The Power of Progressions: Untangling the Knotty Areas of Teaching and Learning Mathematics

Graham Fletcher

gfletchy@gmail.com @gfletchy www.gfletchy.com

Task Instruction:

1.Construct a square with exactly ¼ the area of the original square.

2. Construct a triangle with exactly ¼ the area of the original square.

3. Construct another triangle, also with 1⁄4 the area, that is not congruent to the first one you constructed.

4. Construct a square with exactly ½ the area of the original square.

Name: ______________________________ Date: ______________

1. What did you notice?

2. What do you wonder?

3. Main Question:

4. Make an estimate.

low estimate

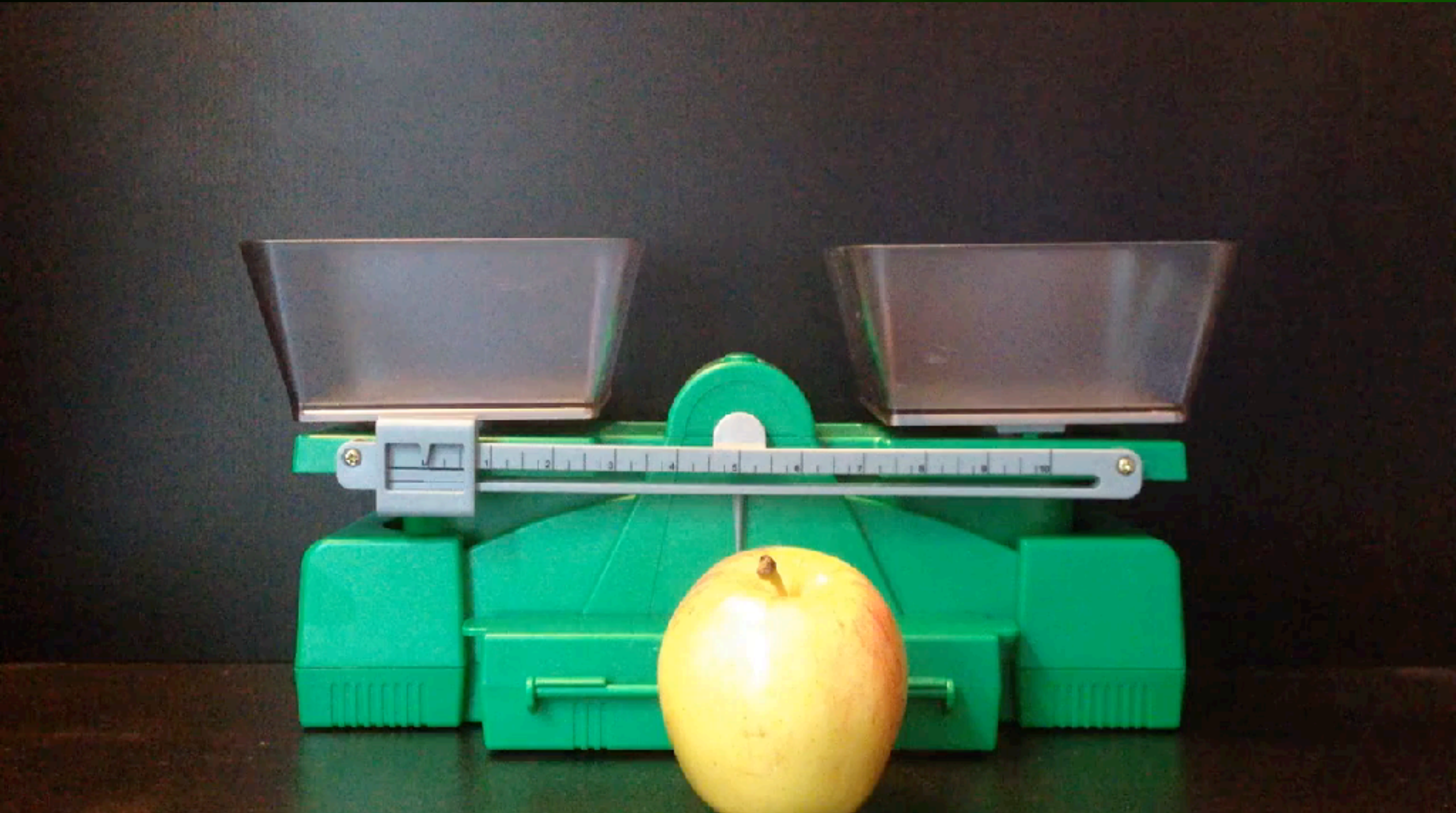
high estimate

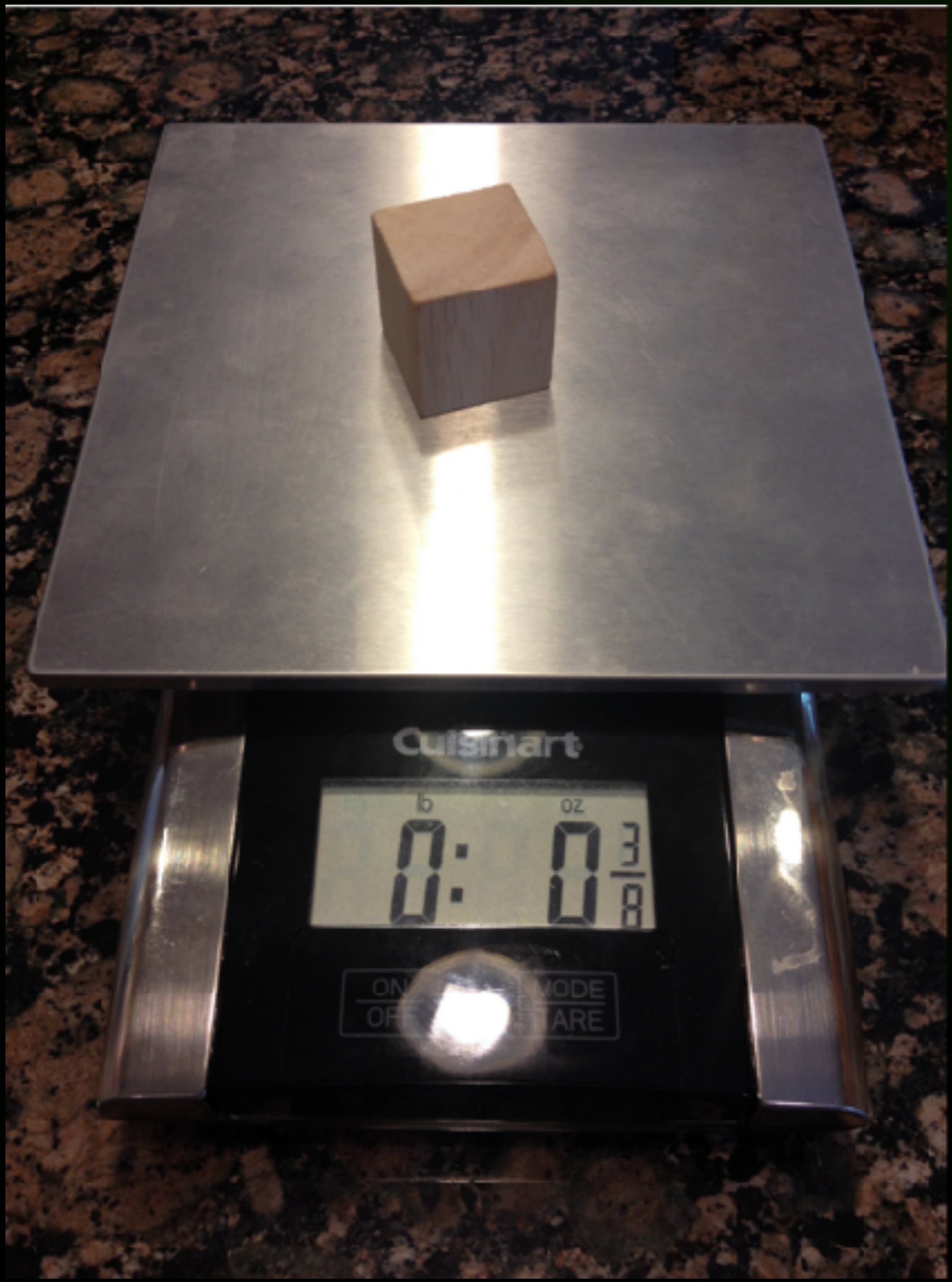
Place an "X" to represent your estimate on the number line.

5. What information do you need?

6. Construct a viable argument or share a reflection:

Answer




Cuisinart
lb oz
0: 0 3/8
ON/OFF MODE/TARE


Cuisinart
lb oz
0: 5 1/4
ON/OFF MODE/TARE


0:00 / 0:14

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Standards for Mathematical Practice

4. Model with mathematics.

What is mathematical modeling?

What is modeling with mathematics?

What ISN'T mathematical modeling

- The use of manipulatives does not ensure that modeling with mathematics is taking place.
- If the mathematics is not contextualized, modeling with mathematics cannot exist.
- Modeling with mathematics does not mean, "I do, we do, you do."

Model with Mathematics

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Model with Mathematics

Mathematically proficient students can apply the mathematics they know to **solve problems arising in everyday life**, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. Mathematically proficient students who can apply what they know are comfortable **making assumptions and approximations** to simplify a complicated situation, realizing that these may need revision later. They are able to **identify important quantities** in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can **analyze those relationships** mathematically to draw conclusions. They routinely **interpret their mathematical results** in the context of the situation and **reflect on whether the results make sense**, possibly improving the model if it has not served its purpose.

Mathematical Modeling in the Elementary Grades

<u>Contextualized</u>

Real-World Problem

Validate Answer

<u>Decontextualized</u>

Make assumptions & estimates

Identify missing variables

Define a solution path using variables

Solve & interpret results

Mathematical Modeling in the Elementary Grades

Contextualized

Real-World Problem

Validate Answer

Decontextualized

Make assumptions & estimates

Identify missing variables

Define a solution path using variables

Solve & interpret results

Mathematical Modeling in the Elementary Grades

Contextualized

Decontextualized

Real-World Problem

Make assumptions & estimates

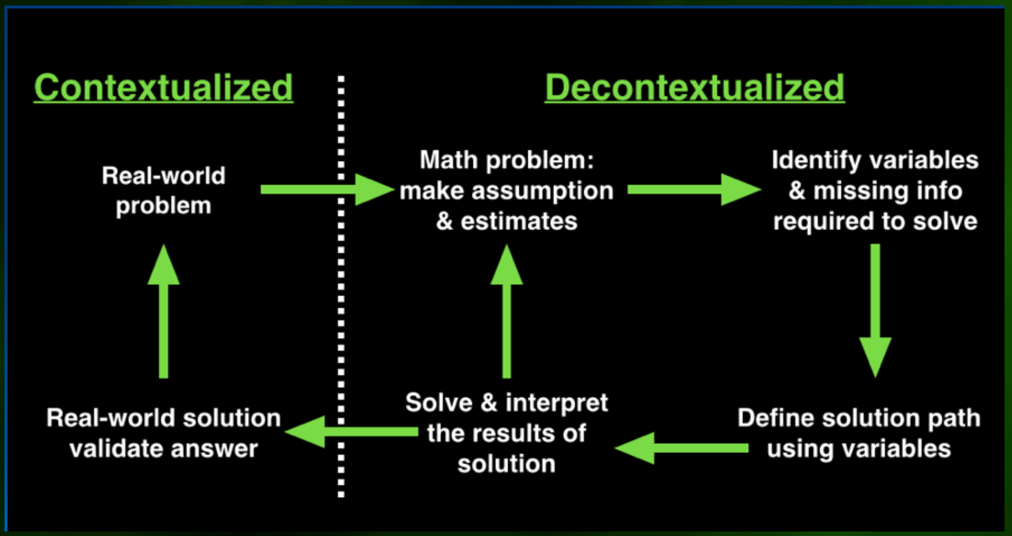
Identify missing variables

Define a solution path using variables

Solve & interpret results

Validate Answer

3-Act Tasks



Act 1:

- Real world problem or scenario presented
- What do you notice? What do you wonder?
- Make estimates

Act 2:

- Identify missing variables and missing variables to solve
- Define solution path using variables

Act 3:

- Solve and interpret results of the solution
- Validate answer

Most asked questions:

- How often should we use 3-Act Tasks?
- When should we use 3-Act tasks? How do they fit into the scope of a unit?
- How long does one task usually take?
- What if we don’t have the time?

Orchestrating Discussions

Five practices constitute a model for effectively using student responses in whole-class discussions that can potentially make teaching with high-level tasks more manageable for teachers.

Margaret S. Smith, Elizabeth K. Hughes, Randi A. Engle, and Mary Kay Stein

Margaret S. Smith, pegs@pitt.edu, is an associate professor of mathematics education at the University of Pittsburgh. Over the past decade, she has been developing research-based materials for use in the professional development of mathematics teachers and studying what teachers learn from the professional development in which they engage. **Elizabeth K. Hughes**, elizabeth.hughes@uni.edu, recently finished her doctorate in mathematics education at the University of Pittsburgh. Her areas of interest include preservice secondary mathematics teacher education and the use of practice-based materials in developing teachers' understanding of what it means to teach and learn mathematics. **Randi A. Engle**, raengle@berkeley.edu, is an assistant professor of mathematics education and the social context of learning at the University of California Berkeley. She is interested in developing practical theories for how mathematics teachers can create discussion-based learning environments that promote strong student engagement, learning, and transfer. **Mary Kay Stein**, mkstein@pitt.edu, is a professor of learning sciences and policy and the director of the Learning Policy Center at the University of Pittsburgh. Her research focuses on instructional practice and the organizational and policy conditions that shape it.

Discussions that focus on cognitively challenging mathematical tasks, namely, those that promote thinking, reasoning, and problem solving, are a primary mechanism for promoting conceptual understanding of mathematics (Hatano and Inagaki 1991; Michaels, O'Connor, and Resnick forthcoming). Such discussions give students opportunities to share ideas and clarify understandings, develop convincing arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things from other perspectives (NCTM 2000).

Although discussions about high-level tasks provide important

The 5 practices are:

1. **Anticipating** student responses to challenging mathematical tasks;
2. **Monitoring** students' work on and engagement with the tasks;
3. **Selecting** particular students to present their mathematical work;
4. **Sequencing** the student responses that will be displayed in a specific order and;
5. **Connecting** different students' responses and connecting the responses to key mathematical ideas.

MTMS: Vol. 14, No. 9, May 2009-5 Practices for Orchestrating Productive Mathematics Discussions

Task Planning Page

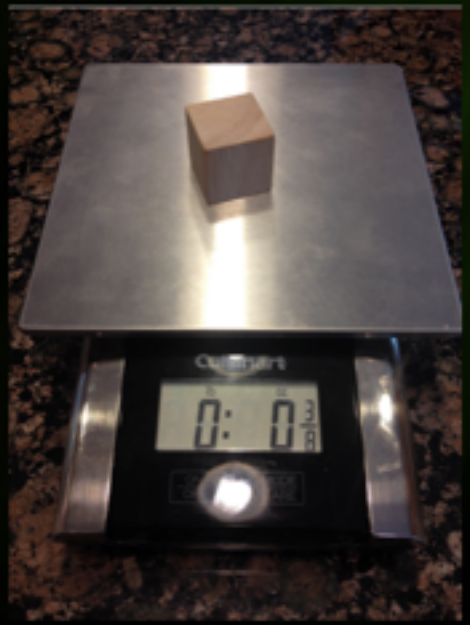
Learning Target:

Questions and Look-Fors:

Strategy	Who and What	Order

Notes:

Anticipating → Monitoring → Selecting → Sequencing → Connecting



Task Planning Page

Learning Target:

Questions and Look-Fors:

Strategy	Who and What	Order

Notes:

Anticipating → Monitoring → Selecting → Sequencing → Connecting

Task Planning Page

Learning Target:

Questions and Look-Fors:

Strategy	Who and What	Order

Notes:

Sequence the order students will share during the closing.

Anticipating → Monitoring → Selecting → Sequencing → Connecting

Task Planning Page

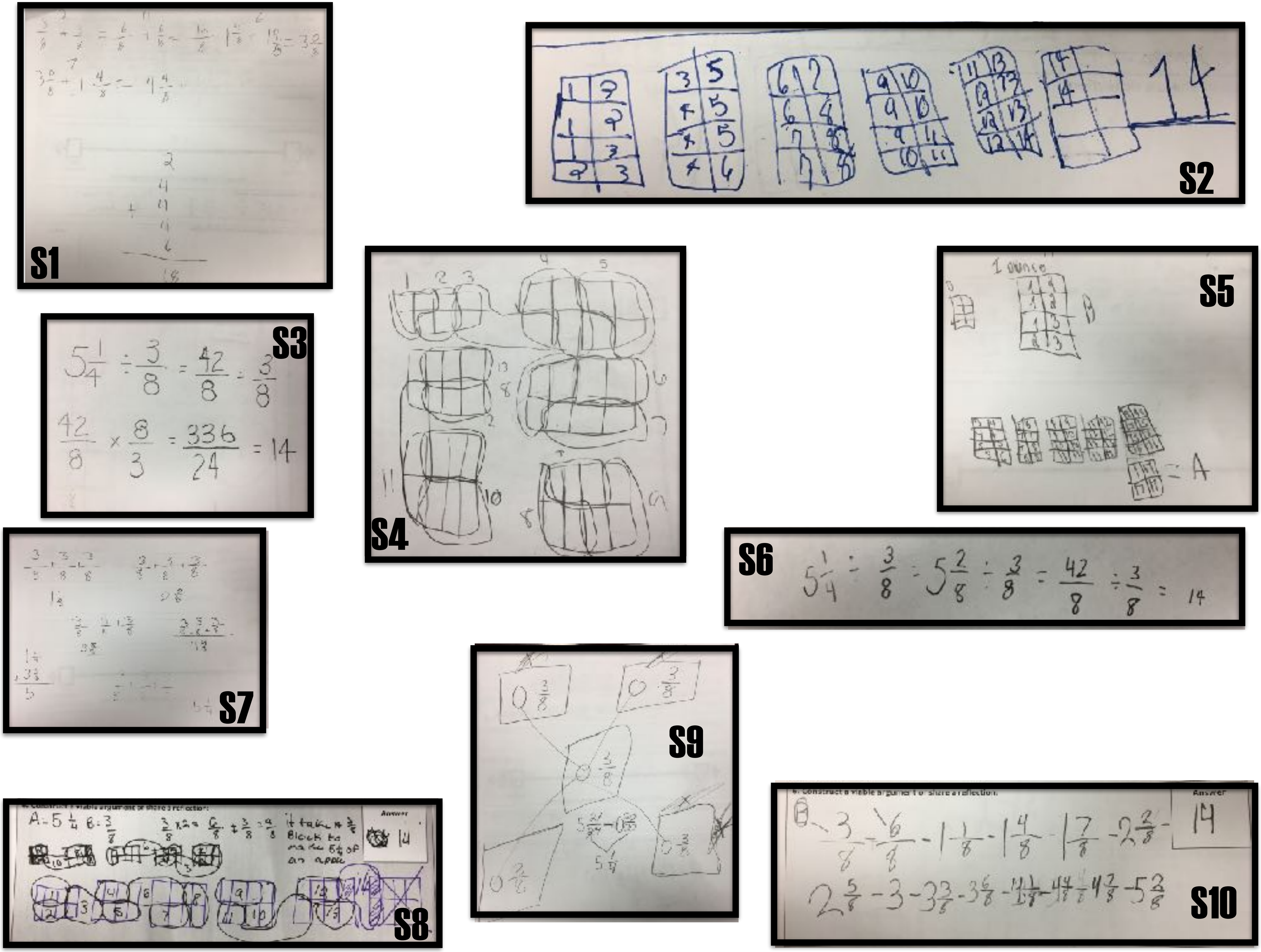
Learning Target:

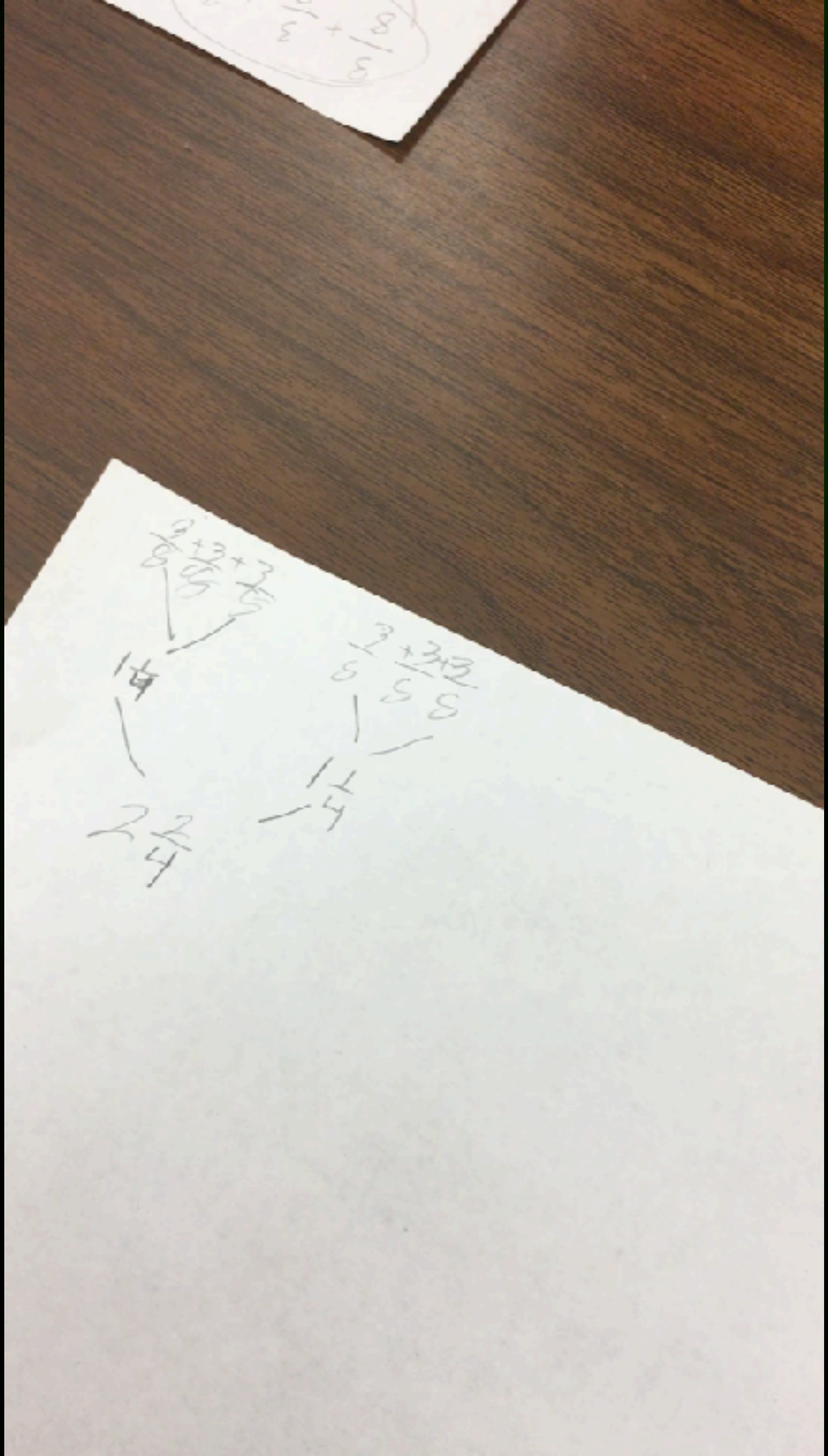
Questions and Look-Fors:

Strategy	Who and What	Order

Notes:

Anticipating → Monitoring → Selecting → Sequencing → Connecting


S1
S2
14
S3
5 1/4 ÷ 3/8 = 42/8 ÷ 3/8
42/8 × 8/3 = 336/24 = 14
S4
S5
1 ounce
S6
5 1/4 ÷ 3/8 = 5 2/8 ÷ 3/8 = 42/8 ÷ 3/8 = 14
S7
S8
Answer
14
S9
S10
Answer
14



Be the teacher…

$\frac{3}{8} + \frac{3}{8} + \frac{3}{8}$

$\frac{3 + 3 + 3}{8 \quad 8 \quad 8}$

$1\frac{1}{4} + 1\frac{1}{4}$

$\frac{3 + 3 + 3}{8 \quad 8 \quad 8}$

$1\frac{1}{4}$

$+ 3\frac{3}{4}$

$$\frac{3}{8}+\frac{3}{8}+\frac{3}{8}$$

$1\frac{1}{8}$

$$\frac{3}{8}+\frac{3}{8}+\frac{3}{8}$$

$2\frac{2}{8}$

$$\frac{3}{8}+\frac{3}{8}+\frac{3}{8}$$

$3\frac{3}{8}$

$$\frac{3}{8}+\frac{3}{8}+\frac{3}{8}$$

$4\frac{4}{8}$

$$\begin{array}{r} 1\frac{1}{4} \\ +\,3\frac{3}{8} \\ \hline 5 \end{array}$$

$$\frac{3}{8}+\frac{3}{8}+\frac{3}{5}$$

$5\frac{1}{4}$

$$5\frac{1}{4} \div \frac{3}{8} = \frac{42}{8} \div \frac{3}{8}$$

$$\frac{42}{8} \times \frac{8}{3} = \frac{336}{24} = 14$$

$$5\frac{1}{4} \div \frac{3}{8} = 5\frac{2}{8} \div \frac{3}{8} = \frac{42}{8} \div \frac{3}{8} = 14$$

$$\frac{3}{8} \overset{2}{+} \frac{3}{8} = \frac{6}{8} \overset{4}{+} \frac{6}{8} = \frac{12}{8} \quad 1\frac{4}{8} \overset{6}{+} 1\frac{4}{8} = 3\frac{0}{8}$$

$$3\frac{0}{8} \overset{7}{+} 1\frac{4}{8} = 4\frac{4}{8} + \frac{12}{8}$$

6. Construct a viable argument or share a reflection:

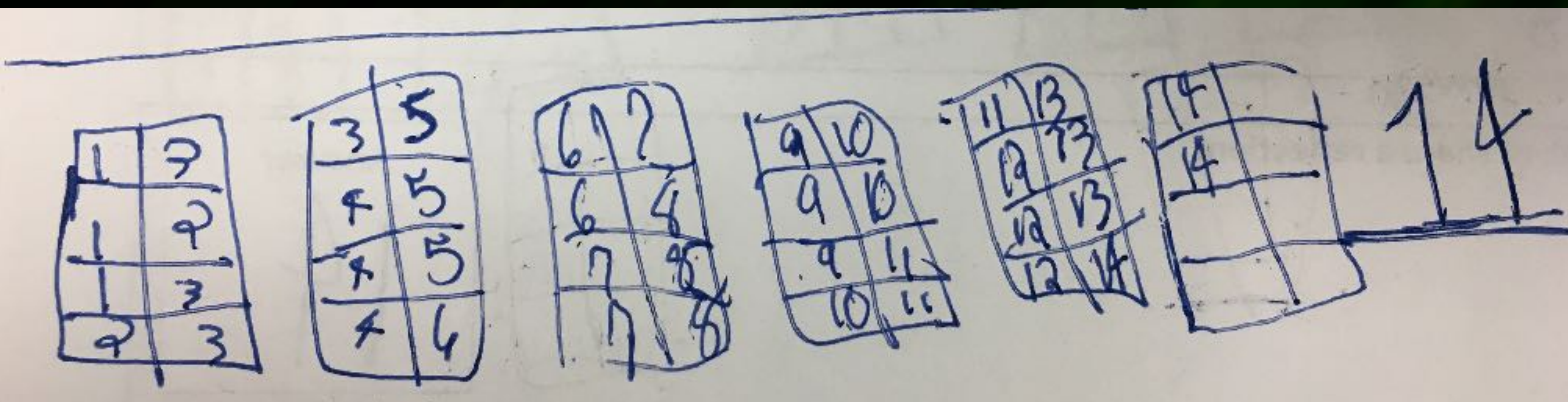
Answer

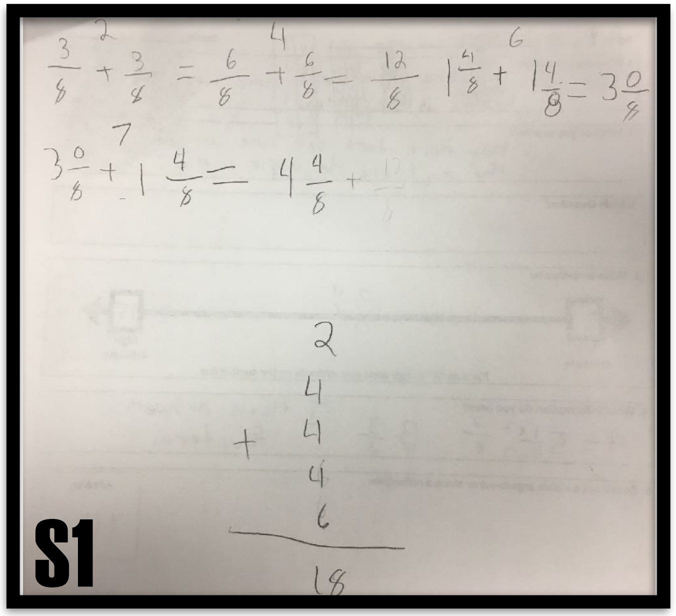
14

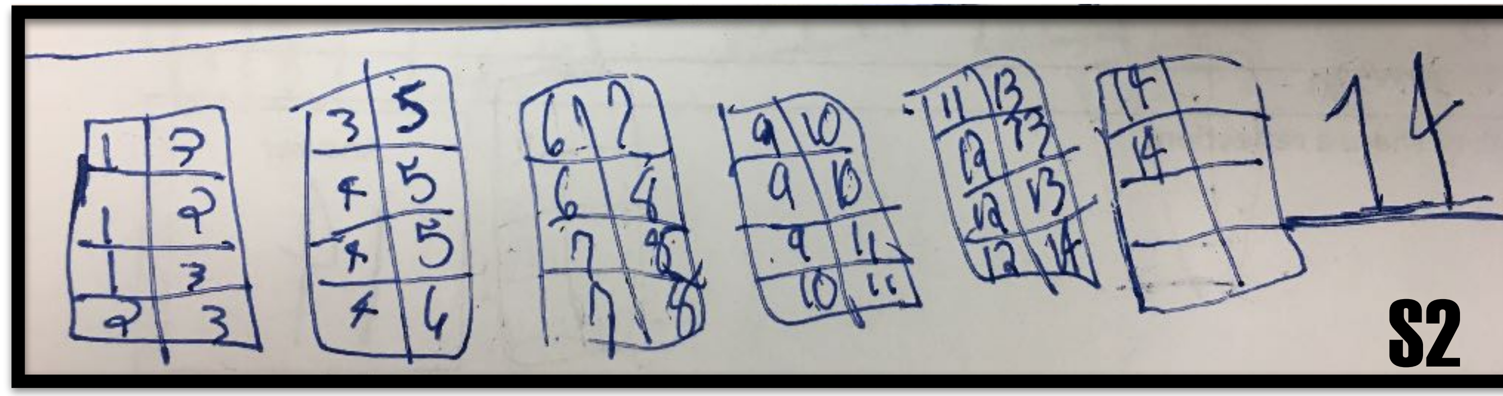
B

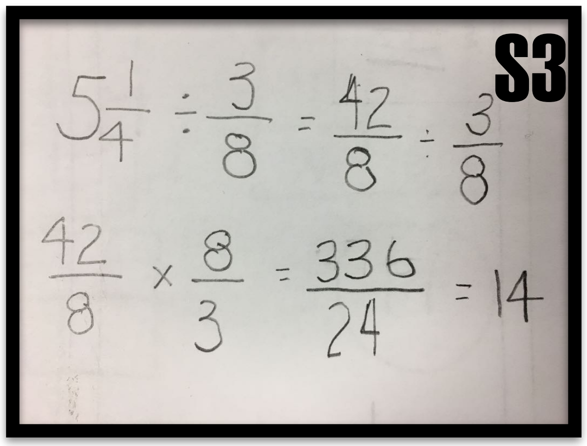
$\frac{3}{8} - \frac{6}{8} - 1\frac{1}{8} - 1\frac{4}{8} - 1\frac{7}{8} - 2\frac{2}{8} -$

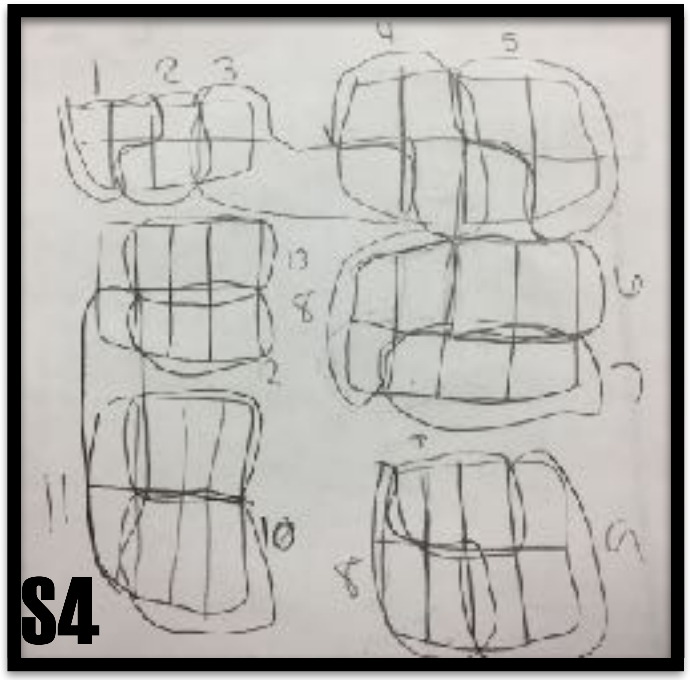
$2\frac{5}{8} - 3 - 3\frac{3}{8} - 3\frac{6}{8} - 4\frac{1}{8} - 4\frac{4}{8} - 4\frac{7}{8} - 5\frac{2}{8}$

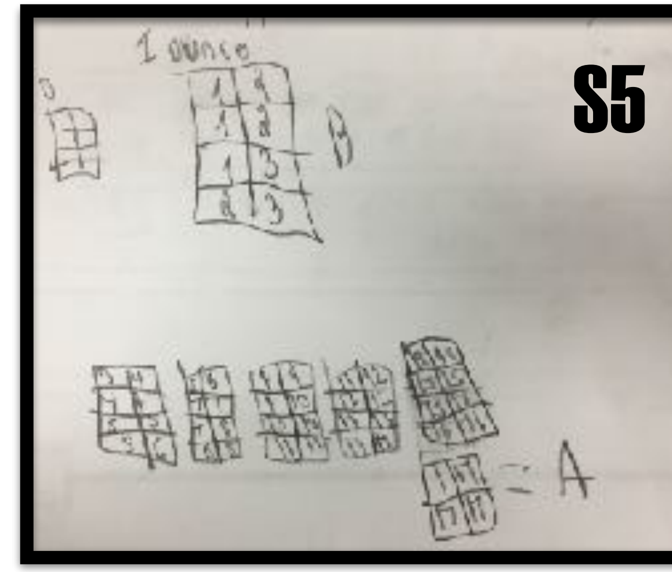


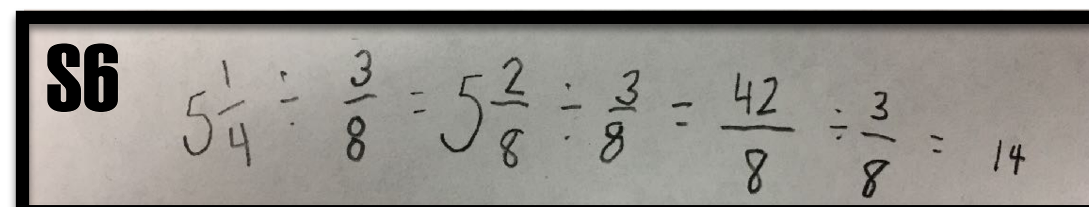

S1

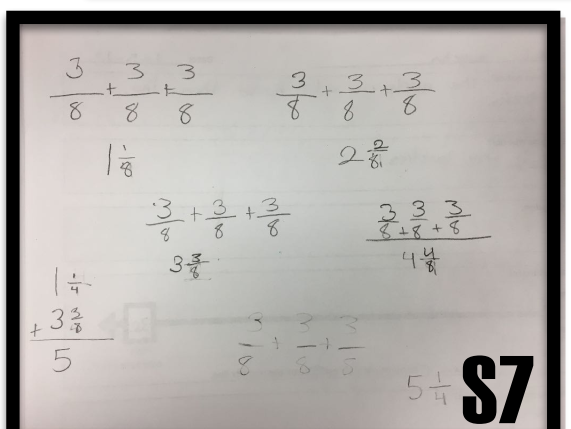

S2
14

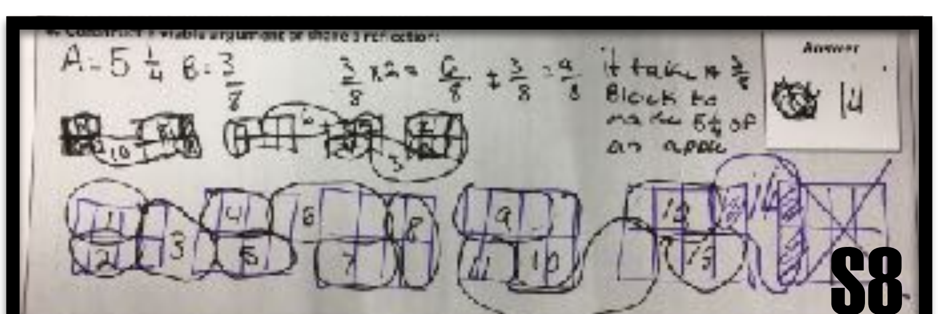

S3
5 1/4 ÷ 3/8 = 42/8 ÷ 3/8
42/8 × 8/3 = 336/24 = 14

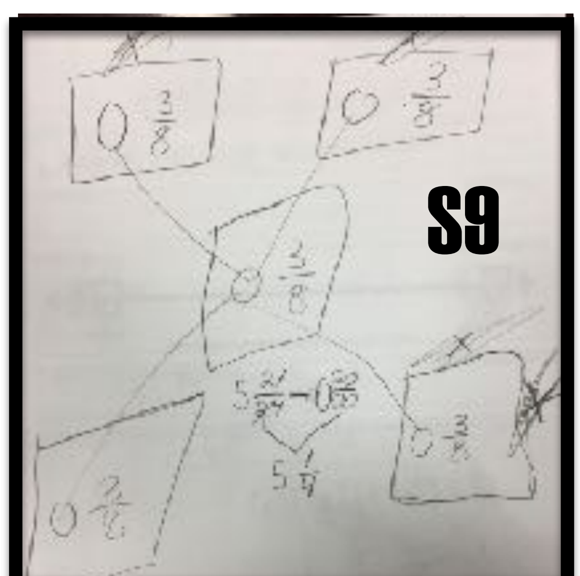

S4

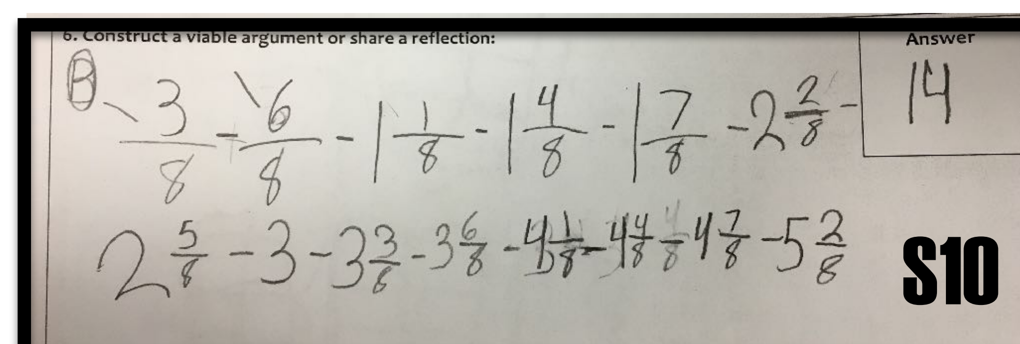

S5
1 ounce
B
= A


S6
5 1/4 ÷ 3/8 = 5 2/8 ÷ 3/8 = 42/8 ÷ 3/8 = 14


S7


Answer
14
S8


S9

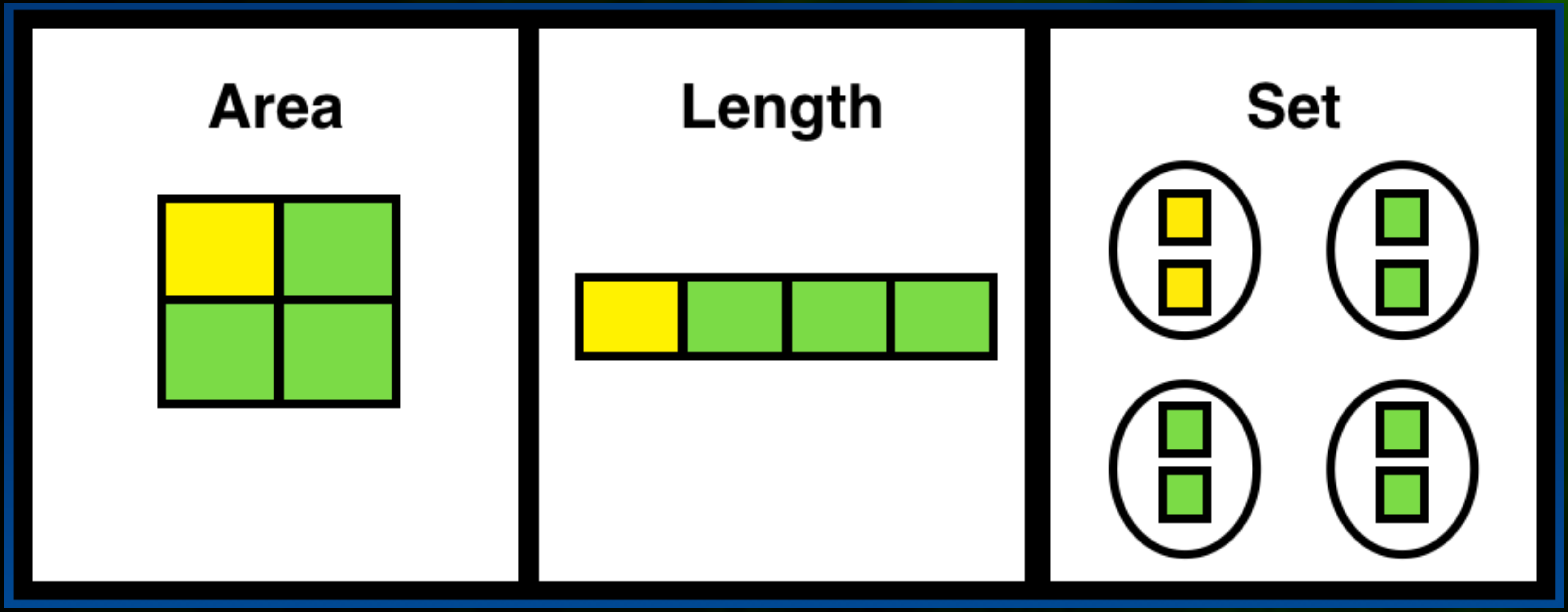

6. Construct a viable argument or share a reflection:
Answer
14
S10

The Progression of Fractions

Concept	Notes
Meaning of Unit Fractions 3.NF.1	
Equivalent Fractions 3.NF.3 & 4.NF.1	
Comparing Fractions 3.NF.3 & 4.NF.2	

Unit Fractions

Representation of a Fraction



unit fraction $\frac{1}{a}$

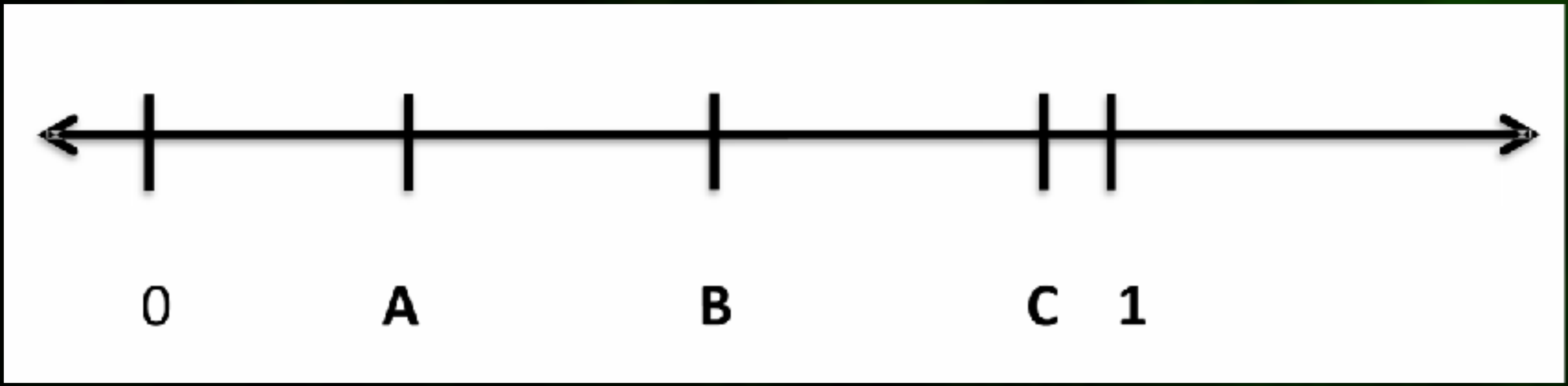
Say this fraction

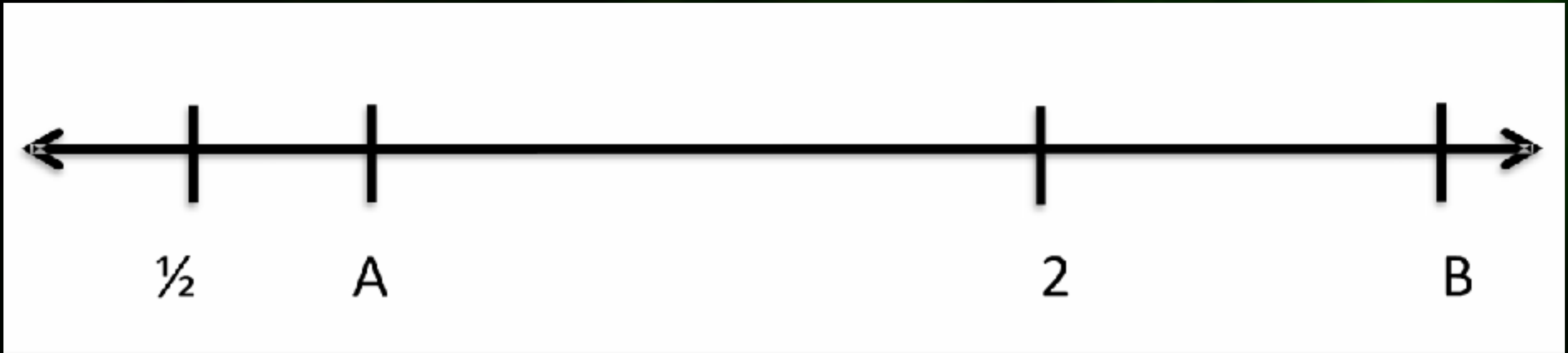
$$\frac{3}{5}$$

three-fifths

three one-fifths

$$\frac{3}{5}$$


0
A
B
C
1

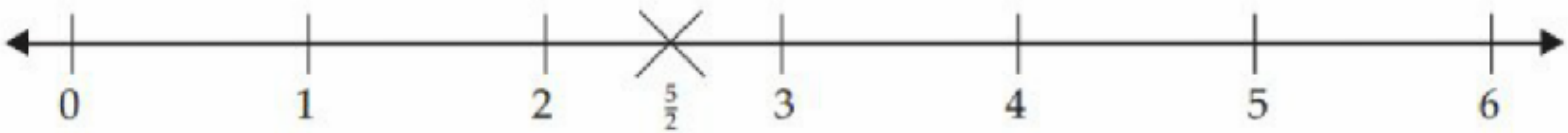

½
A
2
B

ESTIMATION 180


We Will Rock You
Queen — Greatest Hits I
0:00
QUEEN
GREATEST HITS
ESTIMATION180.COM

Dotty Pairs Game

The students play in pairs. One student takes dots, the other takes crosses. Place the cards (cards 1–6, two lots, see Material Master 4-1) face down in a pile. The players take turns turning over two cards. The numbers are used to form a fraction, e.g., 2 and 5 are turned over, so $\frac{5}{2}$ or $\frac{2}{5}$ can be made. One fraction is chosen, made with the fraction pieces, if necessary, and marked on a 0–6 number line with the player's identifying mark (dot or cross).



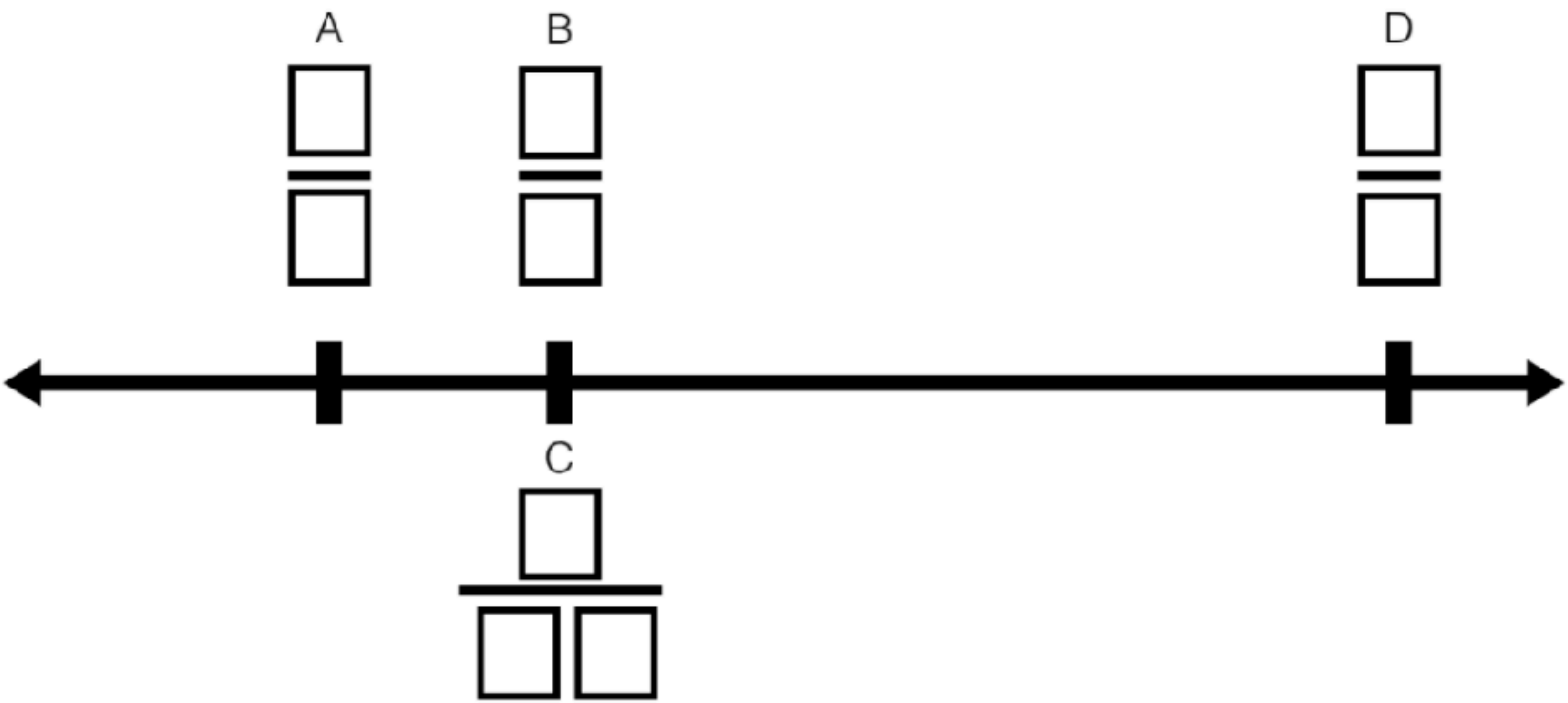
Players take turns. The aim of the game is to get three of their marks uninterrupted by their opponent's marks on the number line. If a player chooses a fraction that is equivalent to a mark that is already there, they miss that turn.

NB: A fraction such as $\frac{4}{1}$ can be made using the cards. Students may not be familiar with fractions in this form and the meaning of the numerator and denominator will need to be explored with the fraction circles.

→ **random dice roller**

COMPARING AND IDENTIFYING FRACTIONS ON A NUMBER LINE

Directions: Using the whole numbers 1-9 once each, create and place 4 fractions on the number line in the correct order. (fractions B & C are equal)



$\frac{1}{20}$	$\frac{20}{25}$
$\frac{2}{3}$	$\frac{5}{4}$

http://wodb.ca/numbers.html

Set 1 - Try to model with only drawing to start. You can label your drawing with numbers but try to use no calculations or algorithms.

---

1. Macey and Bryson have 13 cookies. If they share the cookies equally, how many cookies would each person get?

2. There are 11 yards of ribbon for 4 people to share. How many yards of ribbon can each person get if they share the ribbon equally?

3. 12 children in art class have to share 8 packages of clay so that everyone gets the same amount. How much clay can each child have?

Making Sense Series

The Progression of Fractions
Meaning, Equivalence, & Comparison

created by Graham Fletcher
@gfletchy
www.gfletchy.com

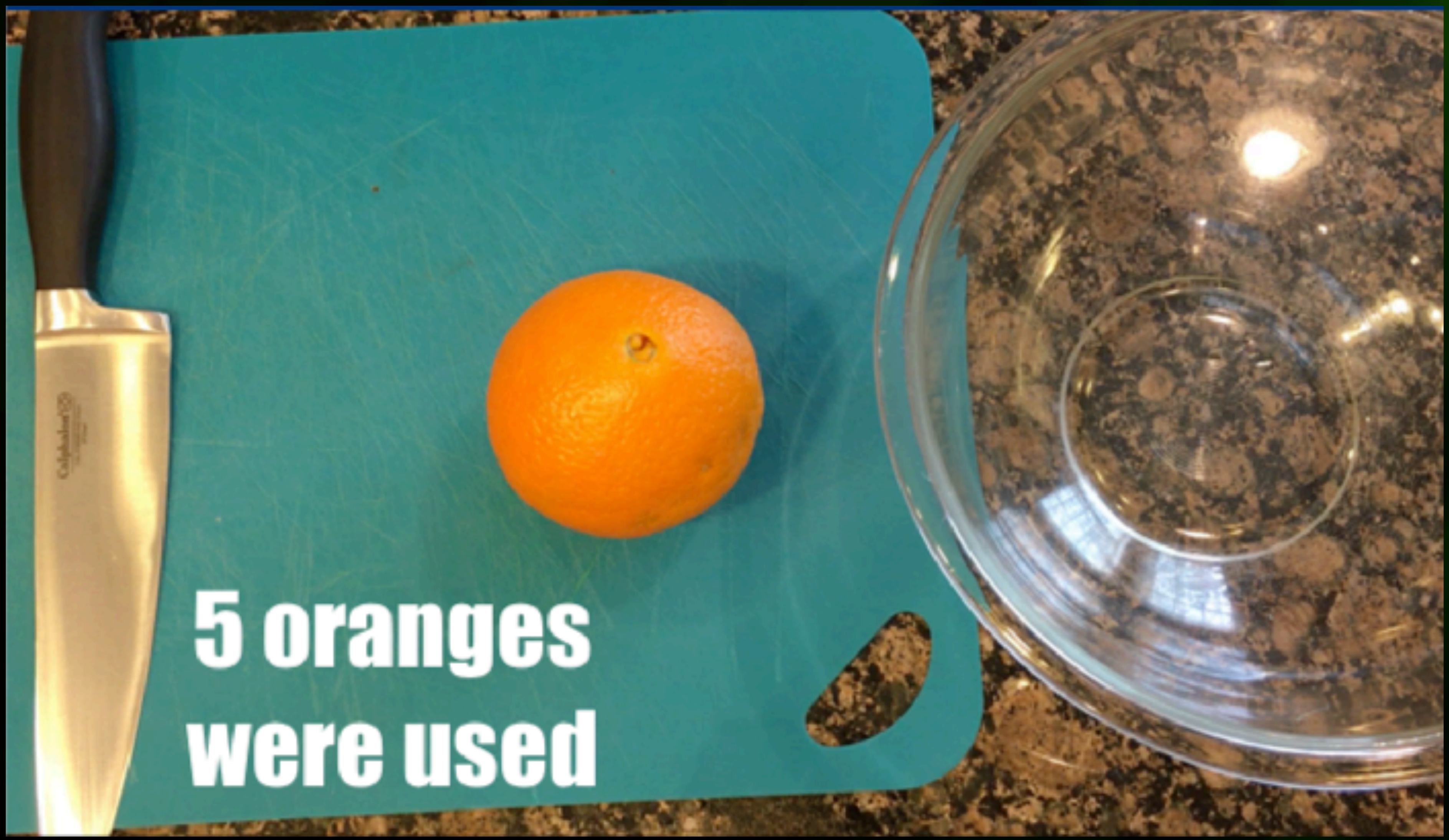
What do you wonder?

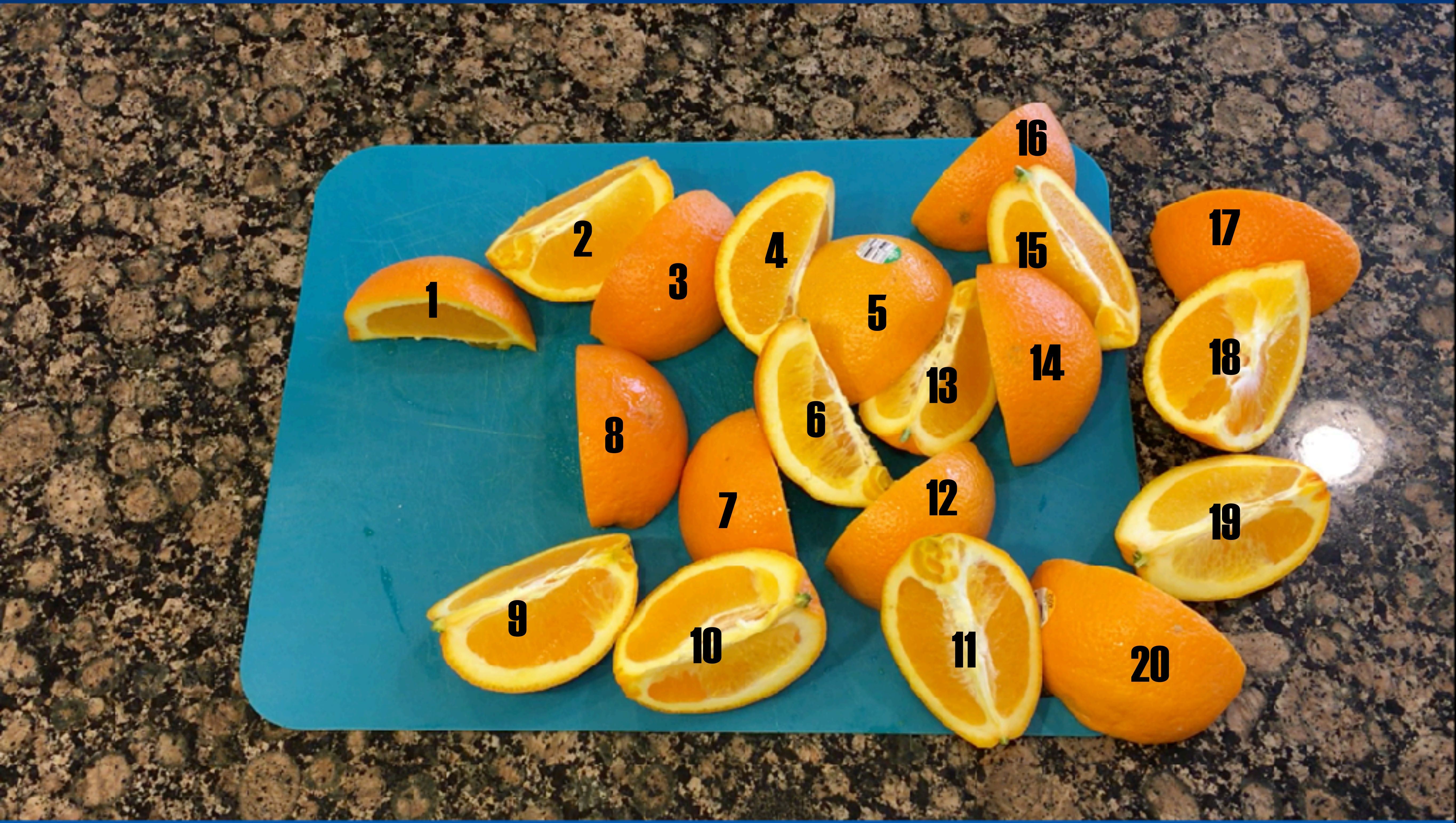
How many oranges?

How many wedges?

Alternate Tasks-How many orange wedges?

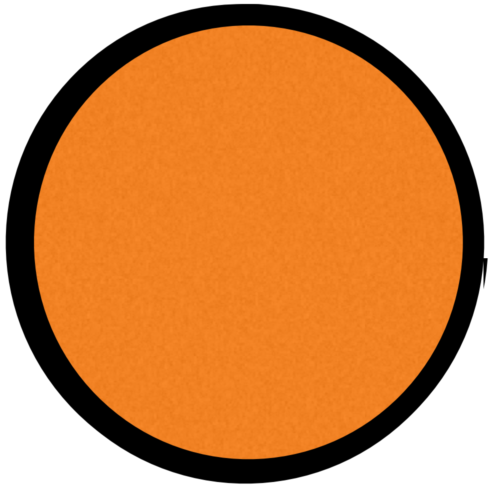
$$5 \div \frac{1}{4}$$



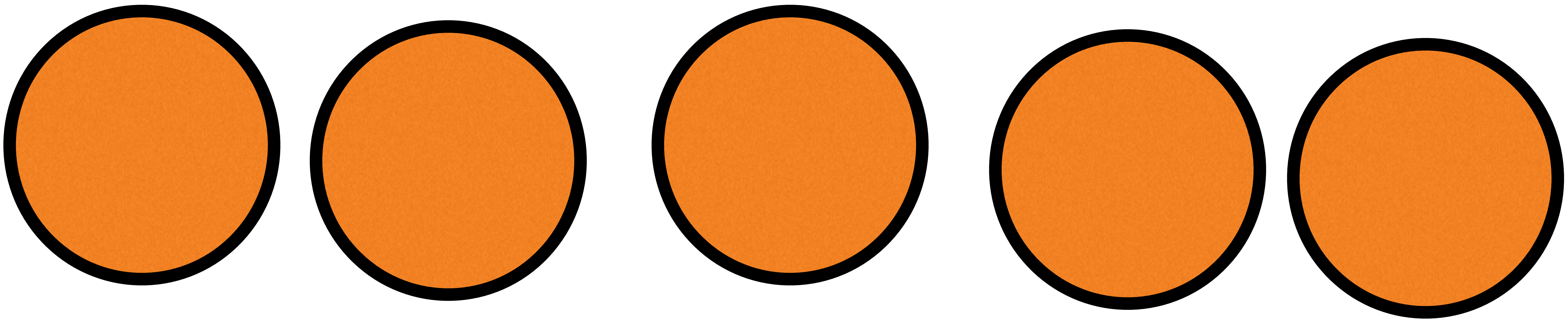

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

1 Task

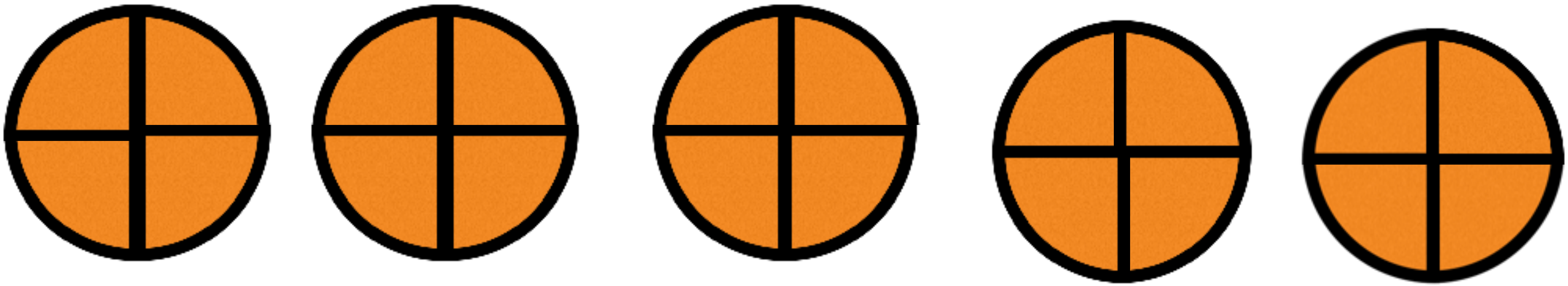
2 Stories



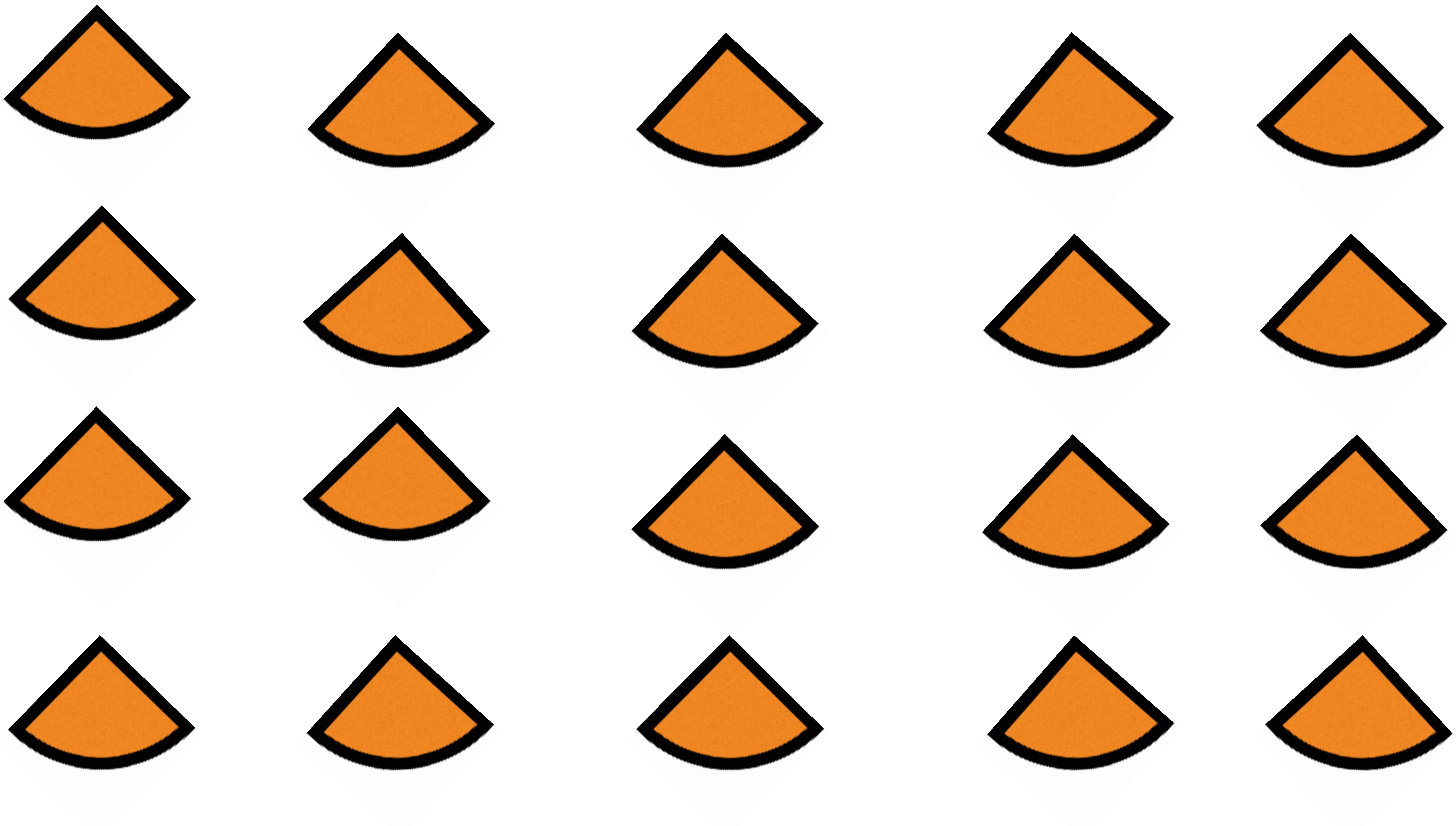
1 orange



5 oranges



$$5 \div \frac{1}{4} = \frac{20}{1}$$

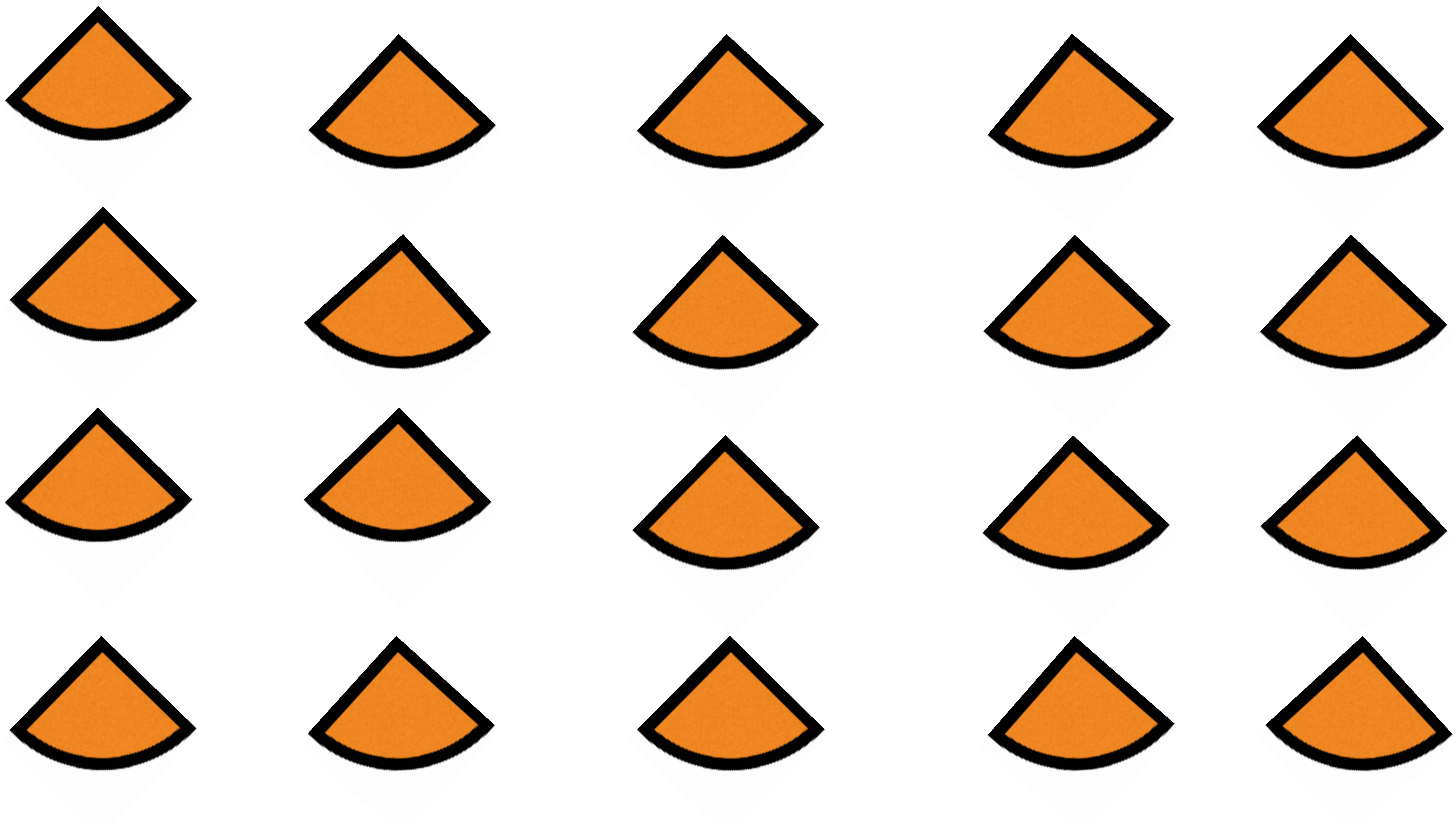


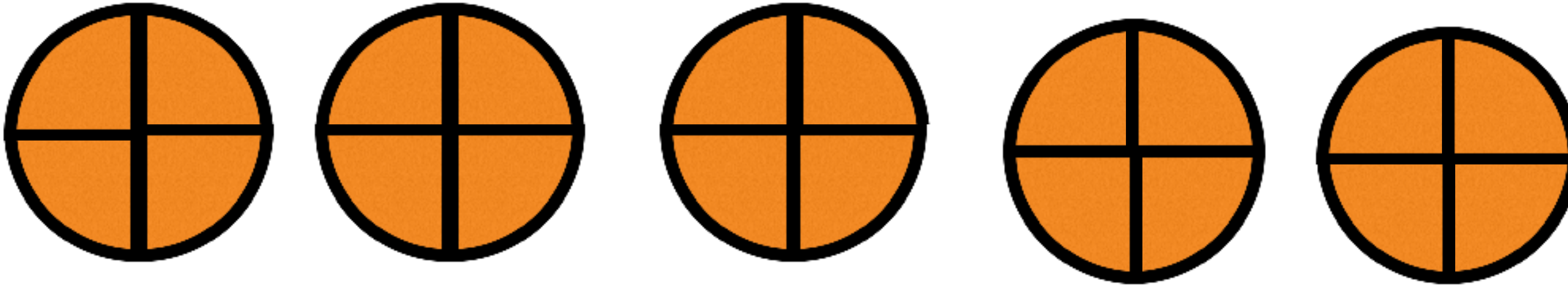
Graham had 5 oranges and cut them into quarters.

How many orange wedges did Graham have?

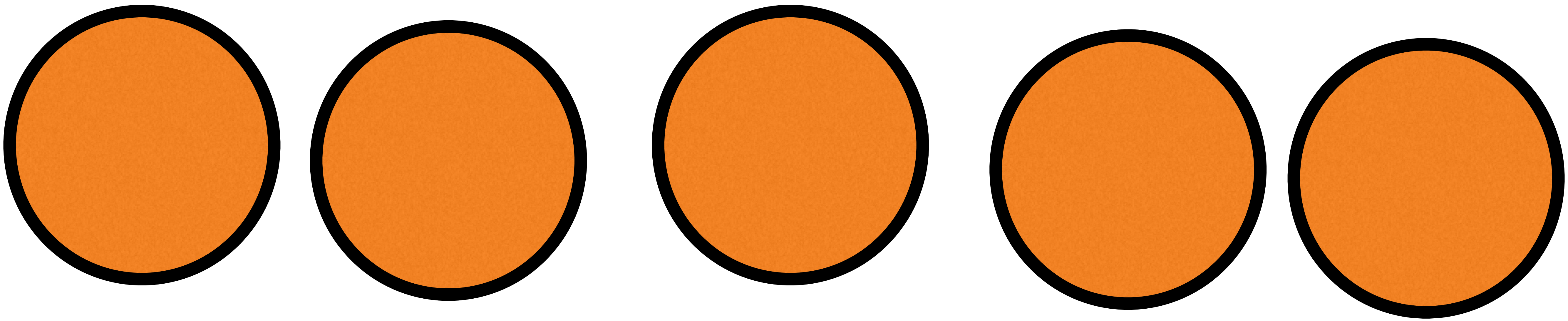
one fourth

$$\frac{1}{4} \times 20 = \frac{20}{4}$$





$$\frac{20}{4} = 5$$



5 oranges

Set 2 - Try to model with only drawing to start. You can label your drawing with numbers but try to use no calculations or algorithms.

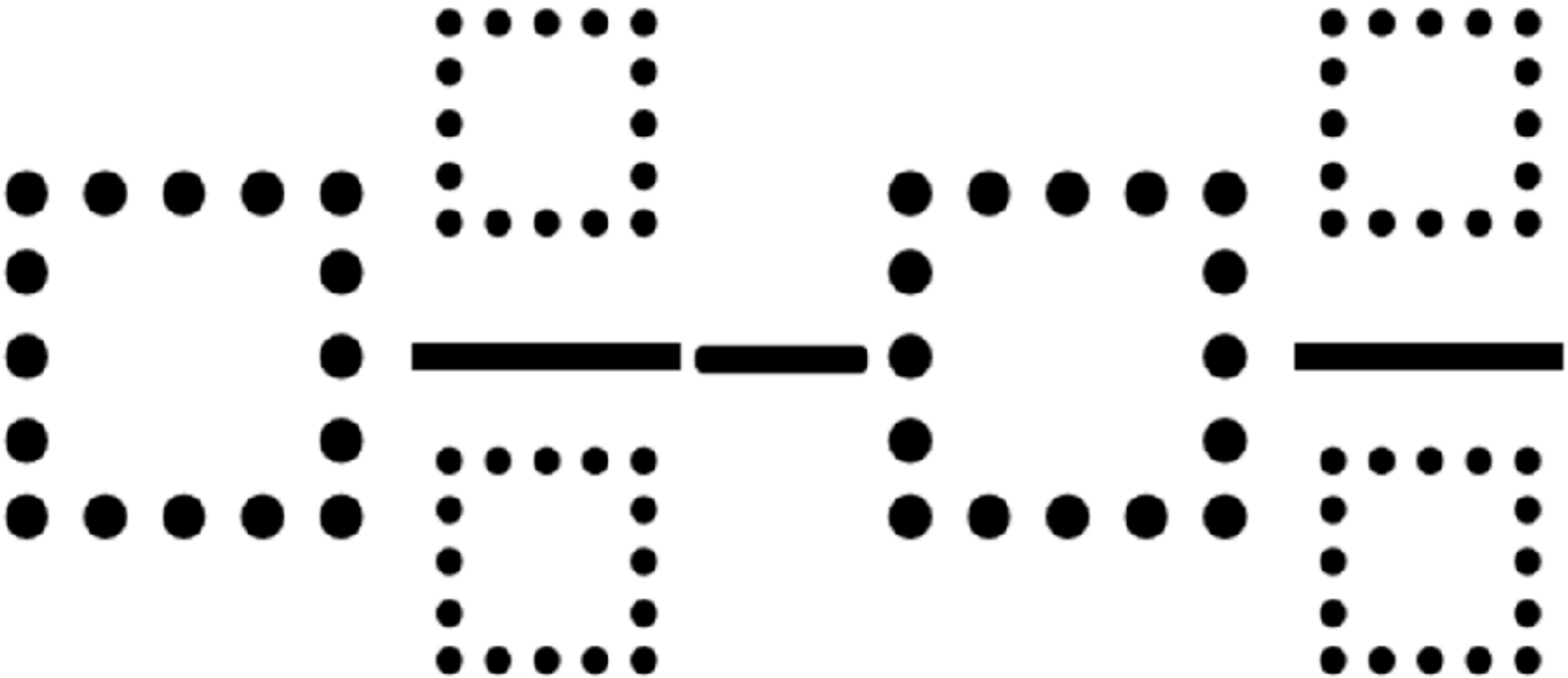
4. Emma drinks 2/3 cups of water for every mile she hikes. Her water bottle holds 4 cups of water. How many miles can she hike before her water runs out?

5. It takes 3/5 yard of ribbon to make a bow. How many bows could you make with 7.5 yards of ribbon?

6. Eve's gecko eats 2/7 jar of baby food a day. She has 10 jars of baby food. How many days can she feed her gecko with this food?

SUBTRACTING MIXED NUMBERS

Directions: Make the smallest difference by filling in the boxes using the whole numbers 1-9 no more than one time each.



http://www.openmiddle.com/subtracting-mixed-numbers/

$$6 \times \frac{5}{8}$$

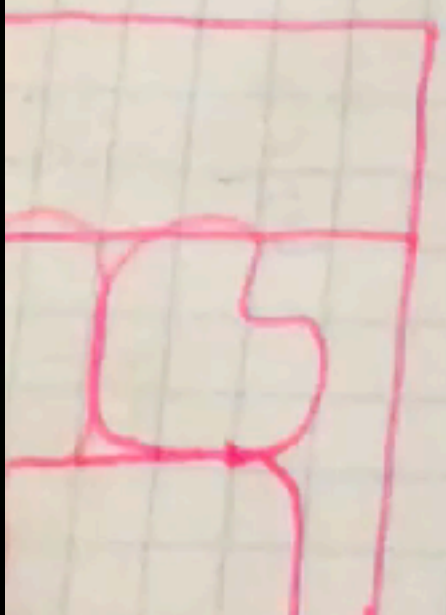
$6 \times \frac{5}{8}$

	10 +	3 +	5/8
60	600	180	

$6 * 5/8 = 3\ 6/8$

	10	+ 3	+ 5/8
0	600	180	

$6 * 5/8 = 3\ 6/8$



$6 \times \frac{5}{8}$

$$6 \times \frac{2}{5}$$

Set 2 - Try to model with only drawing to start. You can label your drawing with numbers but try to use no calculations or algorithms.

---

7. Socks are selling for $5 for 4 pairs. How much will 9 pairs of socks cost at this rate?

8. Which plant can grow faster? A plant that can grow 5 cm in 3 days or a plant that can grow 7 cm in 2 days?

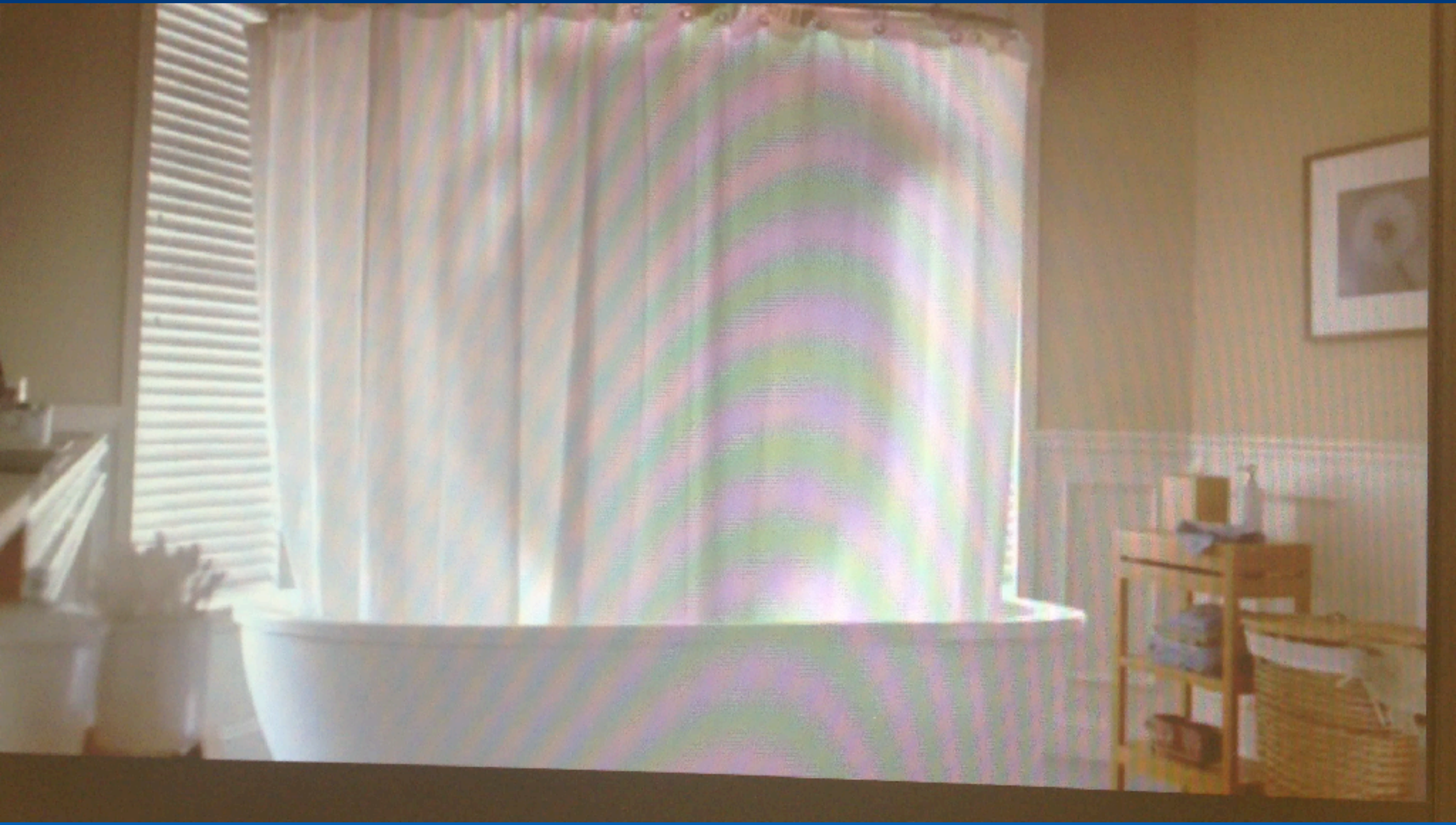
9. David used exactly 24 cups of flour to make 8 loaves of bread. How many loaves of bread can he make with 6 cups of flour?

What's up Dog?

A big dog weighs five times as much as a little dog.

The little dog weighs 2/3 as much as a medium-sized dog.

The medium-sized dog weighs 9 pounds more than the little dog. How much does the big dog weigh?



Harnessing the Power of the Purposeful Task

Graham Fletcher

gfletchy@gmail.com @gfletchy www.gfletchy.com/CS17