counter, hands.
Two wedges outside the bowl.

2. What do you wonder?
How many wedges?

3. Main Question:
How many wedges?

4. Make an estimate.

Place an "X" to represent your estimate on the number line.

5. What information do you need?
Size of wedges! Number of oranges?

6. Construct a viable argument or share a reflection:

$$20 \times \frac{1}{4} = \frac{20}{4} = 5$$

Answer: 20

Name: _____ Date: _____ **S1**

Estimate: 20

Draw a picture to show your thinking:

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

$$\frac{4}{4} + \frac{8}{4} + \frac{12}{4} + \frac{16}{4} = \frac{20}{4}$$

$\frac{20}{4} = \text{wedges}$

Use numbers to show your thinking:

2b-Skip Counting

Answer: 5

Name: _____ Date: _____ **S2**

1. What did you notice?
Owning's glass bowl

2. What do you wonder?
how many oranges?

3. Main Question:
how many wedges?

4. Make an estimate.

Place an "X" to represent your estimate on the number line.

5. What information do you need?
How big is the peels

6. Construct a viable argument or share a reflection:

⊕ ⊕ ⊕ ⊕ ⊕
5 oranges + 10 wedges
4 8 12 16 20

Answer: 20

2a-Skip Counting

Name: _____ Date: _____ **S3**

1. What did you notice?
Bowl oranges oranges wedges

2. What do you wonder?
How many whole oranges?

3. Main Question:
How many wedges?

4. Make an estimate.

Place an "X" to represent your estimate on the number line.

5. What information do you need?
What are the sizes of the wedges?

6. Construct a viable argument or share a reflection:

5 Oranges
Orange cut in fourths
 $5 \times 4 = 20$

Answer: 20

Name: _____ Date: _____ **S4**

Estimate: 6

Draw a picture to show your thinking:

Use numbers to show your thinking:

$$4 + 4 + 4 + 4 + 4 = 20$$

2a-Skip Counting

Answer: 20

Name: _____ Date: _____ **S5**

1. What did you notice?
bowl orange

2. What do you wonder?
why?

3. Main Question:
How many wedges

4. Make an estimate.

Place an "X" to represent your estimate on the number line.

5. What information do you need?

6. Construct a viable argument or share a reflection:

Answer: 20

1-Counting Up

Name: _____ Date: _____ **S6**

1. What did you notice?
Oranges, wedges, bowl, counter, hands.
Two wedges outside the bowl.

2. What do you wonder?
How many wedges?

3. Main Question:
How many wedges?

4. Make an estimate.

Place an "X" to represent your estimate on the number line.

5. What information do you need?
Size of wedges! Number of oranges?

6. Construct a viable argument or share a reflection:

$$20 \times \frac{1}{4} = \frac{20}{4} = 5$$

Answer: 20

Name: _____ Date: _____ **S1**

Estimate: 20

Draw a picture to show your thinking:

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

$$\frac{4}{4} + \frac{8}{4} + \frac{12}{4} + \frac{16}{4} = \frac{20}{4}$$

$\frac{20}{4} = \text{wedges}$

Use numbers to show your thinking:

2b-Skip Counting

Answer: 5

Name: _____ Date: _____ **S2**

1. What did you notice?
Owning's glass bowl

2. What do you wonder?
how many wedges?

3. Main Question
how many wedges?

4. Make an estimate.

5. What information do you need?
How big is the peels

6. Construct a viable argument or share a reflection.

⊕ ⊕ ⊕ ⊕ ⊕

5 oranges + 10 wedges = 20

4 8 12 16 20

Answer: 20

2a-Skip Counting

Name: _____ Date: _____ **S3**

1. What did you notice?
Bowl oranges oranges wedges

2. What do you wonder?
How many whole oranges?

3. Main Question
how many wedges?

4. Make an estimate.

5. What information do you need?
Size of the size of the wedges?

6. Construct a viable argument or share a reflection.

5 Oranges
Orange + 10 wedges = 20

5 x 4 = 20

Answer: 20

3b-Multiplicative Thinking

Name: _____ Date: _____ **S4**

Estimate: 6

Draw a picture to show your thinking:

Use numbers to show your thinking:

$$4 + 4 + 4 + 4 + 4 = 20$$

2a-Skip Counting

Answer: 20

Name: _____ Date: _____ **S5**

1. What did you notice?
bowl orange

2. What do you wonder?
why?

3. Main Question
How many wedges

4. Make an estimate.

5. What information do you need?

6. Construct a viable argument or share a reflection.

Answer: 20

1-Counting Up

Name: _____ Date: _____ **S6**

1. What did you notice?
Oranges wedges, bowl, counter, hands.
Two wedges outside the bowl

2. What do you wonder?
How many wedges?

3. Main Question
How many wedges?

4. Make an estimate.

5. What information do you need?
Size of wedge? Number of oranges?

6. Construct a viable argument or share a reflection.

$20 \times \frac{1}{4} = \frac{20}{4} = 5$

Answer: 20

3a-Multiplicative Thinking

Name: _____ Date: _____

Estimate: 20 S1

Draw a picture to show your thinking:

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

$$\frac{4}{4} + \frac{8}{4} + \frac{12}{4} + \frac{16}{4} = \frac{20}{4}$$

$\frac{20}{4} = \text{wedges}$

Use numbers to show your thinking:

2b-Skip Counting

Answer: 5

Name: _____ Date: _____

1. What do you notice? S3
Both oranges oranges wedges.

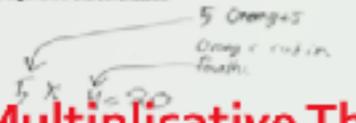
2. What do you wonder?
How many whole oranges?

3. Main Question:
How many wedges?

4. Make an estimate:


Place an "X" to represent your estimate on the number line.

5. What information do you need?
Can we use the size of the wedges?

6. Construct a viable argument or share a reflection:


5 Oranges
Orange = 4 wedges
5 x 4 = 20

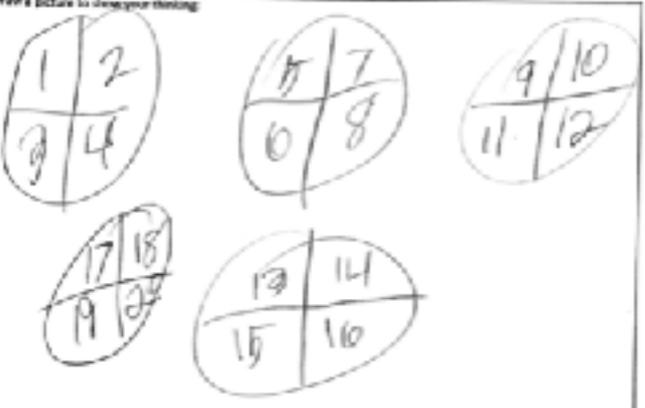
Answer: 20

3b-Multiplicative Thinking

Name: _____ Date: _____

Estimate: 6 S4

Draw a picture to show your thinking:



Use numbers to show your thinking:

$$4 + 4 + 4 + 4 + 4 = 20$$

2a-Skip Counting

Answer: 20

Sequencing and Connecting Student Work

Estimate: 6 S4

Draw a picture to show your thinking:

Use numbers to show your thinking:

$4 + 4 + 4 + 4 + 4 = 20$

2a-Skip Counting

Answer: 20

Estimate: 20 S1

Draw a picture to show your thinking:

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

$$\frac{4}{4} + \frac{8}{4} + \frac{12}{4} + \frac{16}{4} + \frac{20}{4}$$

$\frac{20}{4} = \text{wedges}$

Use numbers to show your thinking:

2b-Skip Counting

Answer: 5

Name: _____ Date: S3

1. What did you notice?
Banks oranges oranges wedges

2. What do you wonder?
How many whole oranges?

3. Main Question
How many wedges?

4. Make an estimate.

5. What information do you need?
What are the sizes of the wedges?

6. Construct a viable argument or share a reflection.

5 Oranges?
Orange cut in fourths.
 $5 \times \frac{1}{4} = 20$

Answer: 20

3b-Multiplicative Thinking

Name: _____ Date: _____ **S1**

Estimate: 20

Draw a picture to show your thinking:

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

$$\frac{4}{4} + \frac{8}{4} + \frac{12}{4} + \frac{16}{4} = \frac{20}{4}$$

$\frac{20}{4} = \text{wedges}$

Use numbers to show your thinking:

2b-Skip Counting

Answer: 5

Name: _____ Date: _____ **S2**

1. What did you notice?
owling's glass bowl

2. What do you wonder?
how many wedges?

3. Main Question
how many wedges?

4. Make an estimate.

Place an "X" to represent your estimate on the number line.

5. What information do you need?
How big is the peis?

6. Construct a viable argument or show a reflection.

⊕ ⊕ ⊕ ⊕ ⊕

5 bowls + 4 owling's

4 8 12 16 20

Answer: 20

2a-Skip Counting

Name: _____ Date: _____ **S3**

1. What did you notice?
Bowl oranges oranges wedges

2. What do you wonder?
How many whole oranges?

3. Main Question
How many wedges?

4. Make an estimate.

Place an "X" to represent your estimate on the number line.

5. What information do you need?
What are the sizes of the wedges?

6. Construct a viable argument or show a reflection.

5 Oranges
Orange cutting fourths

5 x 4 = 20

Answer: 20

3b-Multiplicative Thinking

Name: _____ Date: _____ **S4**

Estimate: 6

Draw a picture to show your thinking:

Use numbers to show your thinking:

$$4 + 4 + 4 + 4 + 4 = 20$$

2a-Skip Counting

Answer: 20

Name: _____ Date: _____ **S5**

1. What did you notice?
bowl orange

2. What do you wonder?
why?

3. Main Question
How many wedges

4. Make an estimate.

Place an "X" to represent your estimate on the number line.

5. What information do you need?

6. Construct a viable argument or show a reflection.

Answer: 20

1-Counting Up

Name: _____ Date: _____ **S6**

1. What did you notice?
Oranges wedges, bowl, counter, hands.
Two wedges outside the bowl.

2. What do you wonder?
How many wedges?

3. Main Question
How many wedges?

4. Make an estimate.

Place an "X" to represent your estimate on the number line.

5. What information do you need?
Size of wedge? Number of oranges?

6. Construct a viable argument or show a reflection.

$20 \times \frac{1}{4} = \frac{20}{4} = 5$

Answer: 20

3a-Multiplicative Thinking

Group 1

Name: _____ Date: _____

Score: **S5**

1. What did you notice?
Paul Orange

2. What do you wonder?
Why?

3. Main Question:
How many wedges?

4. Make an estimate.

5. What information do you need?

6. Construct a viable argument or share a reflection.

1-Counting Up

Group 2

Name: _____ Date: _____

Score: **S1**

Estimate: **20**

Draw a picture to show your thinking:
 $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1$
 $\frac{4}{4} + \frac{8}{4} + \frac{12}{4} + \frac{16}{4} = \frac{20}{4}$
 $\frac{20}{4} = \text{wedges}$

Use numbers to show your thinking:
2b-Skip Counting
 Answer: **5**

Name: _____ Date: _____

Score: **S2**

1. What did you notice?
Dwain's glass ball

2. What do you wonder?
how many oranges?

3. Main Question:
how many wedges?

4. Make an estimate.

5. What information do you need?
How big is the peis?

6. Construct a viable argument or share a reflection.

2a-Skip Counting

Name: _____ Date: _____

Score: **S4**

6

Draw a picture to show your thinking:

Use numbers to show your thinking:
 $4 + 4 + 4 + 4 + 4 = 20$
2a-Skip Counting
 Answer: **20**

Group 3

Name: _____ Date: _____

Score: **S3**

1. What did you notice?
Both designs oranges wedges

2. What do you wonder?
How many whole oranges?

3. Main Question:
How many wedges?

4. Make an estimate.

5. What information do you need?
How big are the sides of the wedges?

6. Construct a viable argument or share a reflection.

3b-Multiplicative Thinking

Name: _____ Date: _____

Score: **S6**

1. What did you notice?
Designs oranges, bowl, orange, bowl
Two wedges outside the bowl

2. What do you wonder?
How many wedges?

3. Main Question:
How many wedges?

4. Make an estimate.

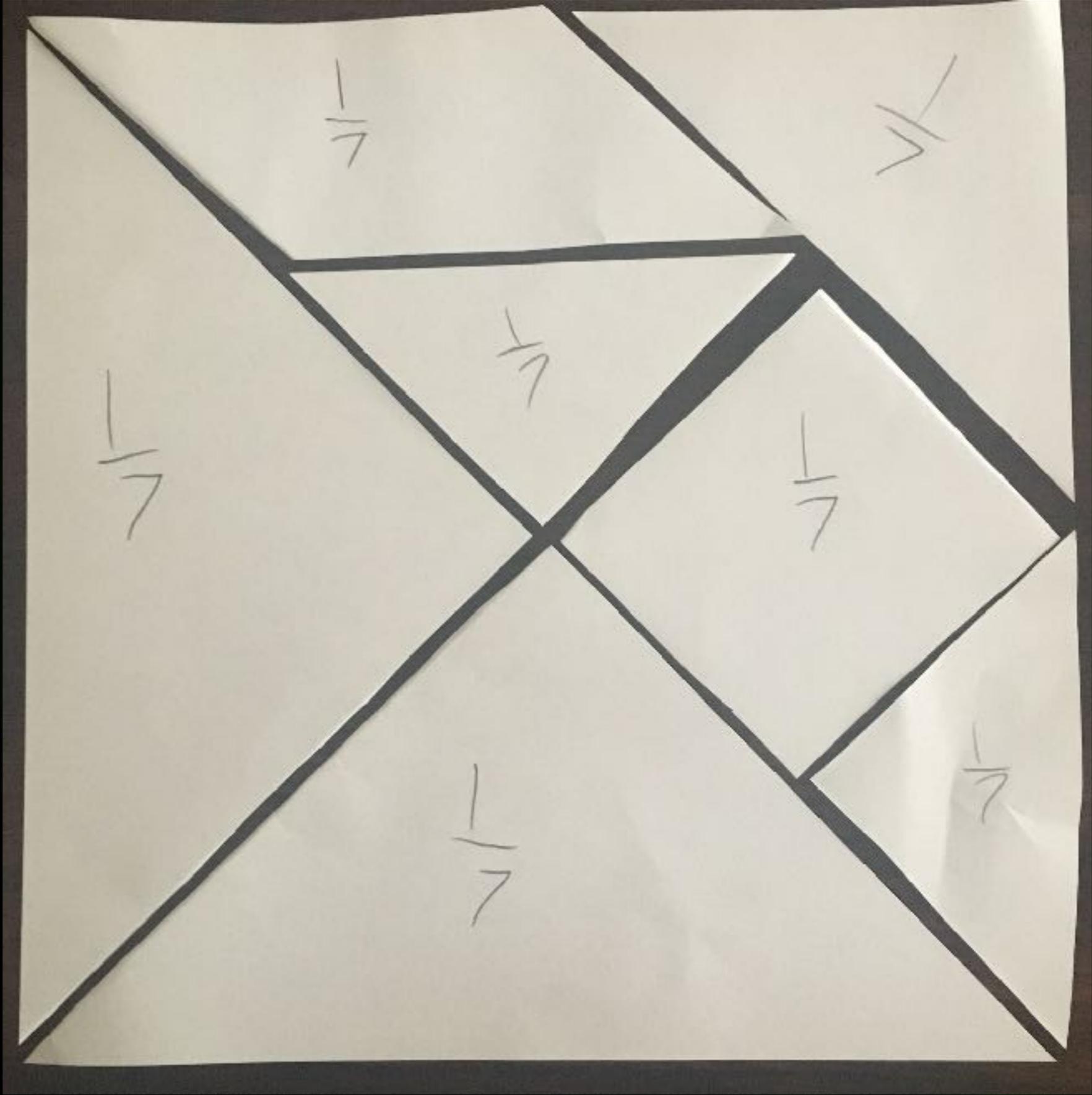
5. What information do you need?
Size of wedge? Number of oranges?

6. Construct a viable argument or share a reflection.
 $20 \times \frac{1}{4} = 5$
3a-Multiplicative Thinking

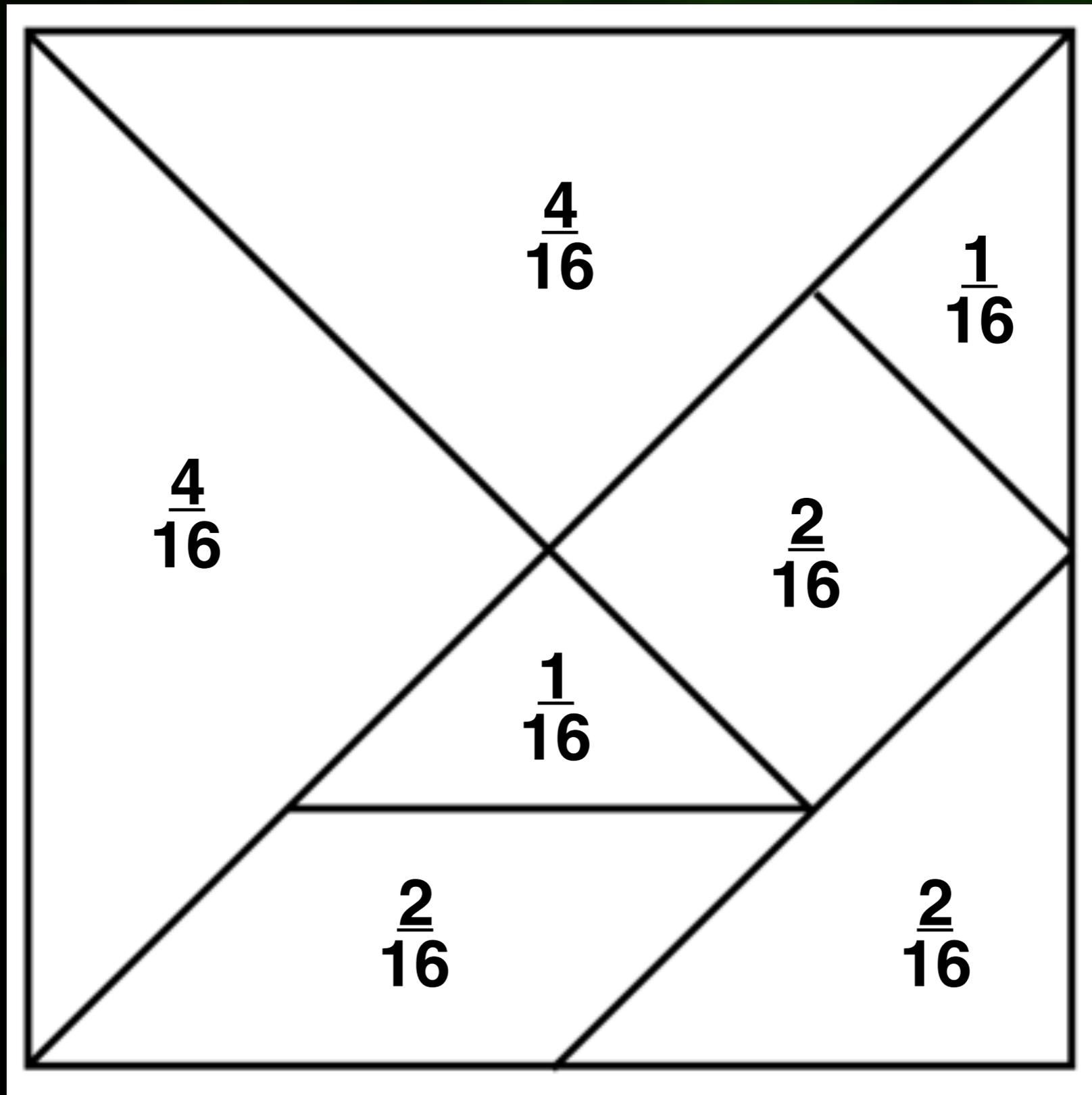
The Progression of Fractions

Concept	Notes
Meaning of Unit Fractions 3.NF.1	
Equivalent Fractions 3.NF.3 & 4.NF.1	
Comparing Fractions 3.NF.3 & 4.NF.2	

Unit Fractions

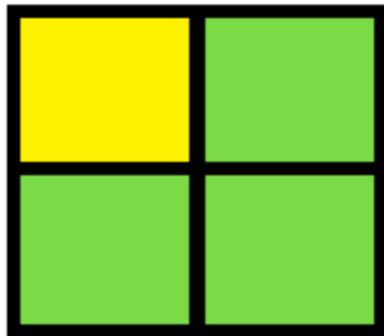


Tangram Challenge



Representation of a Fraction

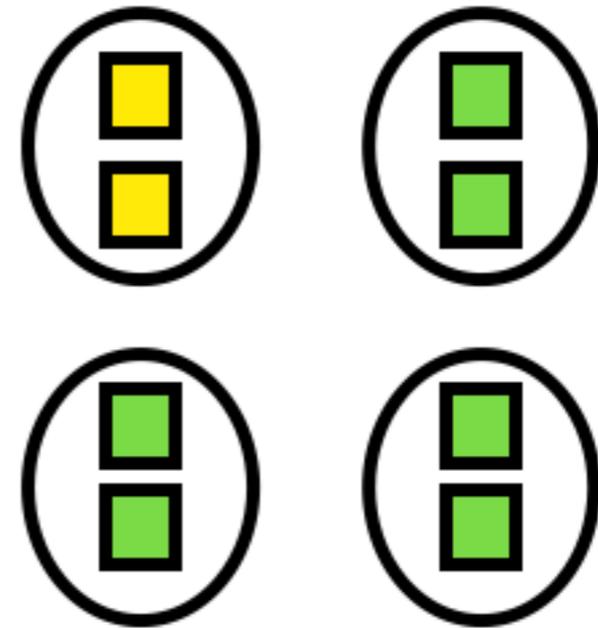
Area



Length



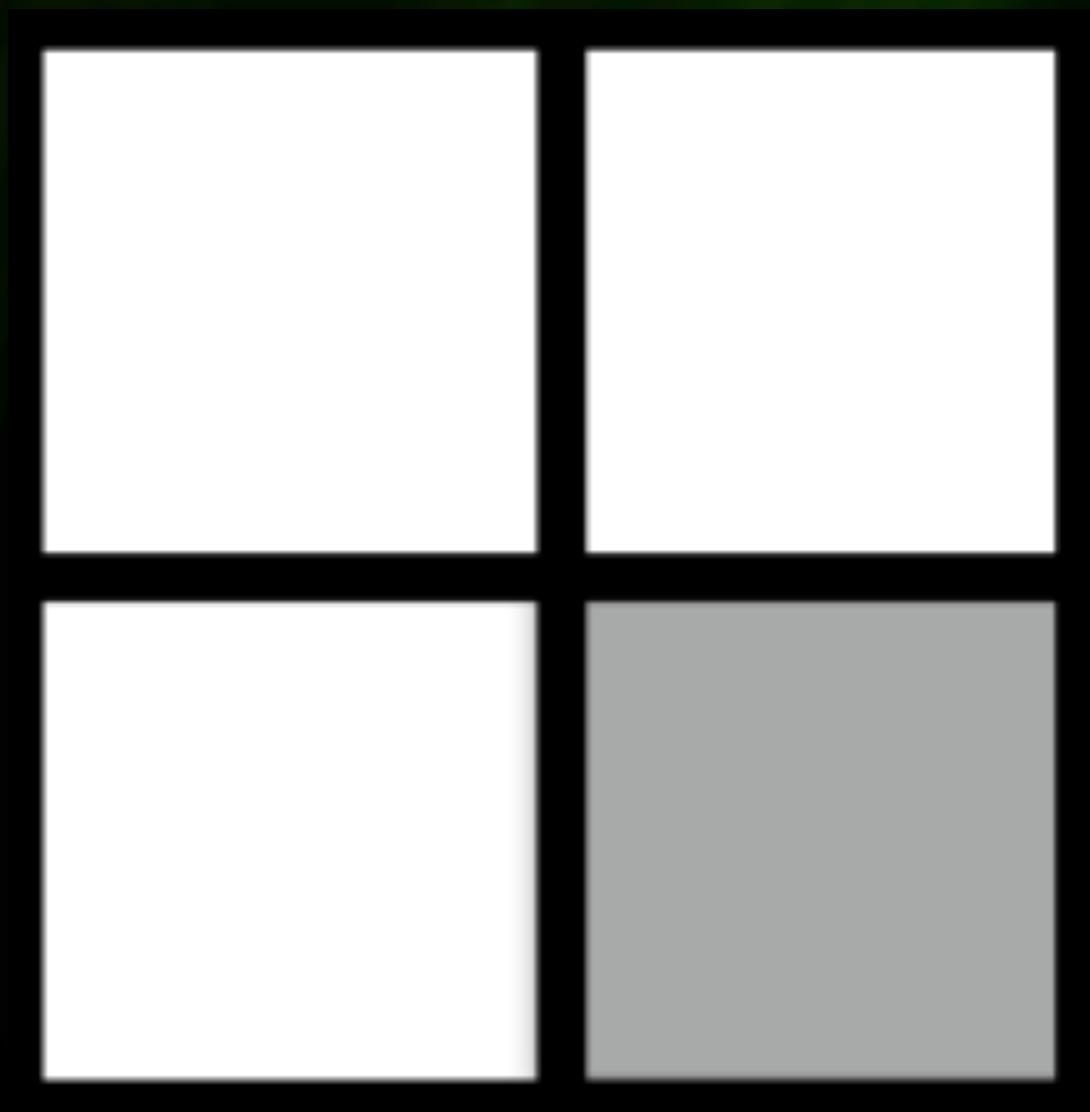
Set



unit fraction $\frac{1}{a}$

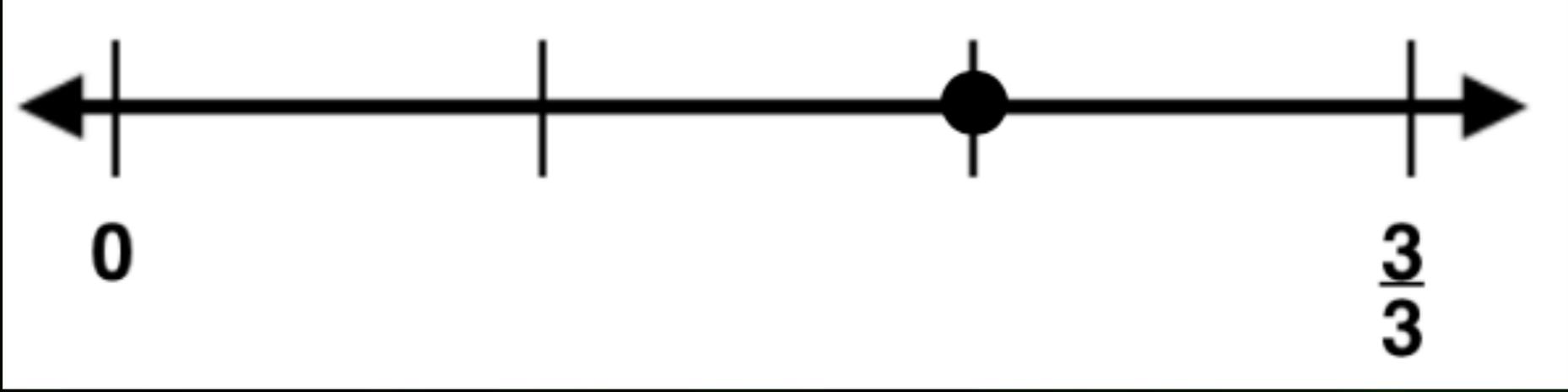
Say this fraction

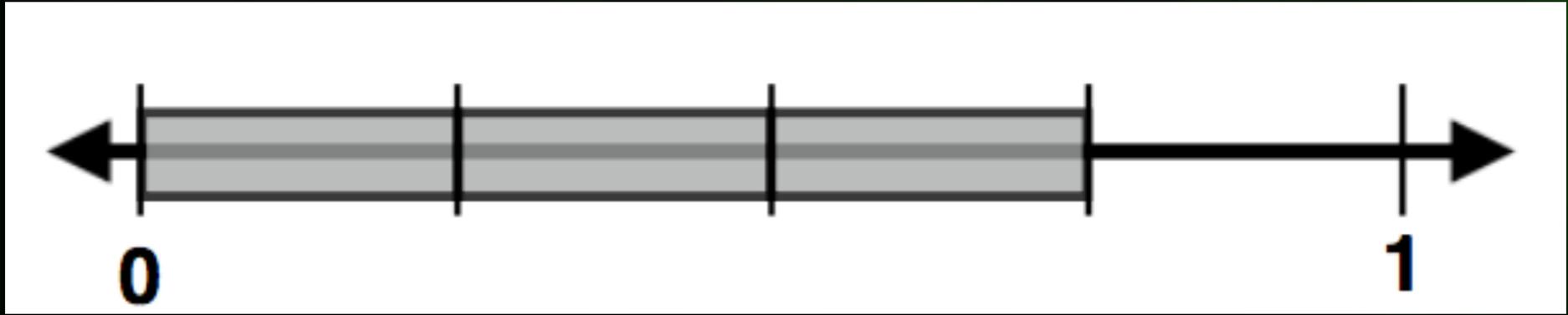
$$\frac{3}{5}$$

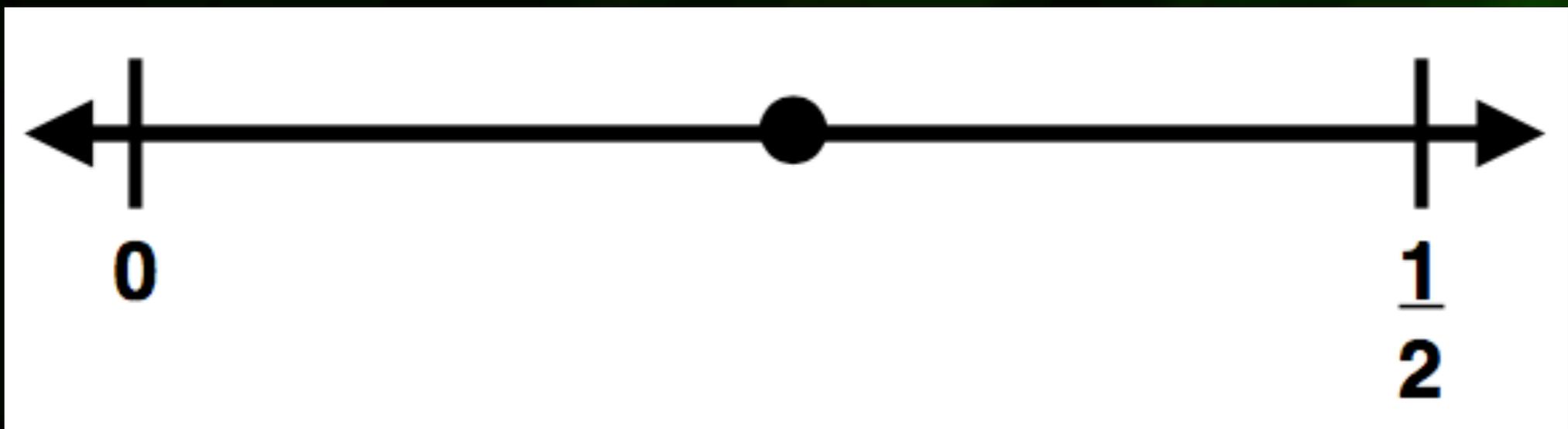


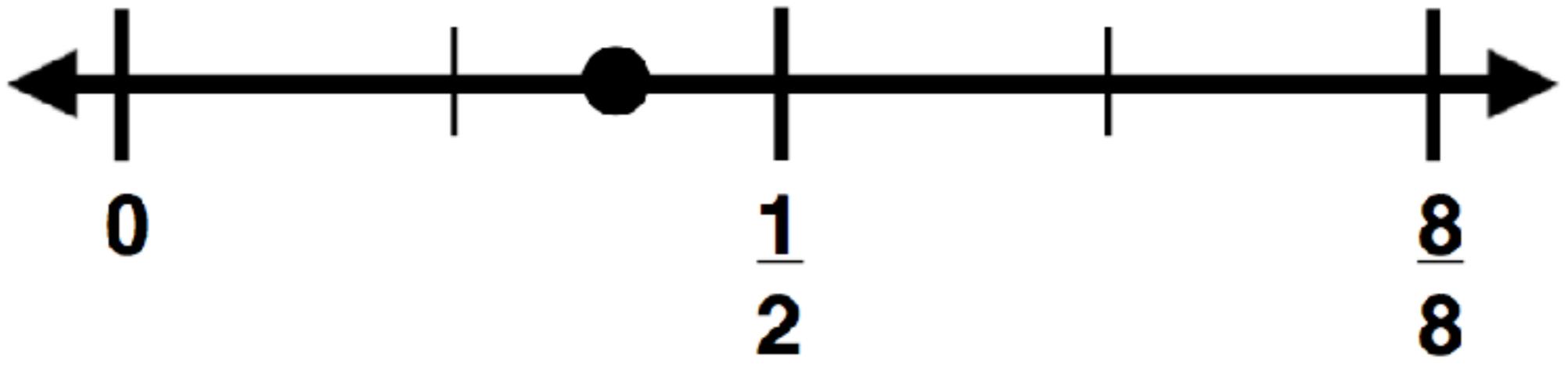




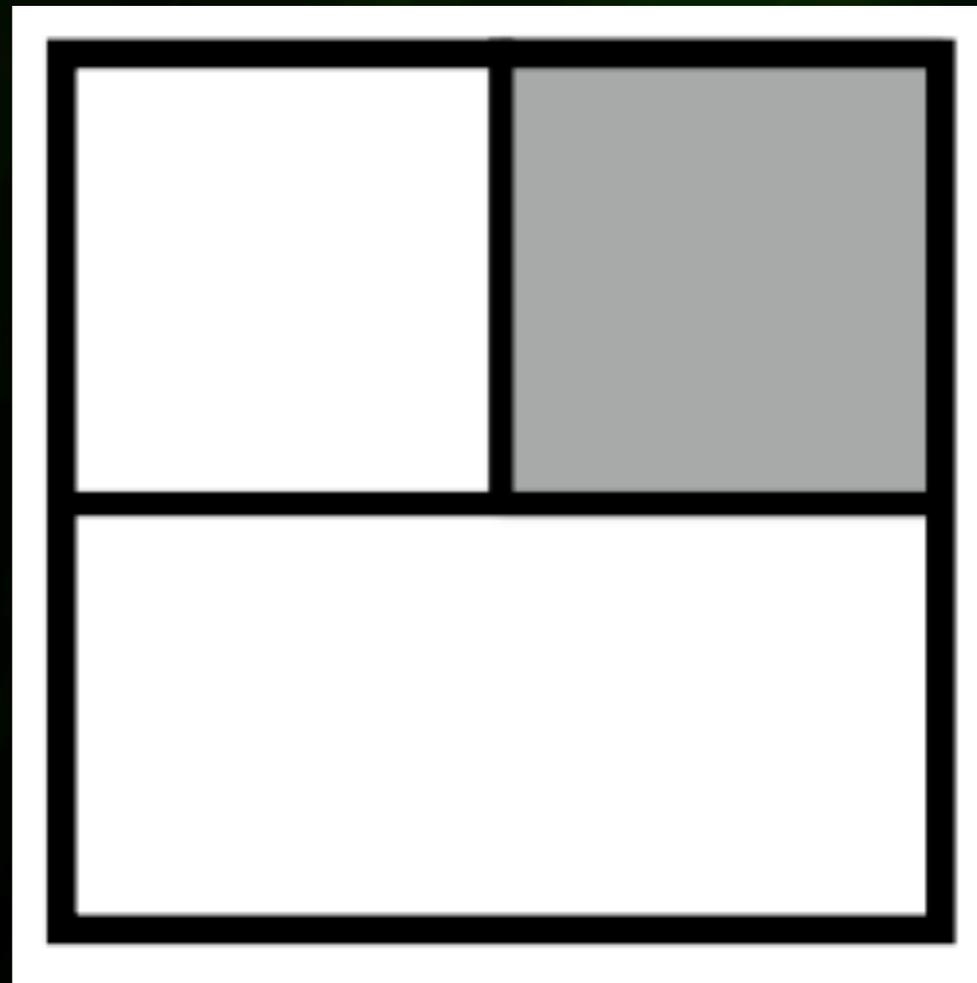


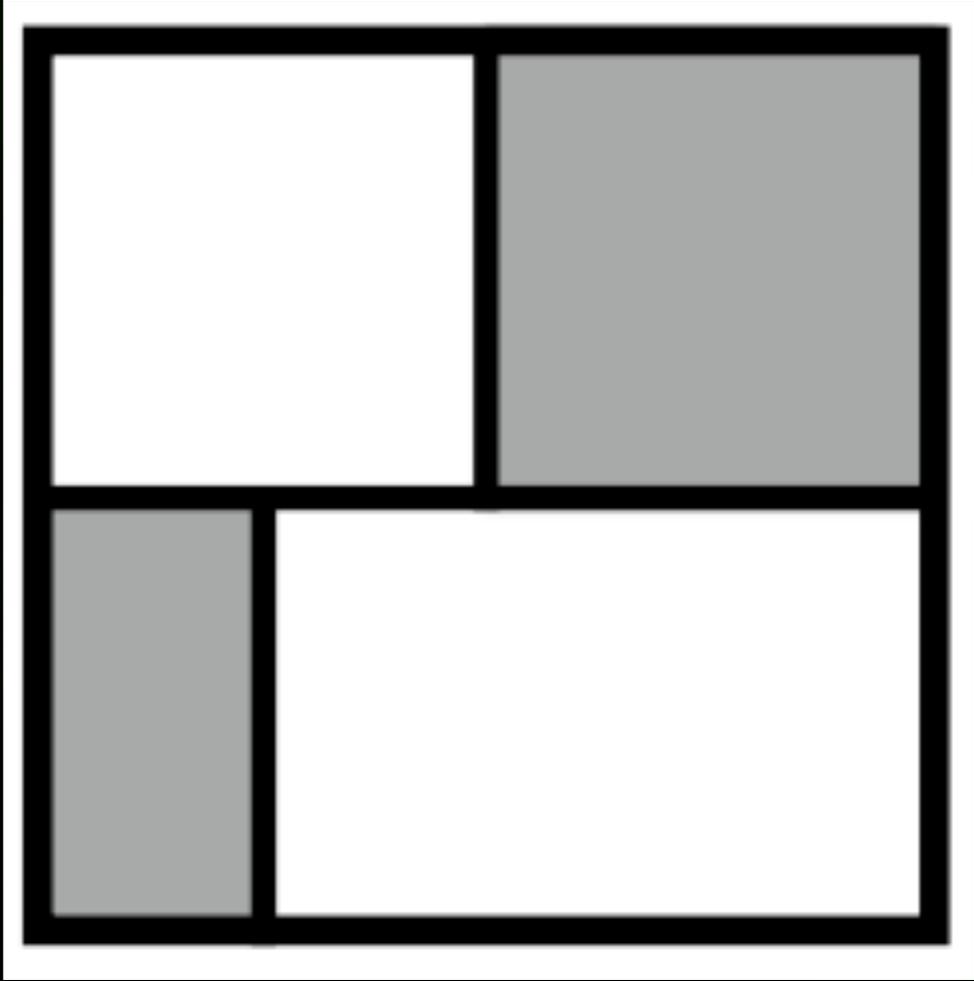












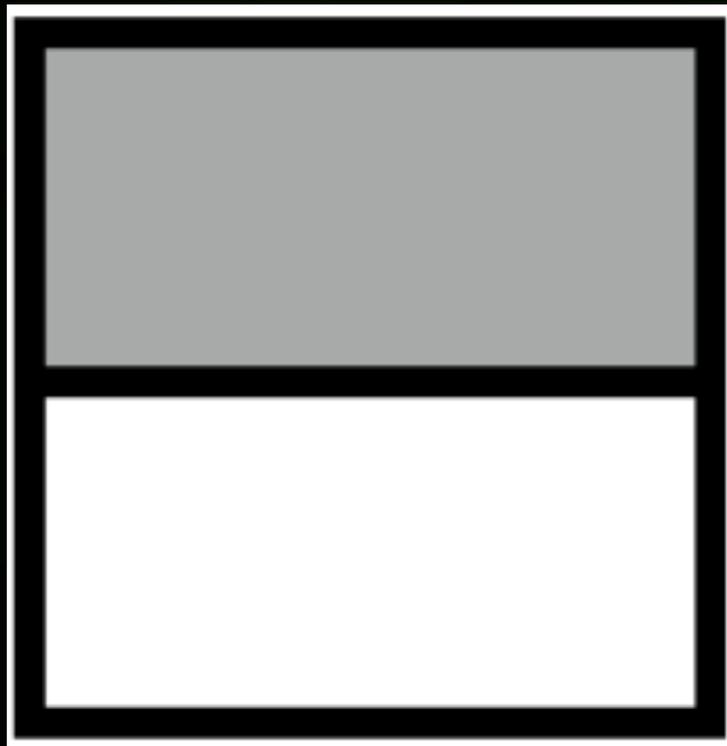
What's the Sum?

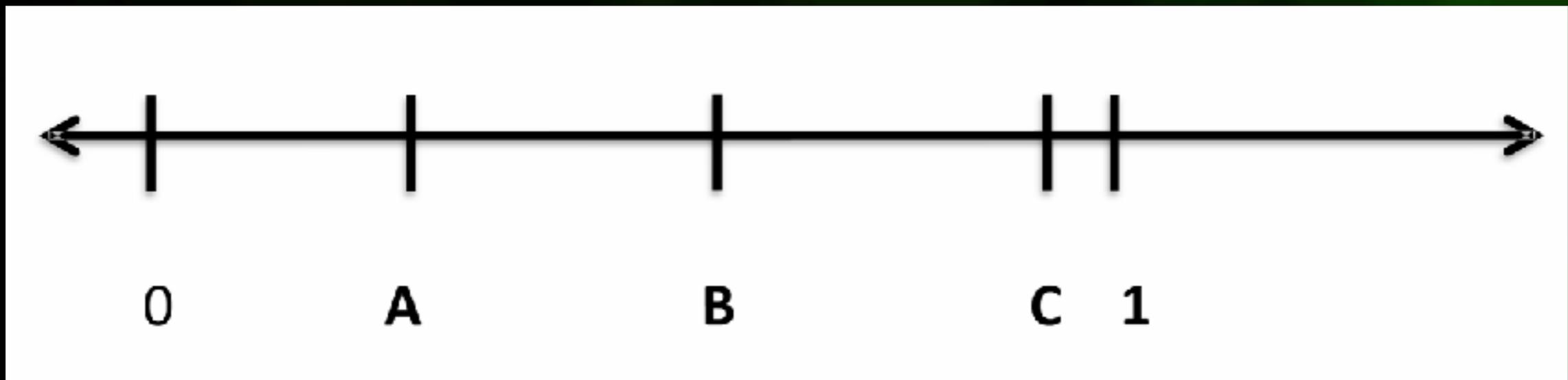


What's the Sum?



What's the Sum?

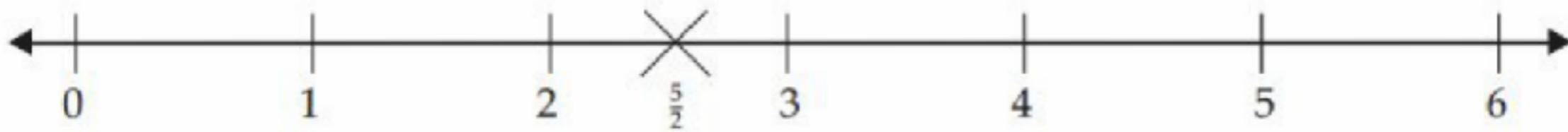






Dotty Pairs Game

The students play in pairs. One student takes dots, the other takes crosses. Place the cards (cards 1–6, two lots, see Material Master 4-1) face down in a pile. The players take turns turning over two cards. The numbers are used to form a fraction, e.g., 2 and 5 are turned over, so $\frac{5}{2}$ or $\frac{2}{5}$ can be made. One fraction is chosen, made with the fraction pieces, if necessary, and marked on a 0–6 number line with the player's identifying mark (dot or cross).



Players take turns. The aim of the game is to get three of their marks uninterrupted by their opponent's marks on the number line. If a player chooses a fraction that is equivalent to a mark that is already there, they miss that turn.

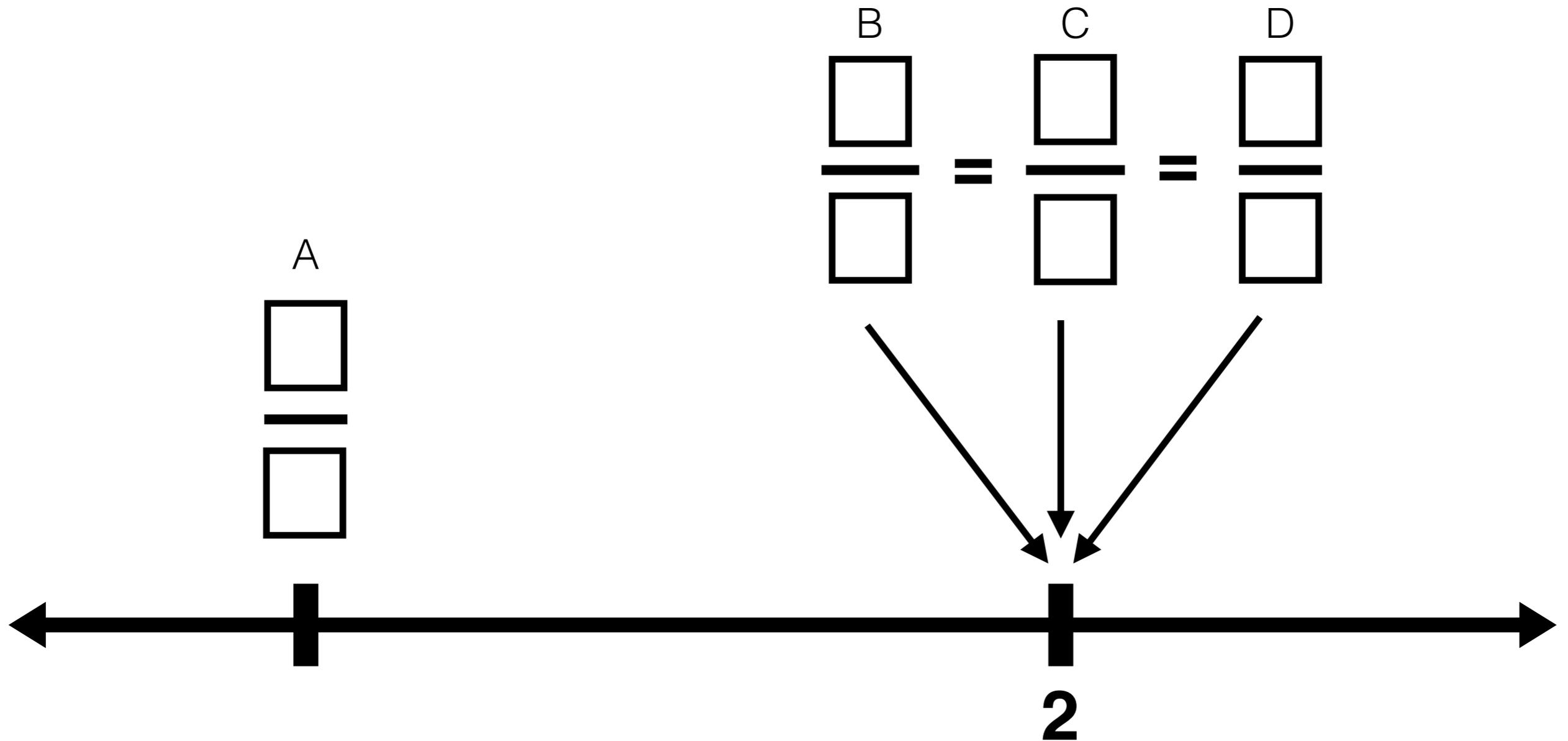
NB: A fraction such as $\frac{4}{1}$ can be made using the cards. Students may not be familiar with fractions in this form and the meaning of the numerator and denominator will need to be explored with the fraction circles.



random dice roller

Open Middle

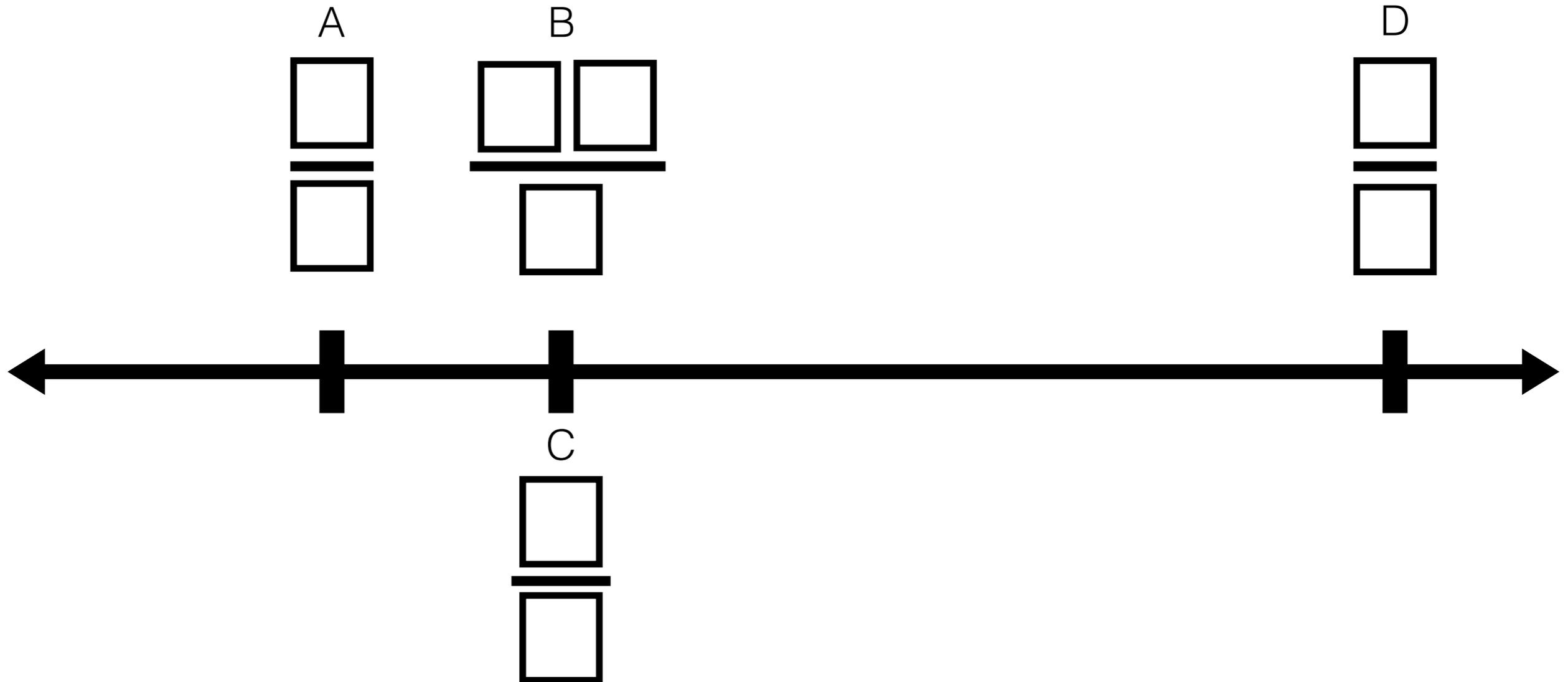
Directions: Using the whole numbers 1-9 no more than one time each, create and place 4 fractions on the number line in the correct order. A is less than 2. Fractions B, C, and D equal 2.



Open Middle

CCSS.MATH.CONTENT.4.NF.A.2

Directions: Using the whole numbers 1-9 once each, create and place 4 fractions *greater than 1* on the number line in the correct order. (*fractions B & C are equal*)

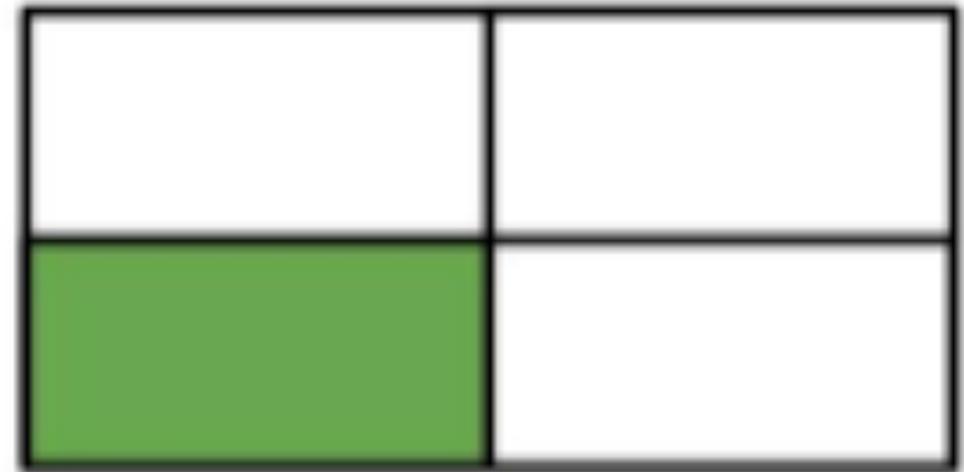


$$\frac{1}{20}$$

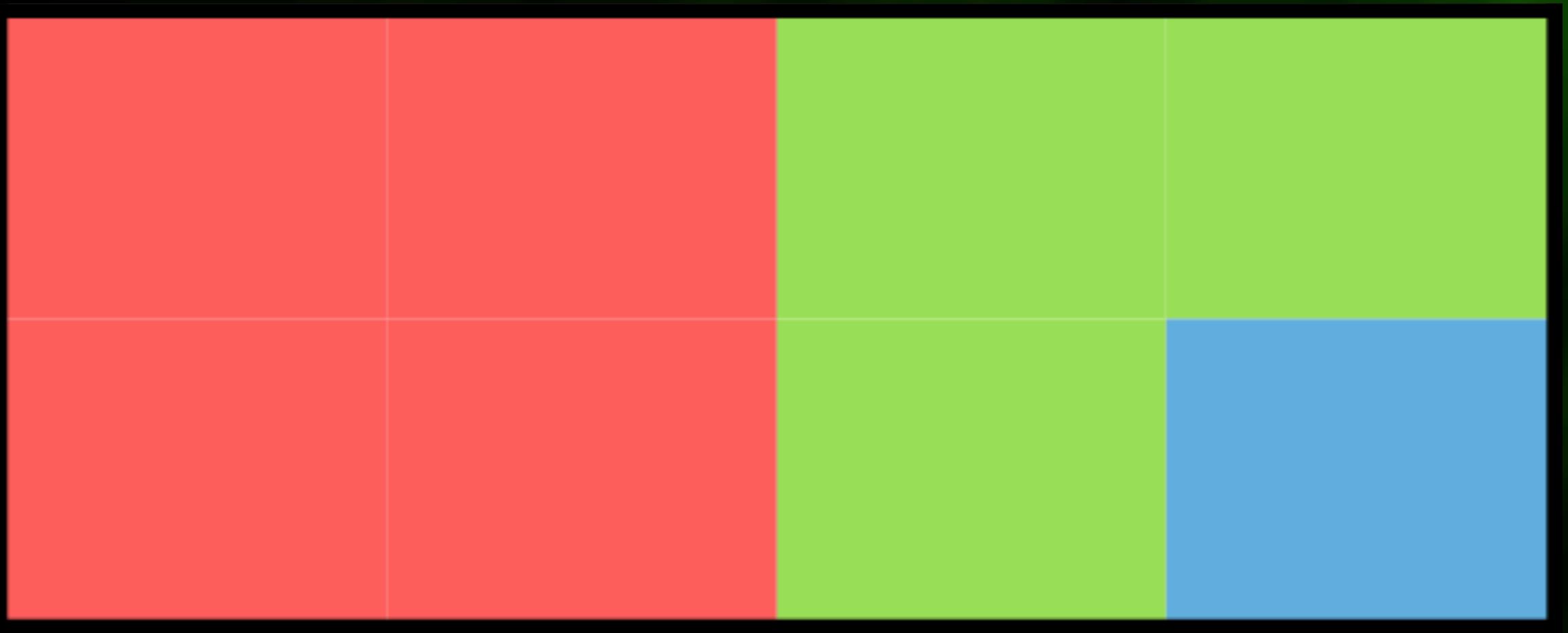
$$\frac{20}{25}$$

$$\frac{2}{3}$$

$$\frac{5}{4}$$



by @DrBrianLack



Pause



Try to model with only a drawing to start. You can label your drawing with numbers but try to use no calculations or algorithms.

1. Macey and Bryson have 13 cookies. If they share the cookies equally, how many cookies would each person get?
2. There are 11 yards of ribbon for 4 people to share. How many yards of ribbon can each person get if they share the ribbon equally?
3. 12 children in art class have to share 8 packages of clay so that everyone gets the same amount. How much clay can each child have?

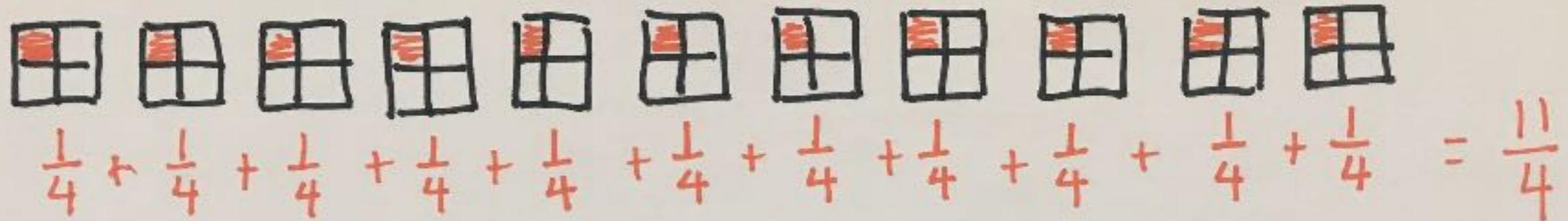
2. There are 11 yards of ribbon for 4 people to share. How many yards of ribbon can each person get if they share the ribbon equally?



$$2 + \frac{1}{2} + \frac{1}{4} = 2\frac{3}{4}$$

Additive Coordination: Sharing one item at a time

2. There are 11 yards of ribbon for 4 people to share. How many yards of ribbon can each person get if they share the ribbon equally?


$$\frac{1}{4} + \frac{1}{4} = \frac{11}{4}$$

$$\frac{11}{4}$$

$$\frac{11}{4} = \frac{1}{4} \times 11 = 2\frac{3}{4}$$

Additive Coordination: Sharing one item at a time

Equivalent Fractions

Equal Fraction

$$\frac{2}{3} = \frac{\square}{\square} \qquad \frac{3}{4} = \frac{\square}{\square}$$

$$\frac{2}{6} = \frac{\square}{\square}$$

Equal Fraction

$$\frac{2}{3} = \frac{5}{6} \quad \frac{3}{4} = \frac{7}{8}$$

$$\frac{2}{6} = \frac{5}{9}$$

Equal Fraction

WTF?

$$\frac{2}{6} = \frac{5}{9}$$

Equal Fraction

What's **T**hat **F**raction **?**

$$\frac{2}{6} = \frac{5}{9}$$

Equal Fraction

$$\frac{2}{3} = \frac{5}{6} \quad \frac{3}{4} = \frac{7}{8}$$

$$\frac{2}{6} = \frac{5}{9}$$

$\frac{3}{6} + \frac{1}{6}$ is equal to which of the following?

A. $\frac{3}{6}$

B. $\frac{4}{12}$

C. $\frac{6}{18}$

D. $\frac{8}{12}$

E. None of the above

Comparing Fractions





Which girl ate more apple?



Which girl ate more apple?

What information do you need to know?

Pause





Slices: 10

Slices: 7



twelfths

Pause

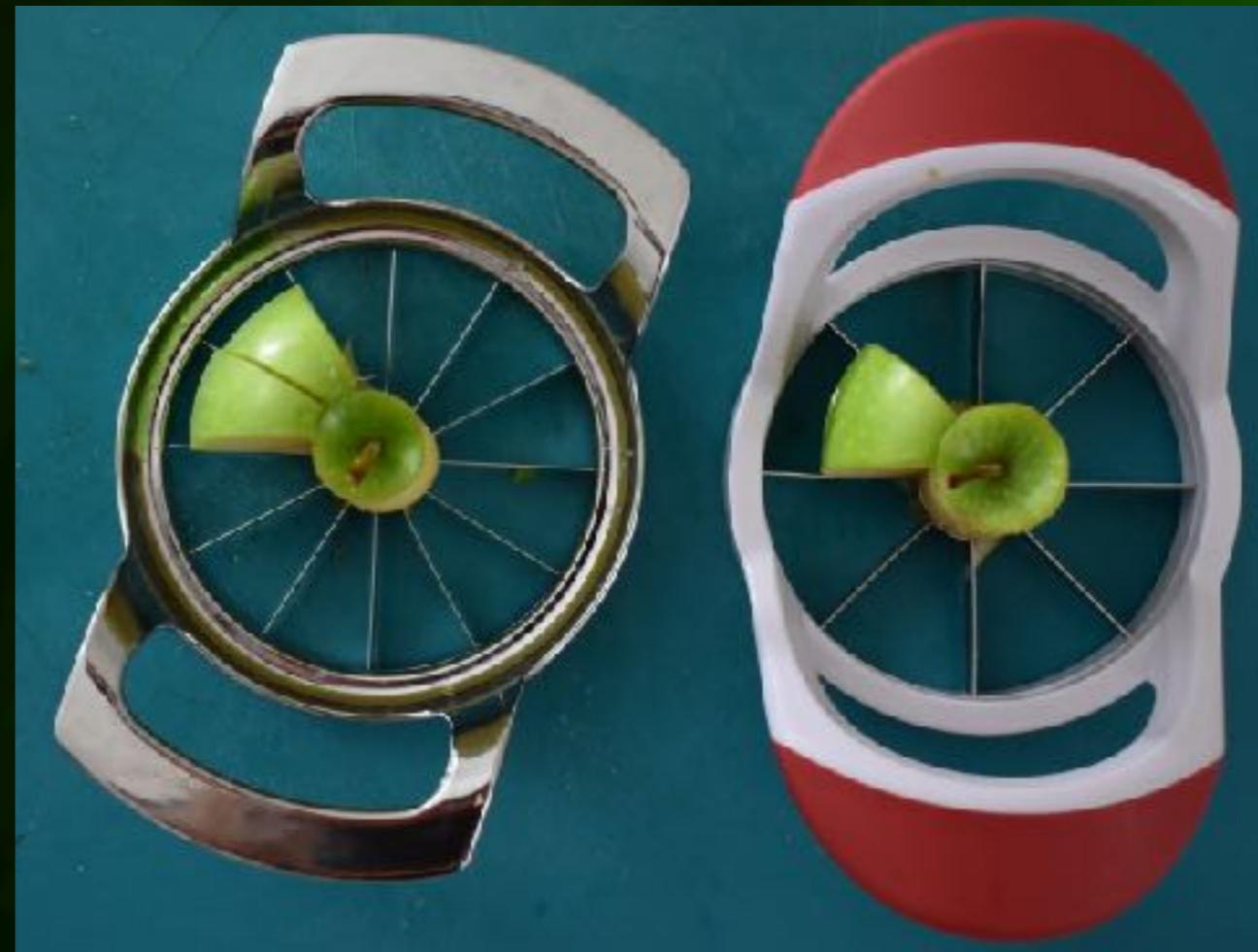
||



eighths

Apple Eat Off

Act-3



Big sister ate $\frac{10}{12}$ of an apple and little sister ate

$\frac{7}{8}$ of an apple. Which sister ate more apple?

The **5** practices are:

1. **Anticipating** student responses to challenging mathematical tasks;
2. **Monitoring** students' work on and engagement with the tasks;
3. **Selecting** particular students to present their mathematical work;
4. **Sequencing** the student responses that will be displayed in a specific order and;
5. **Connecting** different students' responses and connecting the responses to key mathematical ideas.

6. Construct a viable argument or share a reflection:

Answer

$$\frac{10}{12} > \frac{7}{10}$$

ten is more than seven

6. Construct a viable argument or share a reflection:

Answer

$$\frac{10}{12}$$

2 pieces left over

~~$$\frac{1}{8}$$~~

~~1 piece leftover~~
made

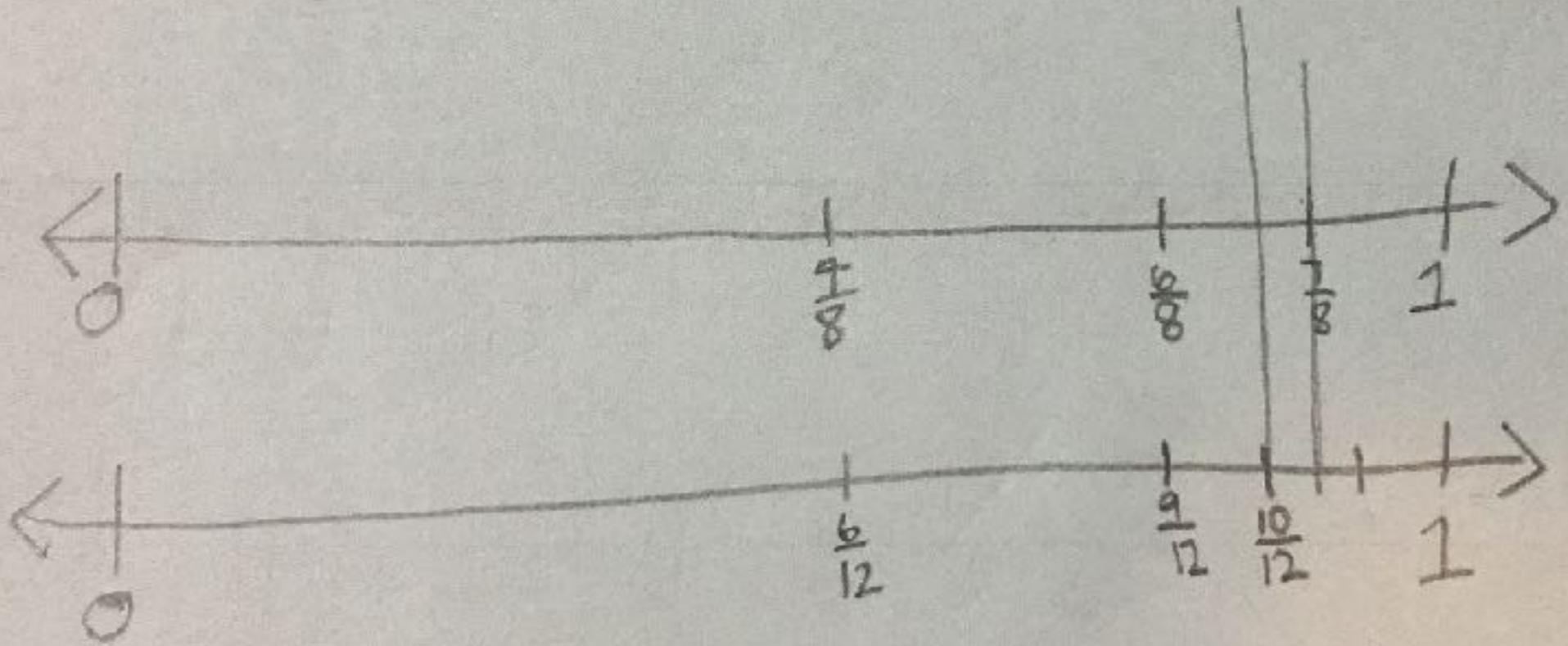
$$\frac{10}{12} > \frac{1}{8}$$

6. Construct a viable argument or share a reflection:

$$\begin{array}{r} 80 < 84 \\ \hline 70 & 7 \\ \hline 12 & 80 \end{array}$$

6. Construct a viable argument or share a reflection:

$$\frac{7}{8} > \frac{10}{12}$$



6. Construct a viable argument or share a reflection:

$$\frac{7}{8}$$

$$\frac{10}{12}$$

$\frac{1}{8}$ is a smaller piece
left over

$$\frac{1}{8}$$

$$\frac{2}{12} = \frac{1}{6}$$

$$\frac{7}{8} > \frac{10}{12}$$

6. Construct a viable argument or share a reflection:

$$\frac{10}{12} \times 2 = \frac{20}{24}$$

$$\frac{7}{8} \times 3 = \frac{21}{24}$$

$$\frac{20}{24} < \frac{21}{24}$$

$$\frac{10}{12} < \frac{7}{8}$$

6. Construct a viable argument or share a reflection:

S1

$\frac{7}{8}$ $\frac{10}{12}$ $\frac{1}{8}$ is a smaller piece left over

$\frac{1}{8}$ $\frac{2}{12} = \frac{1}{6}$ $\frac{7}{8} > \frac{10}{12}$

6. Construct a viable argument or share a reflection:

Answer

S2

$\frac{10}{12}$ 2 pieces left over

$\frac{7}{8}$ 1 piece leftover more

$\frac{10}{12} > \frac{7}{8}$

Pause
||

6. Construct a viable argument or share a reflection:

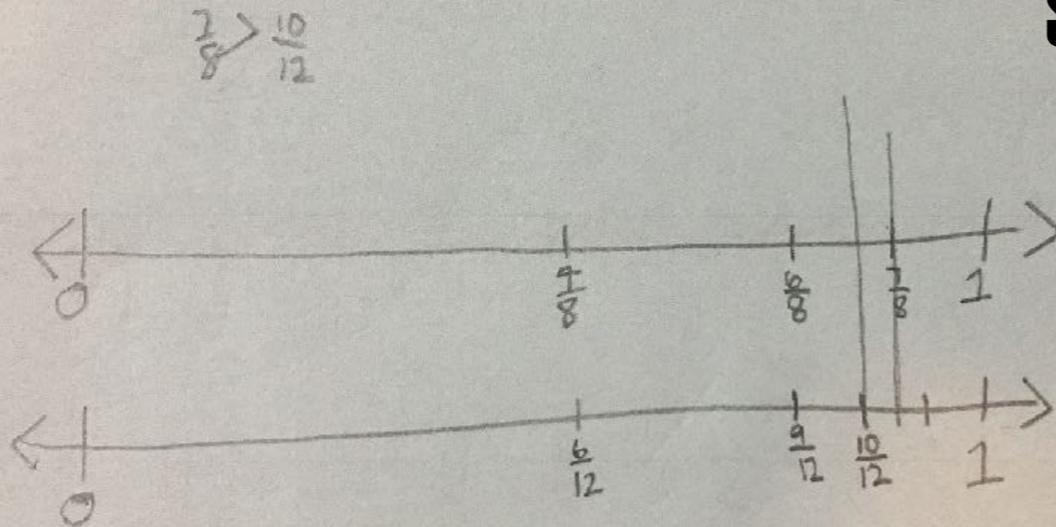
S4

$\frac{10}{12} \times 2 = \frac{20}{24}$ $\frac{26}{24} < \frac{21}{24}$

$\frac{7}{8} \times 3 = \frac{21}{24}$ $\frac{10}{12} < \frac{7}{8}$

6. Construct a viable argument or share a reflection:

S3



6. Construct a viable argument or share a reflection:

Answer

S5

$\frac{10}{12} > \frac{7}{10}$ ten is more than seven

6. Construct a viable argument or share a reflection:

S6

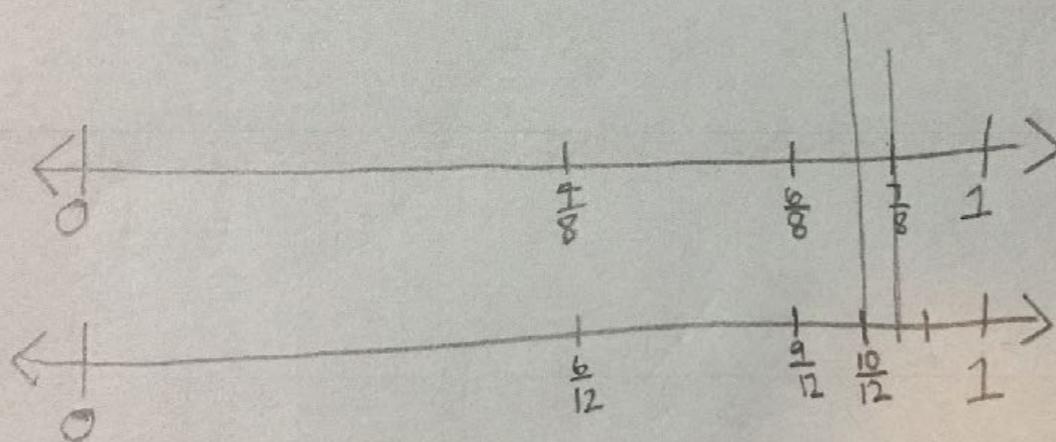
$80 < 84$

~~$\frac{20}{12}$~~ ~~$\frac{7}{8}$~~

6. Construct a viable argument or share a reflection:

S3

$$\frac{7}{8} > \frac{10}{12}$$



6. Construct a viable argument or share a reflection:

S4

$$\frac{10}{12} \times 2 = \frac{20}{24}$$

$$\frac{7}{8} \times 3 = \frac{21}{24}$$

$$\frac{20}{24} < \frac{21}{24}$$

$$\frac{10}{12} < \frac{7}{8}$$

6. Construct a viable argument or share a reflection:

S1

$$\frac{7}{8} \quad \frac{10}{12} \quad \rightarrow \frac{1}{8} \text{ is a smaller piece left over}$$

$$\left(\frac{1}{8}\right) \quad \frac{2}{12} = \frac{1}{6} \quad \frac{7}{8} > \frac{10}{12}$$

What We Know

Misconceptions...

- counting the number of pieces leftover to compare
- seeing numerator & denominator as an individual quantity
- tricks and procedures for comparing fractions

Understanding...

- equally partitioning a number line
- common denominator understanding without multiplying to find a common factor
- equivalent fractions
- comparison of unit fractions

It Takes 3 to Prove it to Me

Comparing Fractions

CCSS.MATH.CONTENT.3.NF.A.3.D

Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

CCSS.MATH.CONTENT.4.NF.A.2

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

Making Sense Series

The Progression of Fractions
Meaning, Equivalence, & Comparison

created by Graham Fletcher

 @gfletchy

www.gfletchy.com

Task Instruction:

1. Construct a square with exactly $\frac{1}{4}$ the area of the original square.
2. Construct a triangle with exactly $\frac{1}{4}$ the area of the original square.
3. Construct another triangle, also with $\frac{1}{4}$ the area, that is not congruent to the first one you constructed.
4. Construct a square with exactly $\frac{1}{2}$ the area of the original square.



BIG PAD

Dream your big idea.

NOTAS GIGANTES

Sueña tu gran idea.

GRANDBLOC

Imaginez de grandes idées.



CONT.



CONT.

30



11 IN/PO x 11 IN/PO (27,9 cm x 27,9 cm)



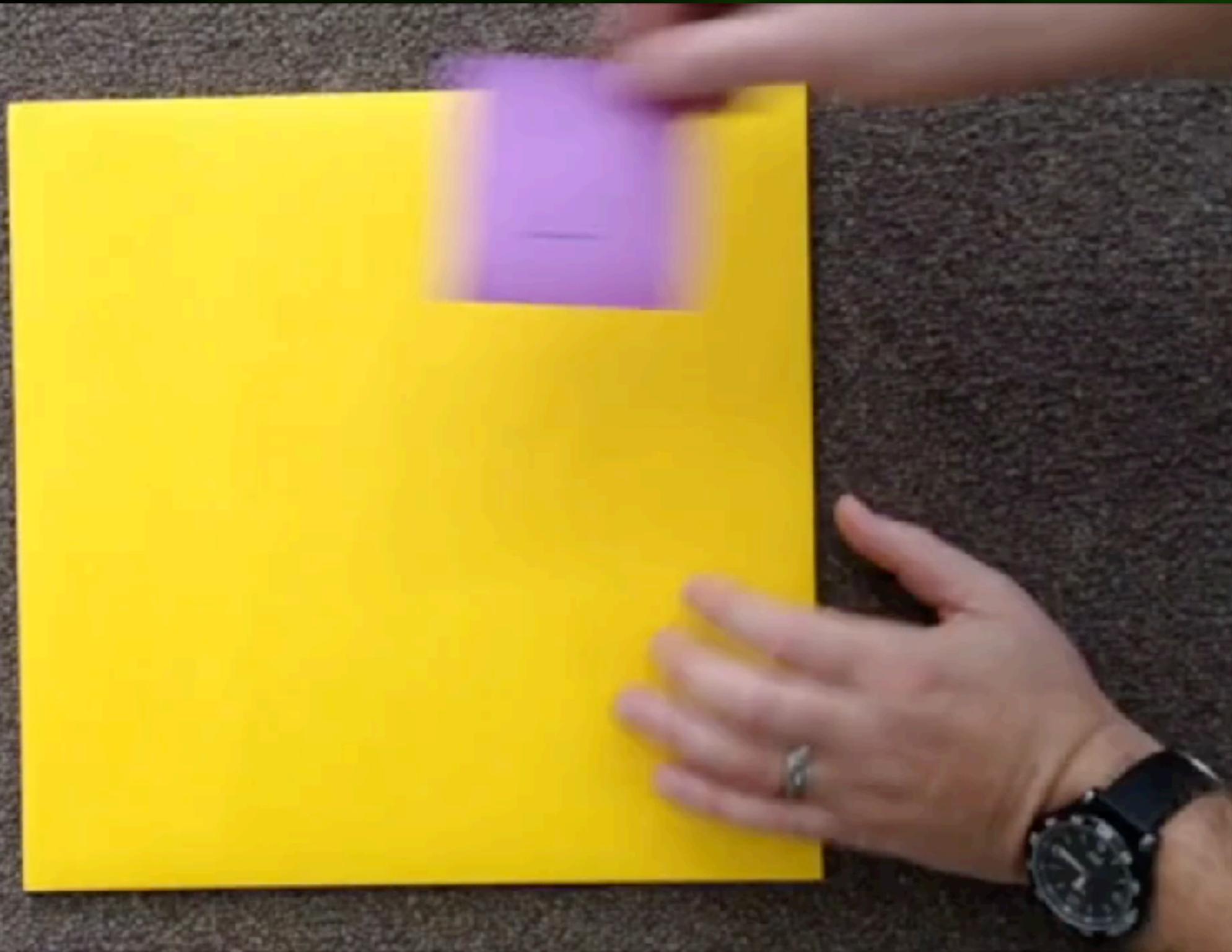
Pop-up Notes

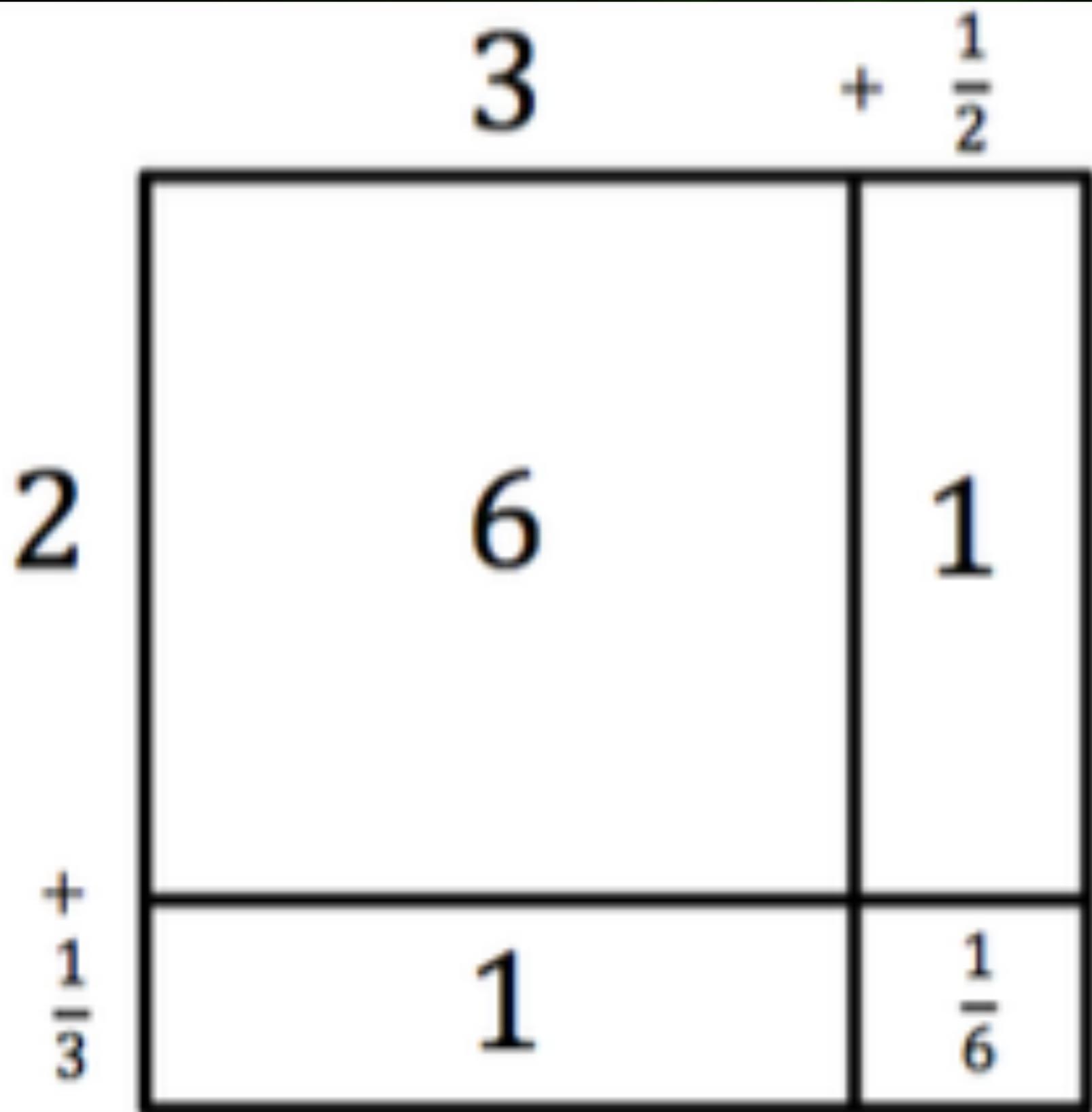


For
Pop-up Note
Dispensers!

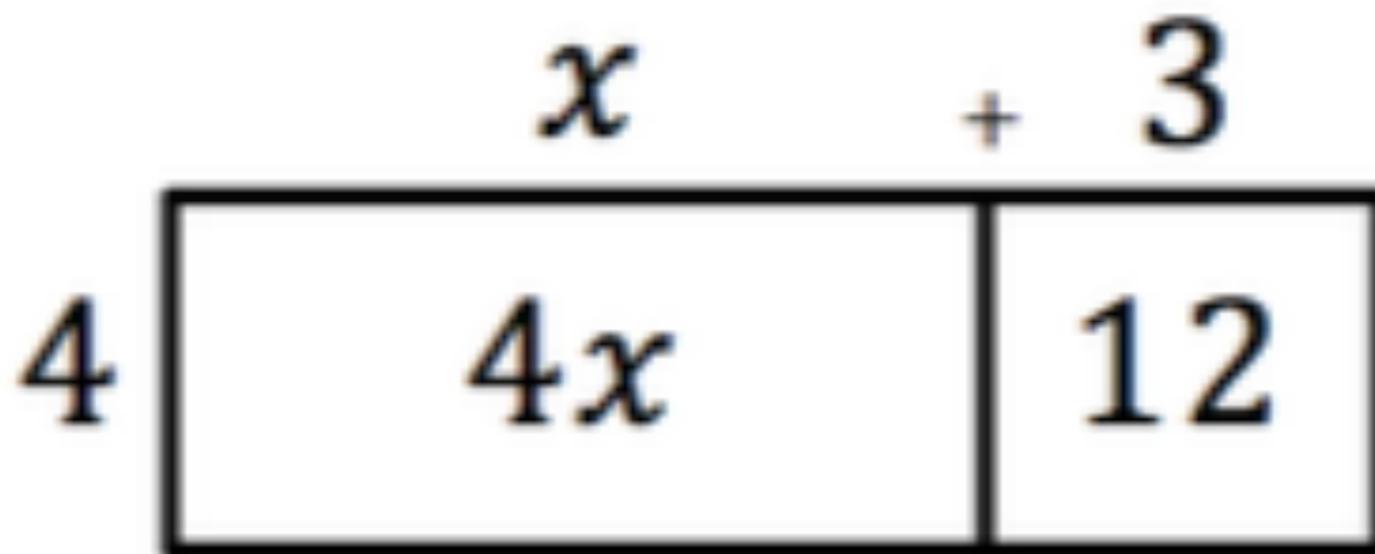
90 SHEETS
3 IN x 3 IN
76 mm x 76 mm







$$4(x + 3)$$



$$(x + 8)(x + 7)$$

	x	$+ 7$
x	x^2	$7x$
$+ 8$	$8x$	56

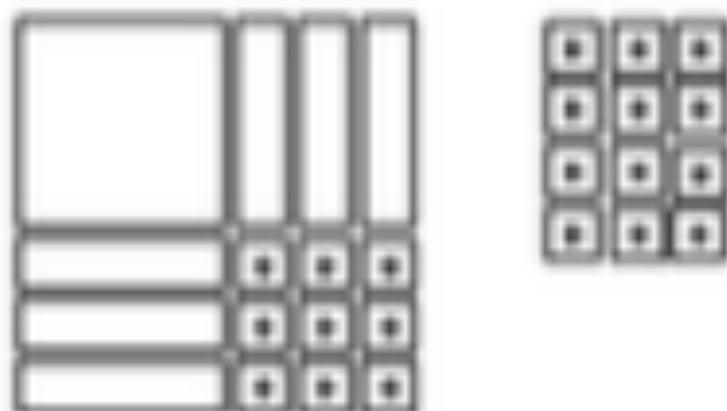
$$x^2 + 6x - 3 = 0$$



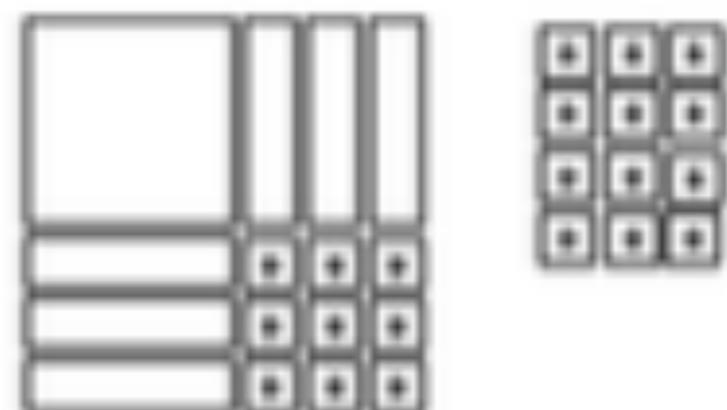
$$x^2 + 6x + \quad = 3$$



$$x^2 + 6x + 9 = 12$$



$$(x + 3)^2 = 12$$



$$x^2 + 6x - 3 = 0$$

Harnessing the Power of Modeling Tasks through the Lens of a Math Progression

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