# Hanover School Division

## April 18th-20th, 2017

by: **Graham Fletcher**
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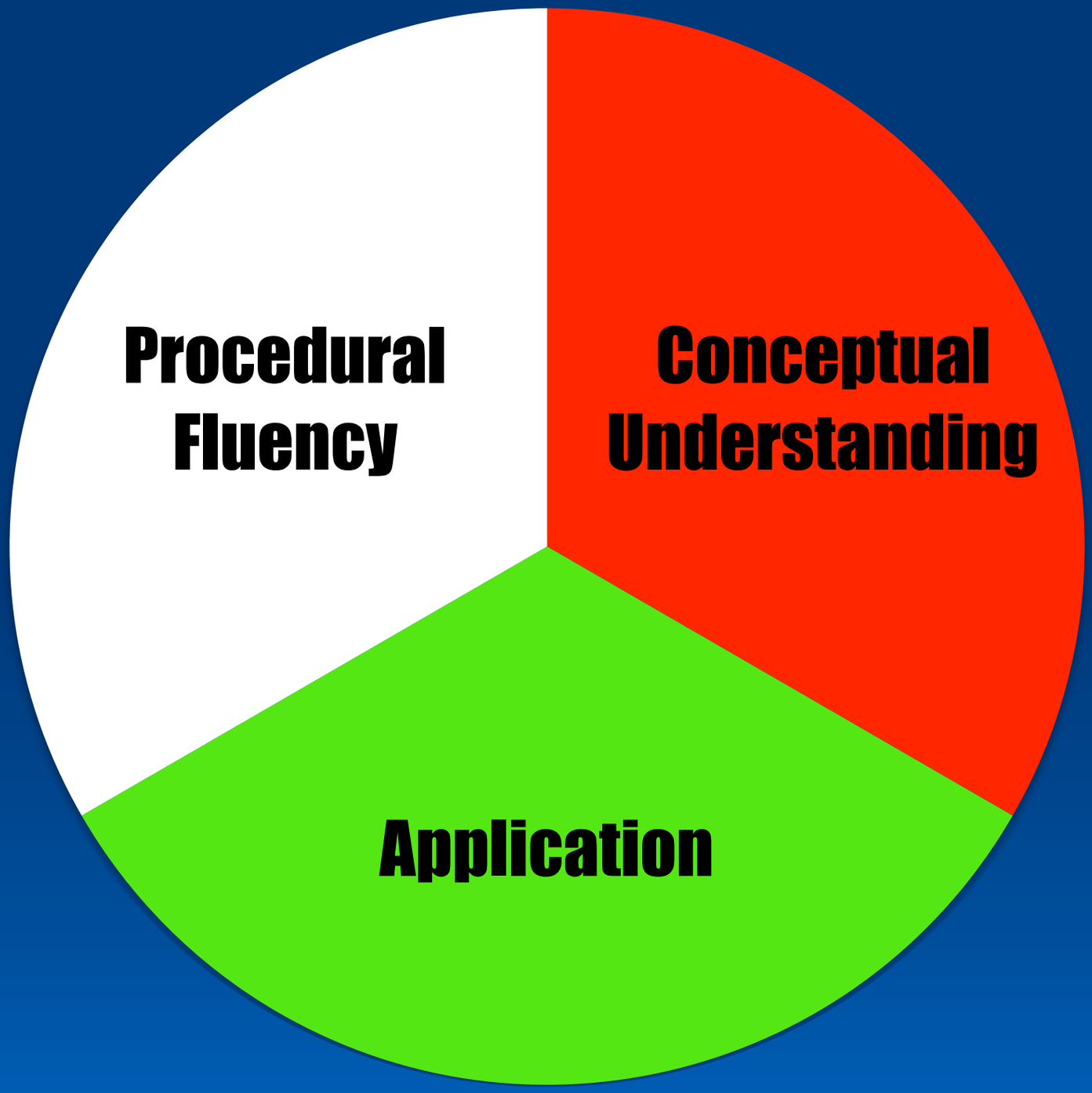
# Broken Squares

- Designate shape keeper
- Share the shapes as evenly as possible between the group
- All the shapes will make 6 congruent squares with no shapes leftover
- Everyone is encouraged to **OFFER** a shape. No one may TAKE or SNATCH.  You may RECEIVE a shape but only if it's OFFERED
- No "community square"
- NO TALKING

## Today's Goals:

- Understand the implementation of a 3-act task and how they can be used as a formative assessment.
- Explore the vertical progression of key mathematical ideas before, during, and after the grade level we teach.
- Value the importance of properly closing a lesson


Procedural Fluency
Conceptual Understanding
Application


NEXT TIME YOU RE AFRAID
TO SHARE IDEAS
REMEMBER SOMEONE
ONCE SAID IN A MEETING
LETS MAKE A FILM WITH A
TORNADO FULL OF SHARKS

*Dan Meyer :*

# “Be Less Helpful”

Demetrius has 17 Skittles which is 12 fewer than Alicia.
How many Skittles does Alicia have?

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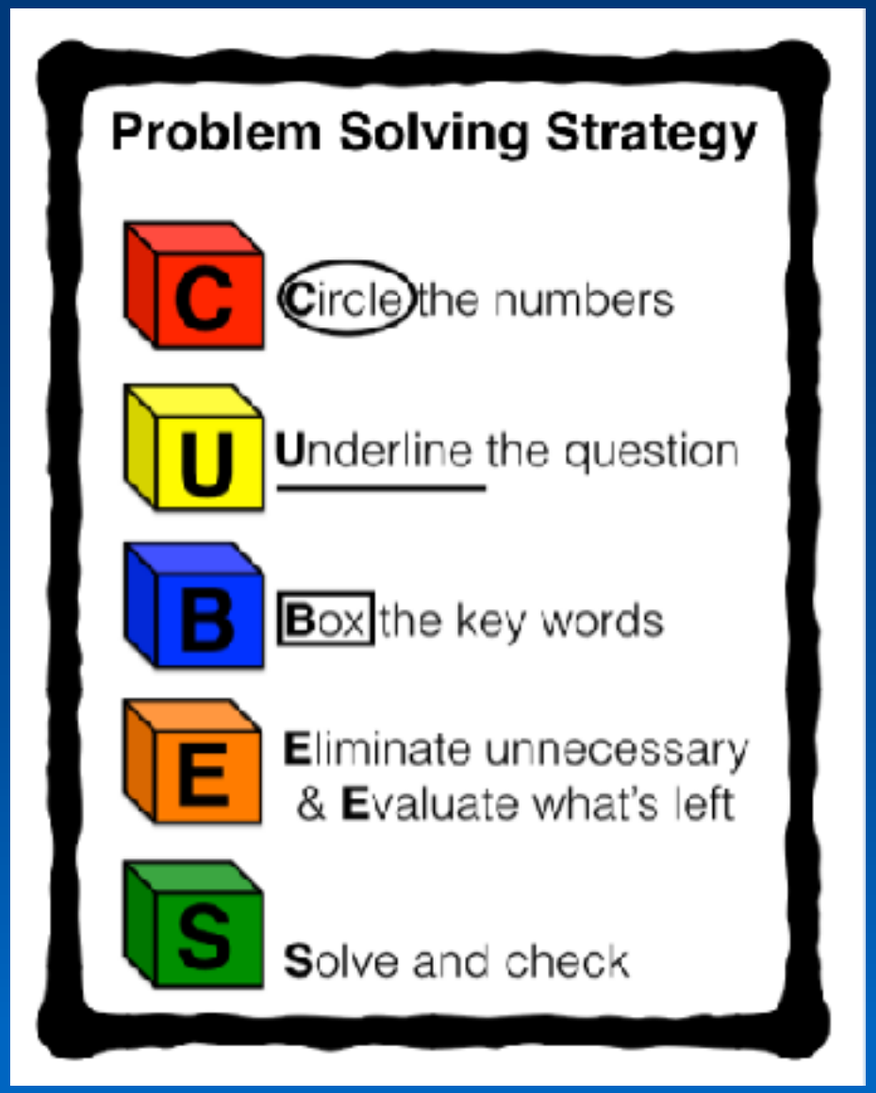

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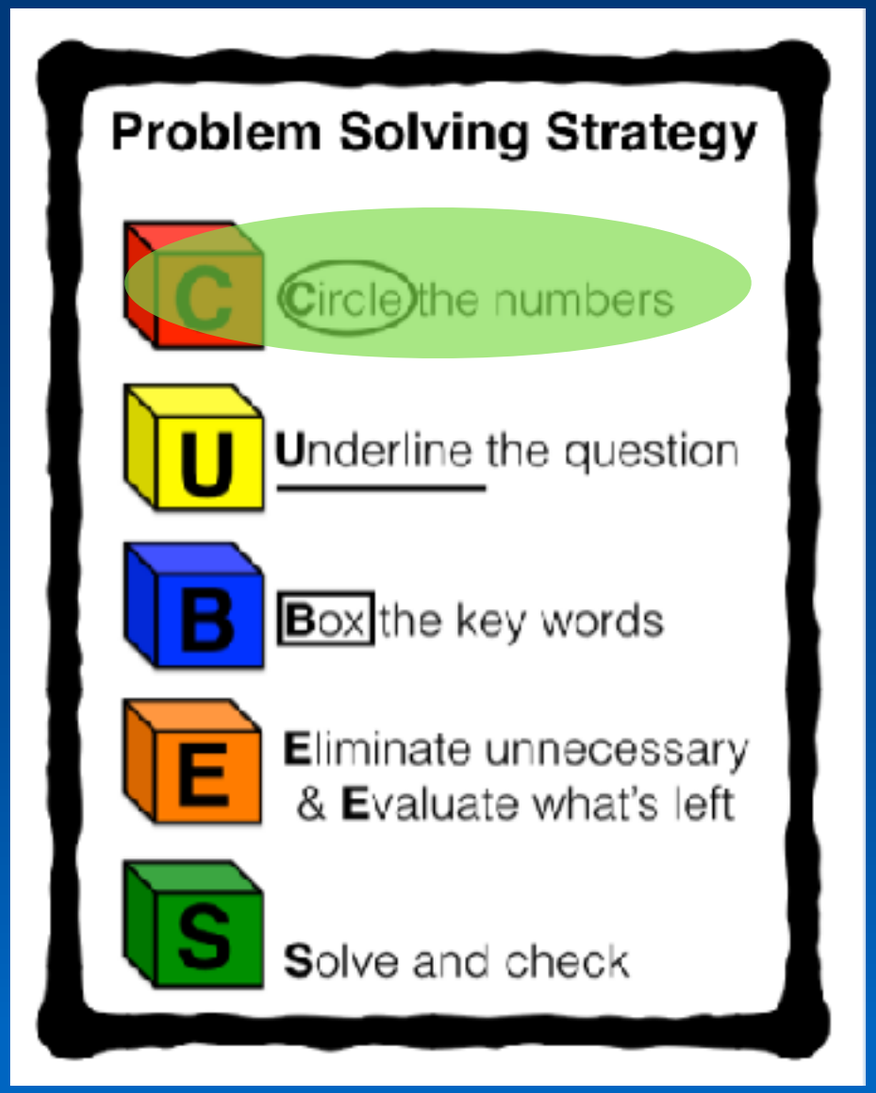

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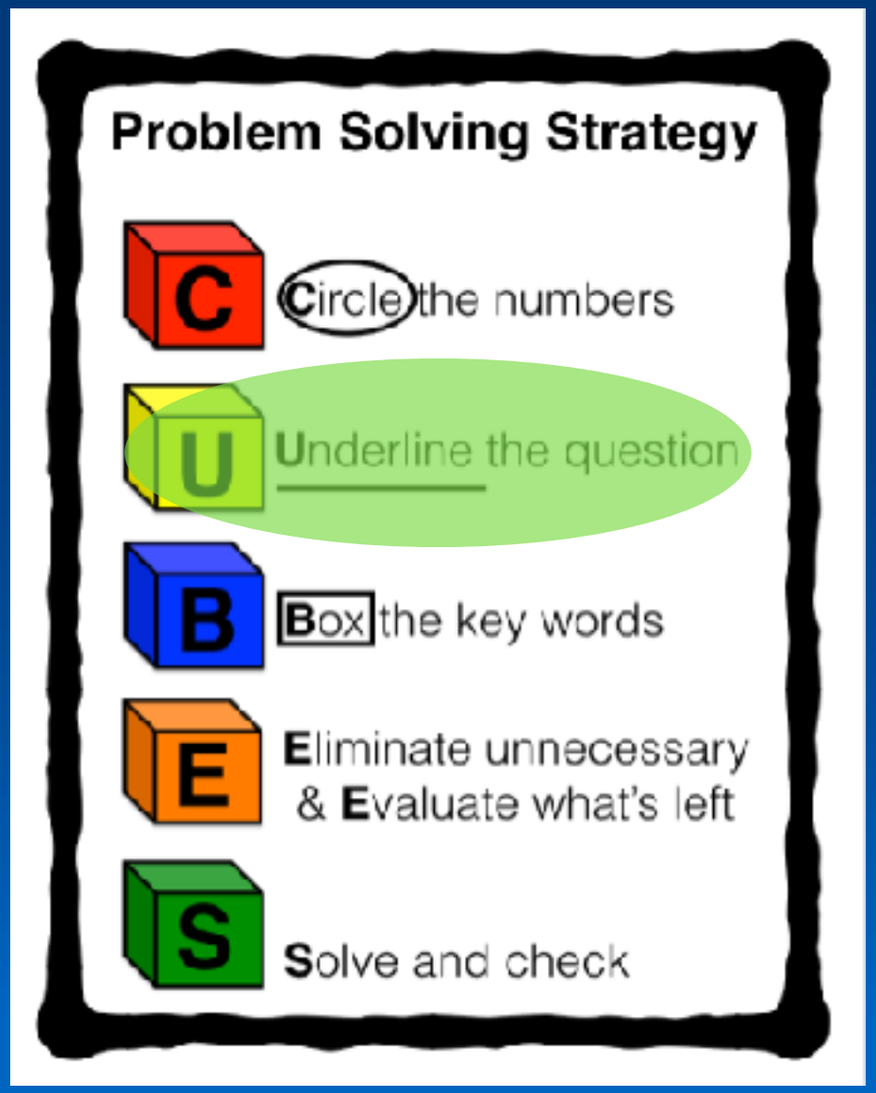

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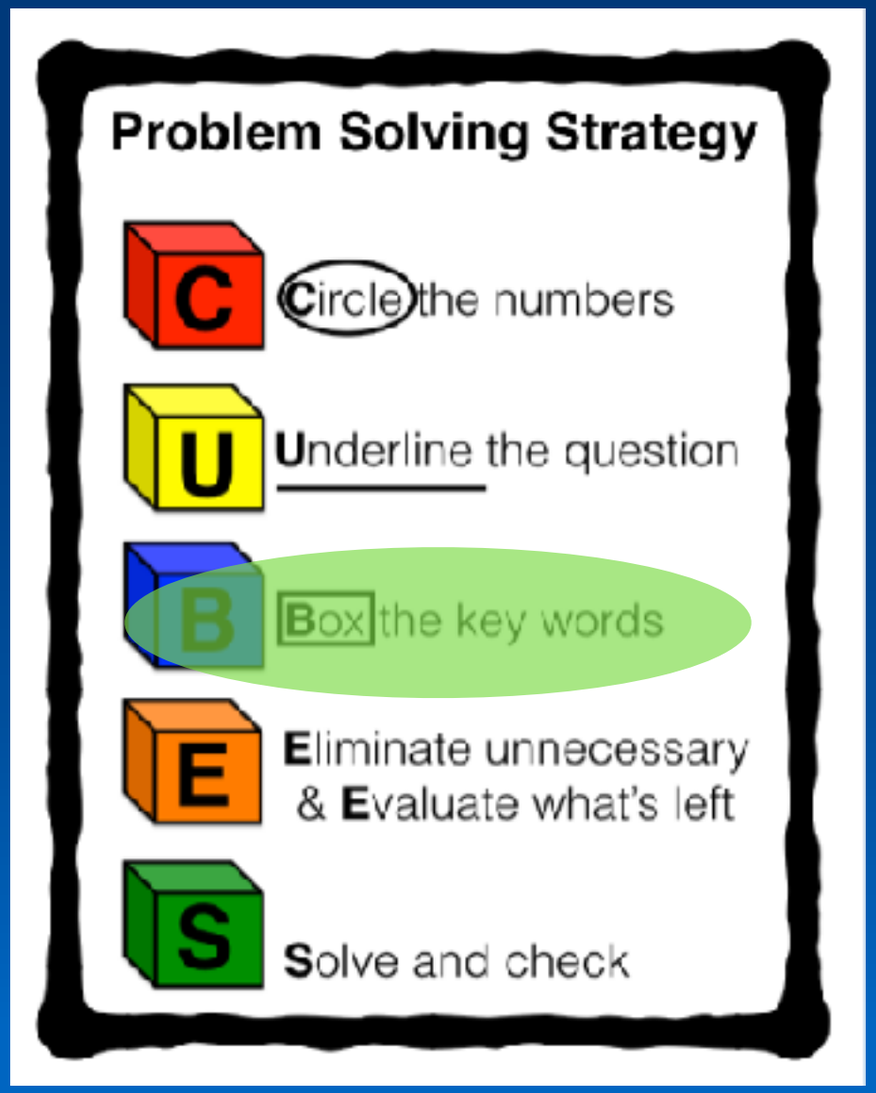
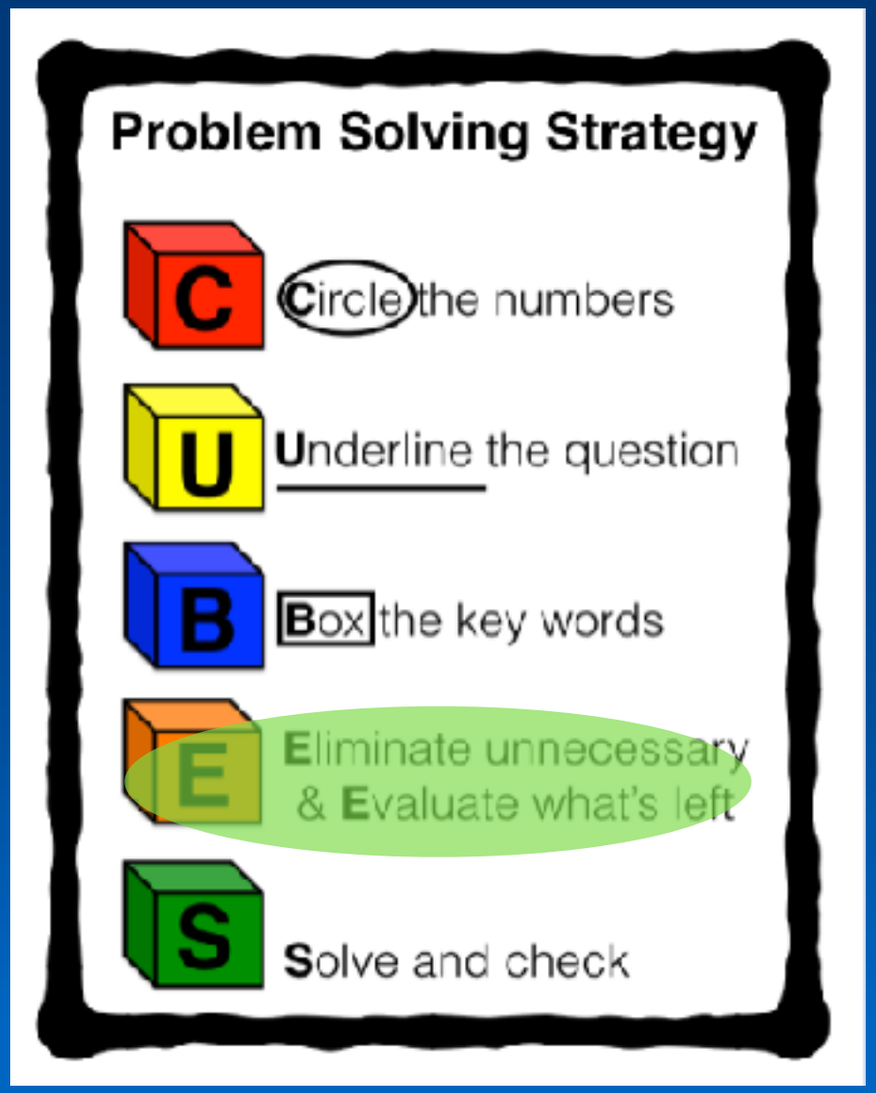


17

12

fewer

# How many Skittles does Alicia have?



17

12

fewer

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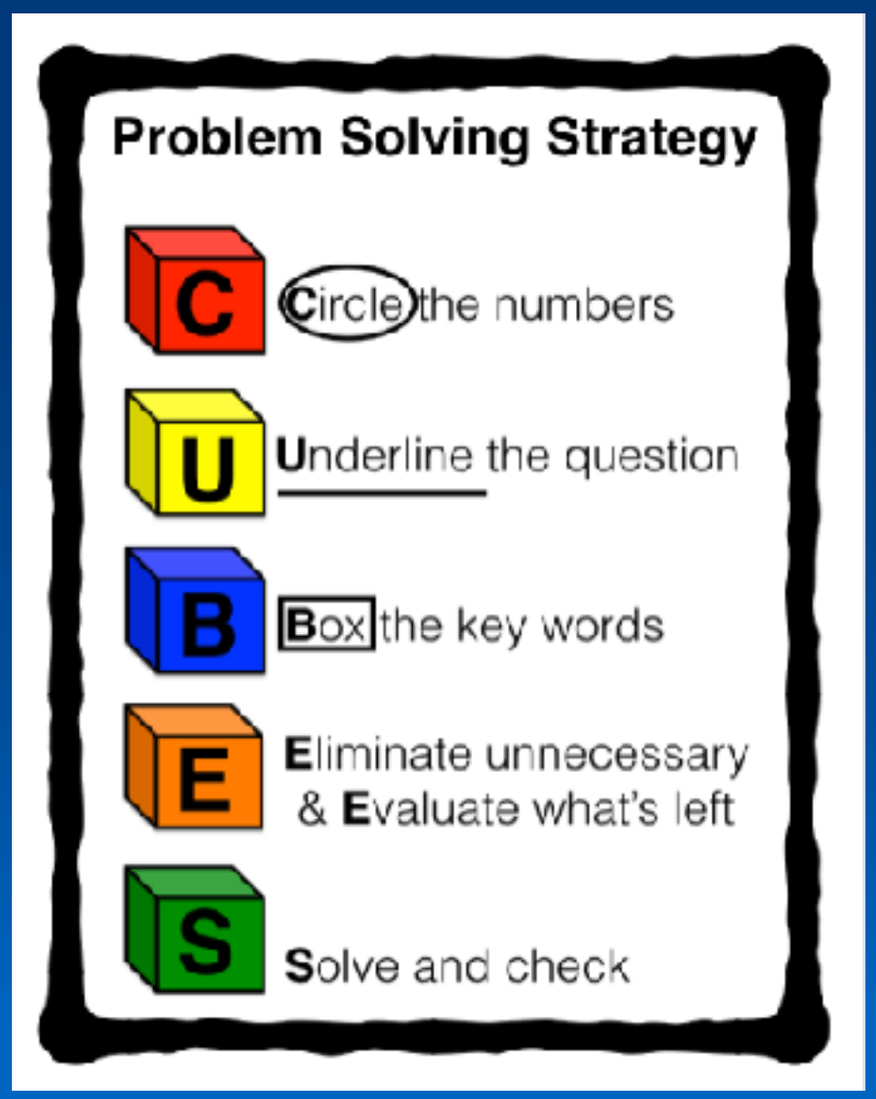
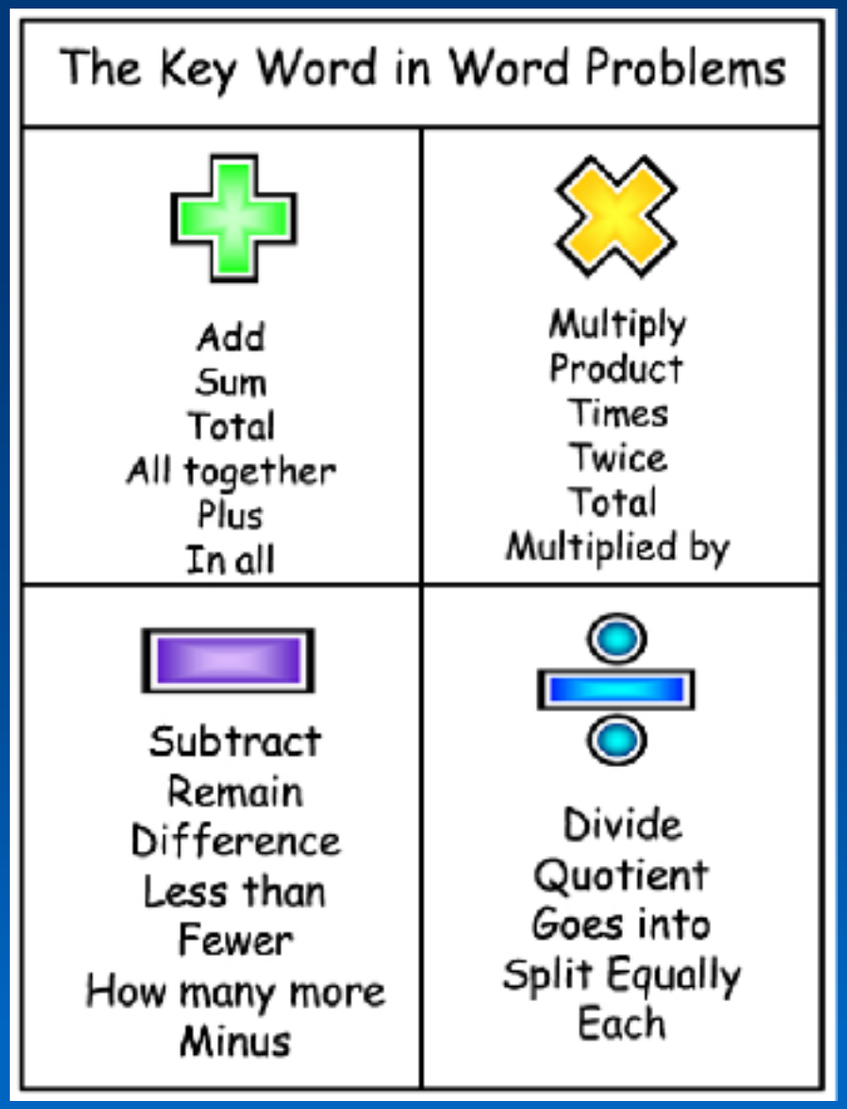


# How many Skittles does Alicia have?

$$17 - 12$$

# WTF?

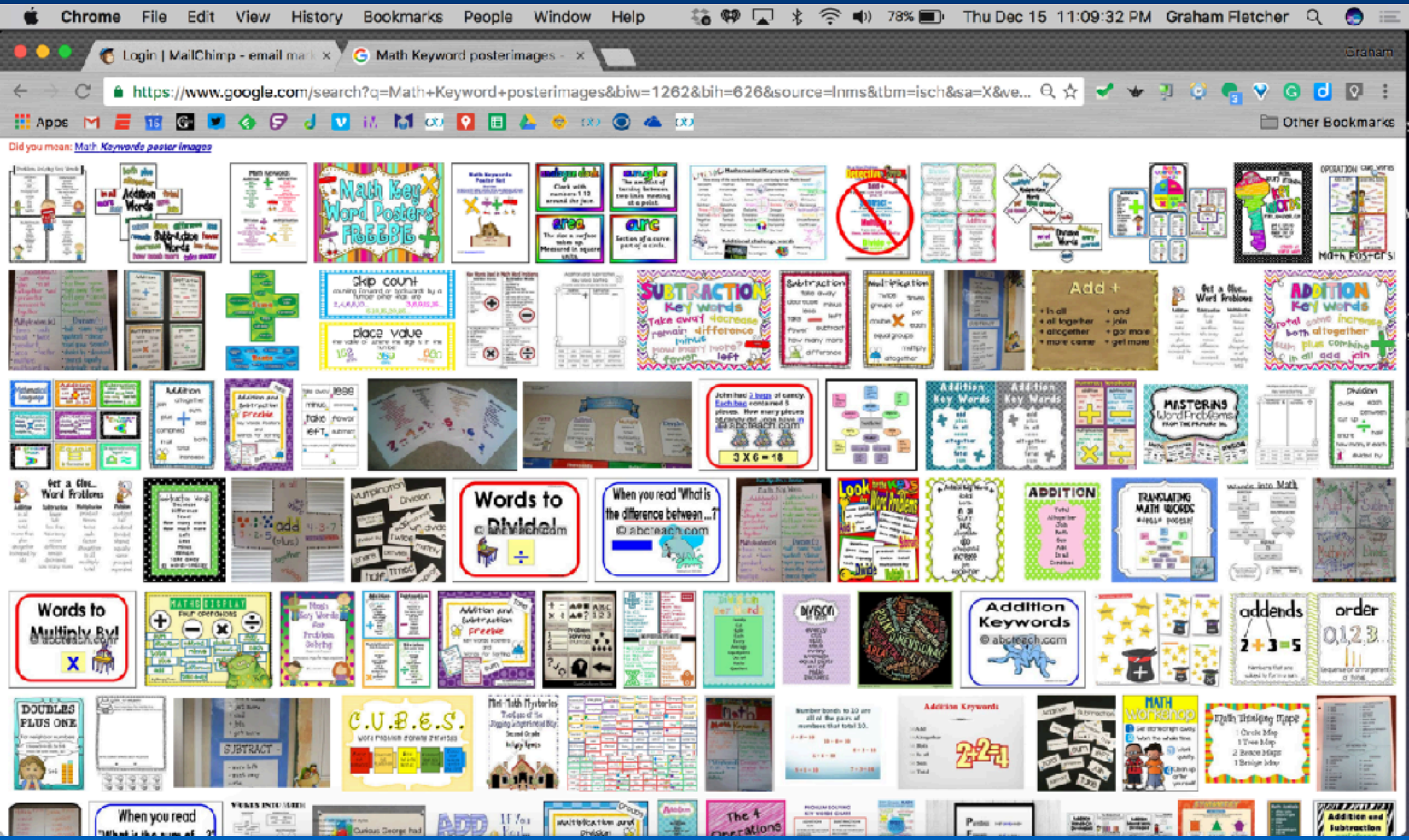
17 – 12

# WTF?

hat's he ive

17 – 12

# Current Research



*Tracy Zager:*

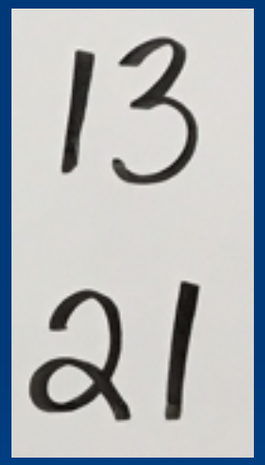
# “How can we break the cycle?”

Joe had some playing cards in his bag. Ashley gave him 13 more cards. Joe now has 21 cards. How many cards did Joe have in his bag?

13

21

# You little plucker!




*John SanGovanni, NCTM '16*

# You little plucker!

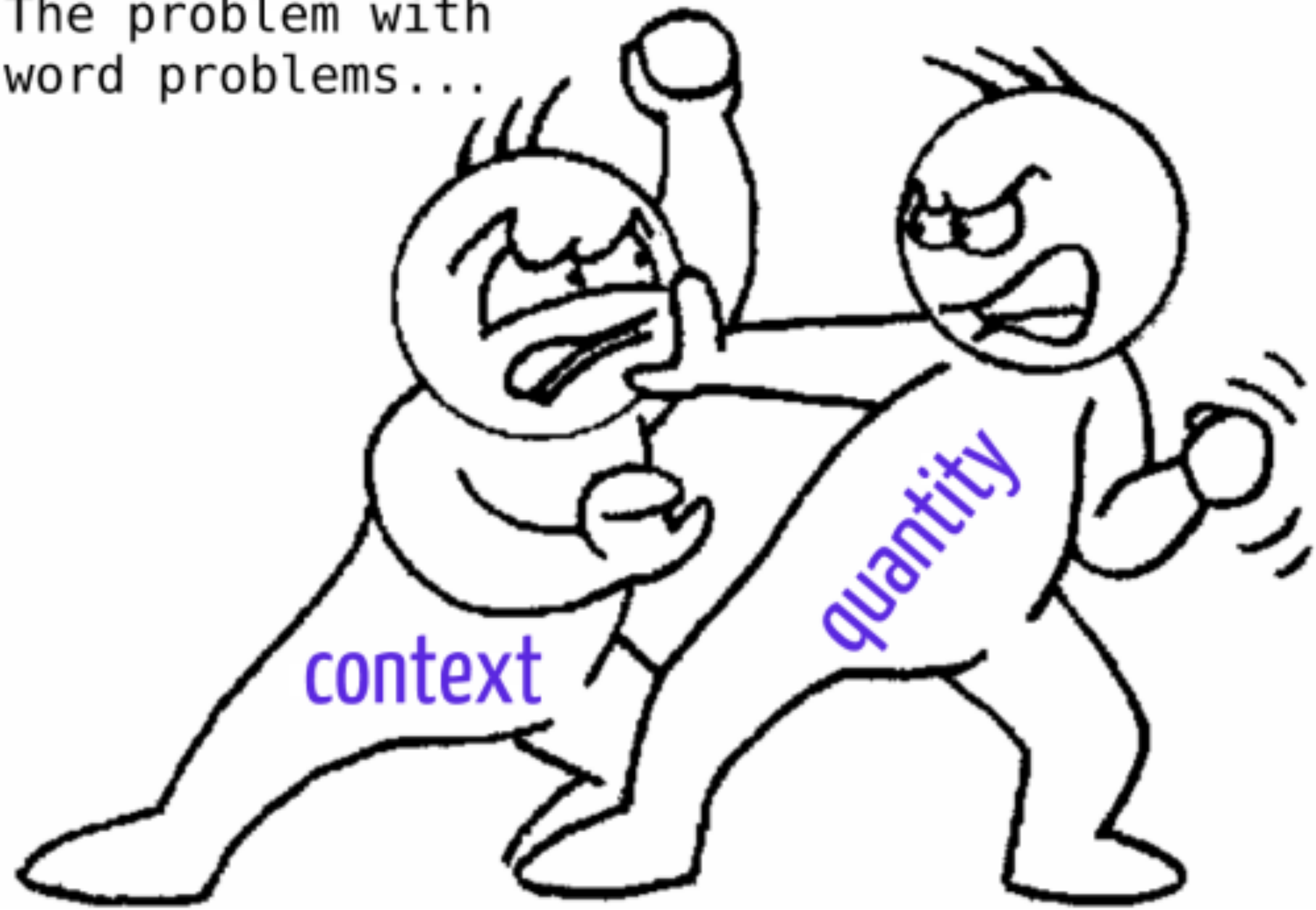
number

13

21

*John SanGovanni, NCTM '16*

The problem with
word problems...




@gfletchy

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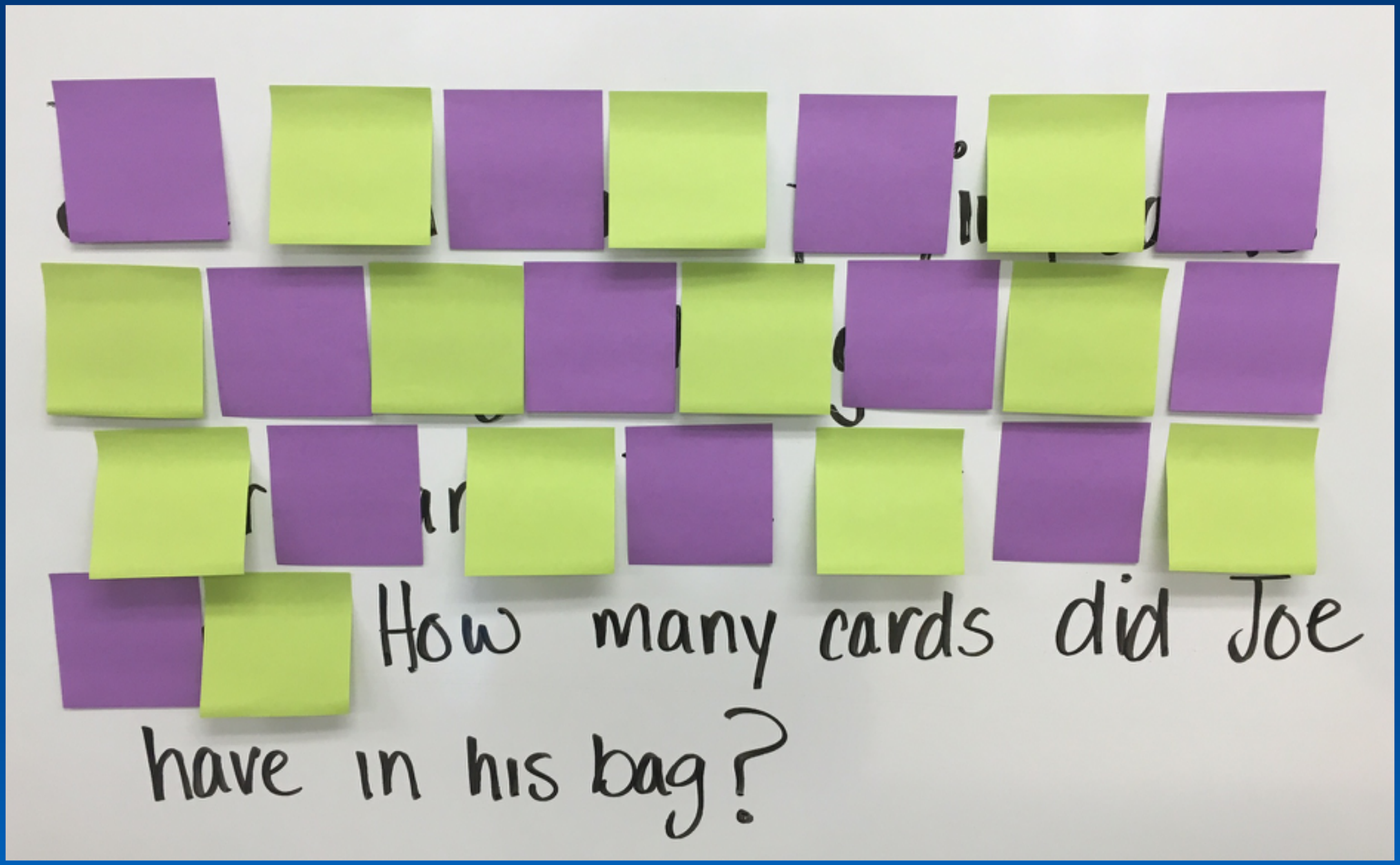

How many cards did Joe
have in his bag?

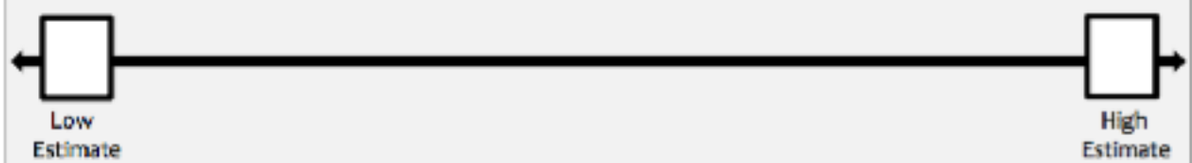
Name: _____ Date: _____

1. What did you notice?

2. What do you wonder?

3. Main Question:

4. Estimate




*place your best estimate on the number line and label*

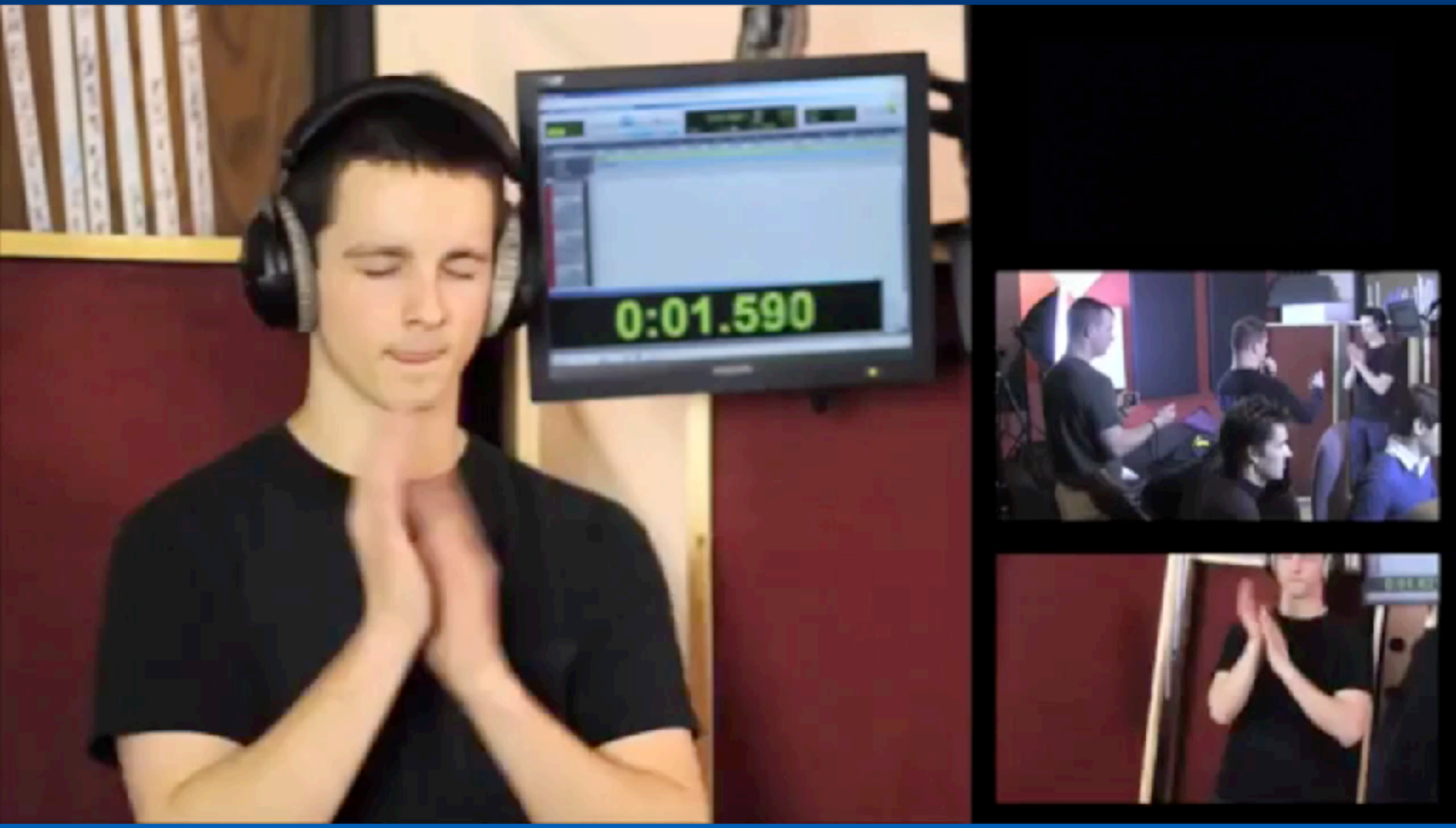
5. What information would you like to know?

6. Answer

# Dots

# Claps

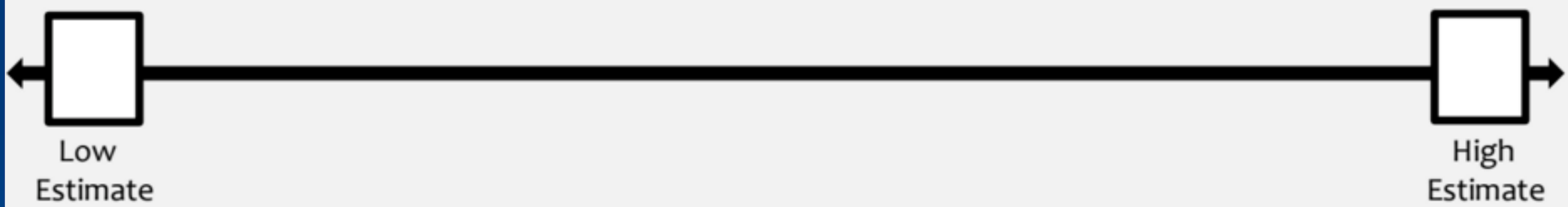
## Do more dots or claps in a minute?


0:01.590

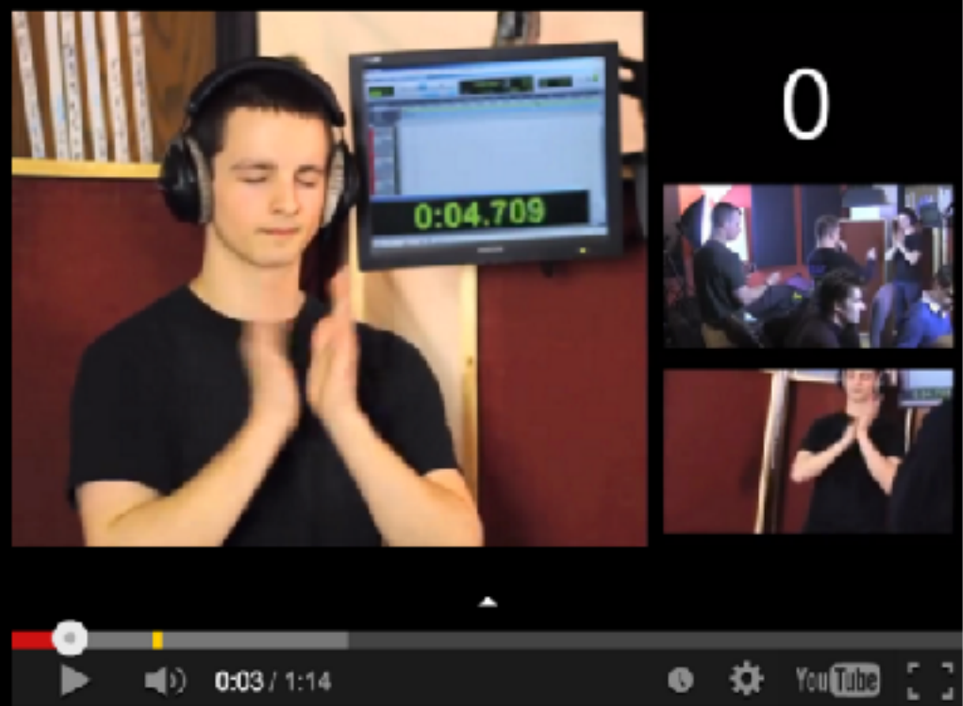
# What do you notice?

# What do you wonder?

4. Estimate



# Questions




0

Started clapping at 4.709 seconds

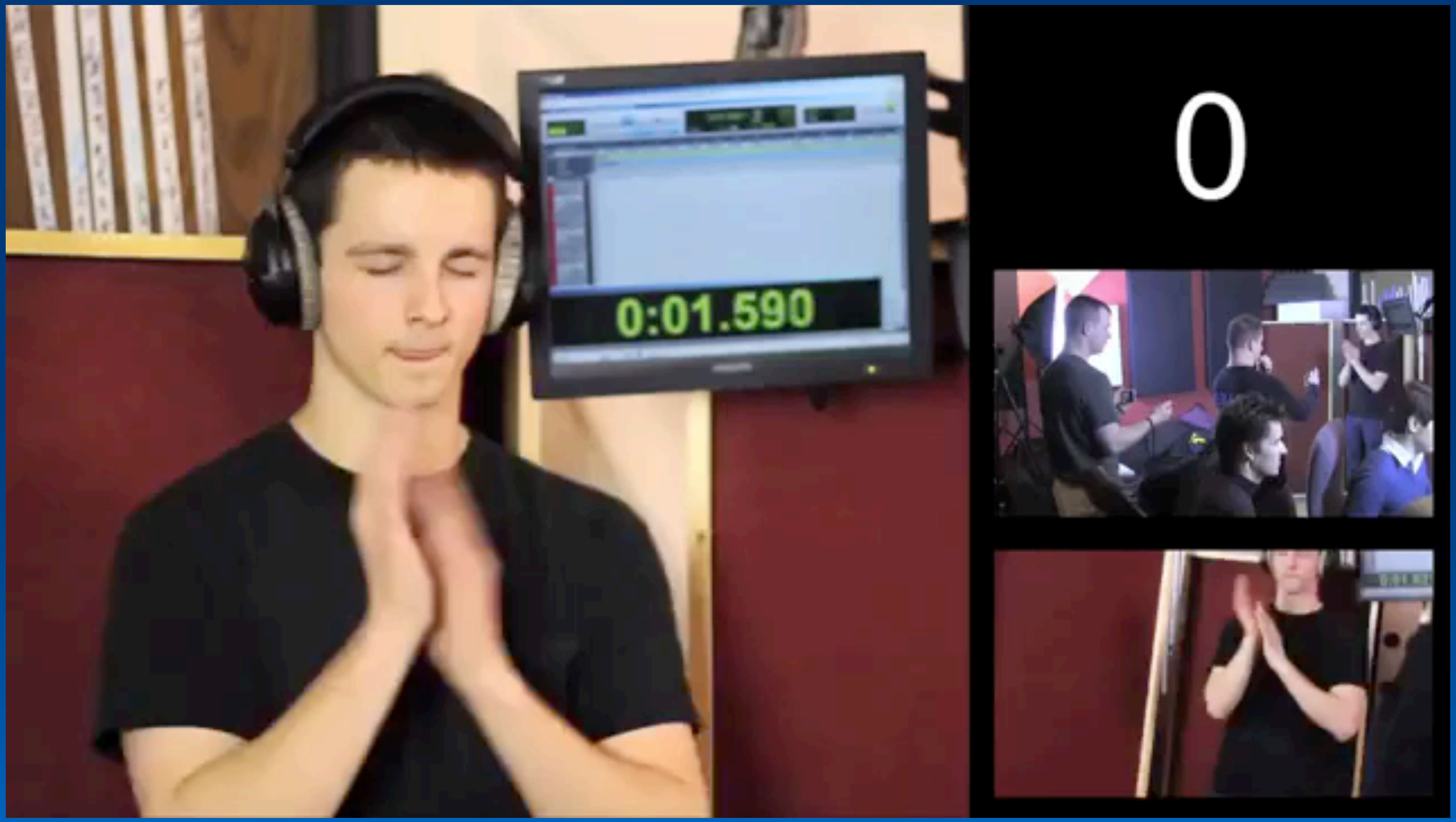
At 12.703 he has clapped 107 times

# Act 2


107

# 3 minutes

individual work time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# Mathematical Modeling

?

# Modeling with Mathematics

## What ISN”T mathematical modeling

- The use of manipulatives does not ensure that modeling with mathematics is taking place.
- If the mathematics is not contextualized, modeling with mathematics cannot exist.
- Modeling with mathematics does not mean, “I do, we do, you do.”

# Model with Mathematics

Mathematically proficient students can apply the mathematics they know to **solve problems arising in everyday life**, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. Mathematically proficient students who can apply what they know are comfortable **making assumptions and approximations** to simplify a complicated situation, realizing that these may need revision later. They are able to **identify important quantities** in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can **analyze those relationships** mathematically to draw conclusions. They routinely **interpret their mathematical results** in the context of the situation and **reflect on whether the results make sense**, possibly improving the model if it has not served its purpose.

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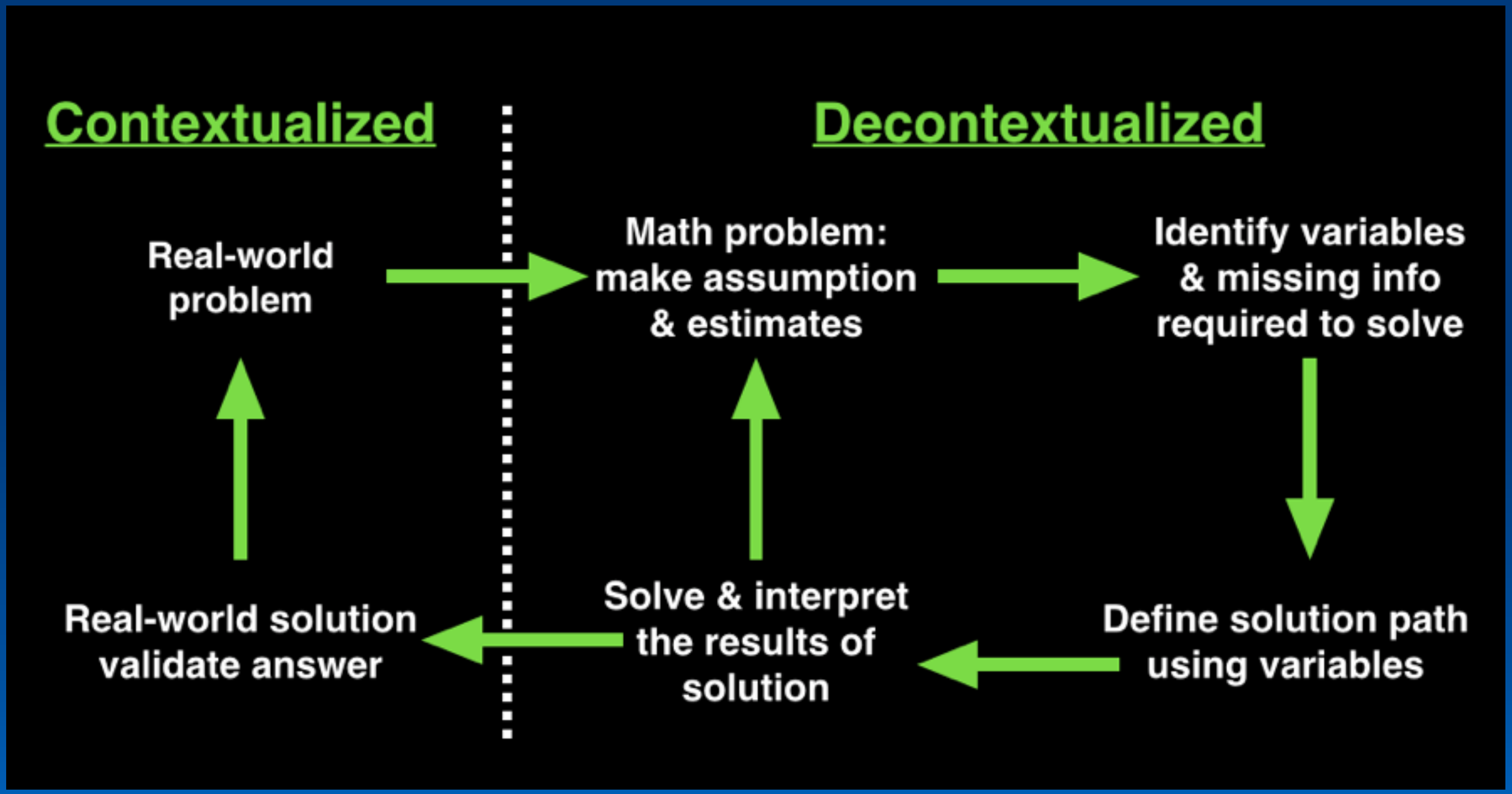
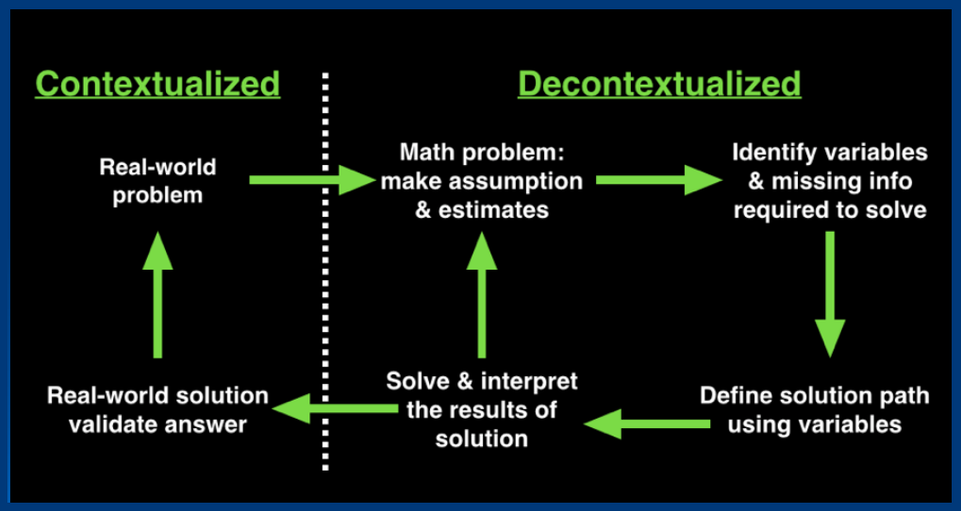
# Mathematical Modeling



# 3-Act Tasks




Act 1:

- Real world problem or scenario presented
- What do you notice? What do you wonder?
- Make estimates

Act 2:

- Identify missing variables and missing variables to solve
- Define solution path using variables

Act 3:

- Solve and interpret results of the solution
- Validate answer

**3 Things on the road to modeling…**

- Identify the problem, or pose a question.
- Make an estimate.
- Identify the variables needed to solve, and answer the problem or question posed.

**Most asked questions:**

- How often should we use 3-Act Tasks?
- When should we use 3-Act tasks? How do they fit into the scope of a unit?
- How long does one task usually take?
- What if we don't have the time?
- Any others?

# Orchestrating Discussions

*Five practices constitute a model for effectively using student responses in whole-class discussions that can potentially make teaching with high-level tasks more manageable for teachers.*

Margaret S. Smith, Elizabeth K. Hughes, Randi A. Engle, and Mary Kay Stein

The **5** practices are:

1. **Anticipating** student responses to challenging mathematical tasks;

Anticipate student solutions for the Whopper Jar task.

Identify and record the types of strategies students will use to solve the task.

# Task Planning Document

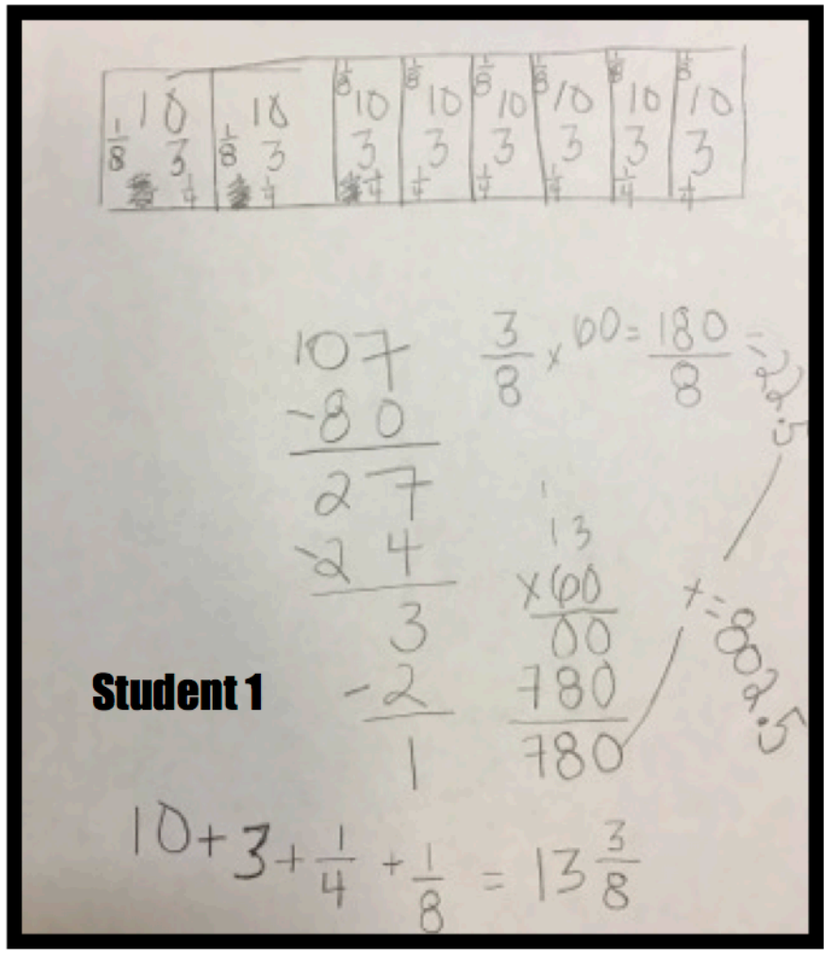
| Task: | | |
|---|---|---|
| Misconceptions: | | |
| **Strategy** | **Who and What (highlight)** | **Order** |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |

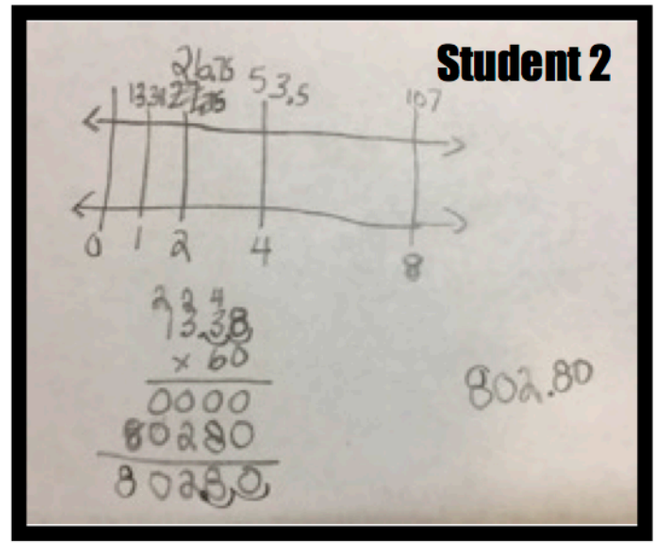
Anticipating → Monitoring → Selecting → Sequencing → Connecting

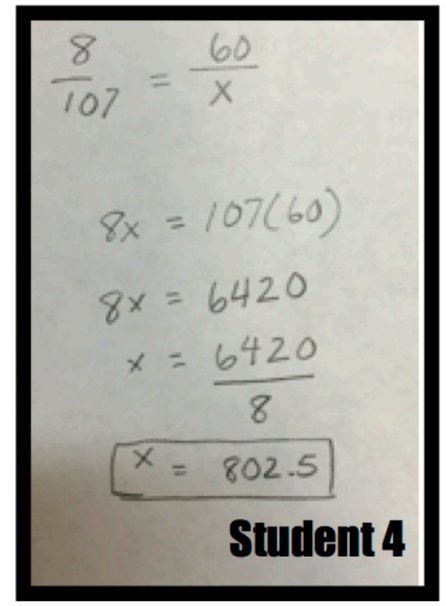
The 5 practices are:

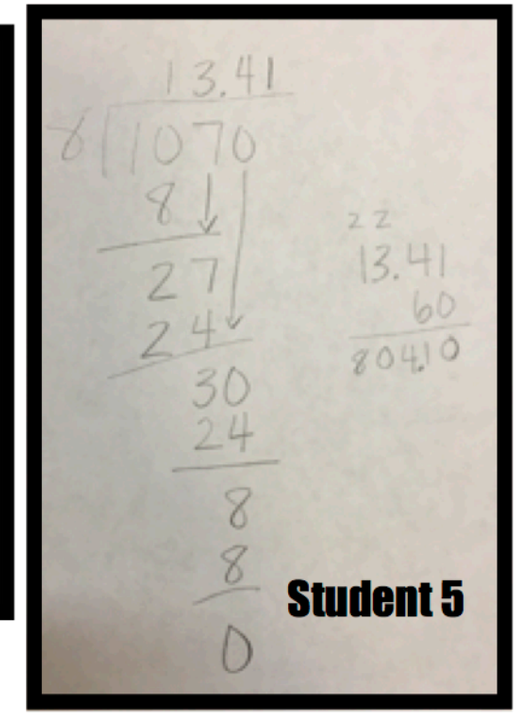
1. **Anticipating** student responses to challenging mathematical tasks;
2. **Monitoring** students' work on and engagement with the tasks;
3. **Selecting** particular students to present their mathematical work;
4. **Sequencing** the student responses that will be displayed in a specific order and;
5. **Connecting** different students' responses and connecting the responses to key mathematical ideas.

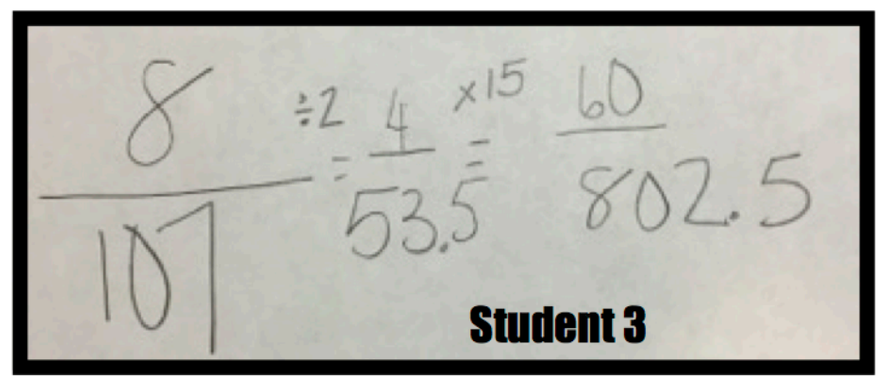
*MTMS: Vol. 14, No. 9, May 2009-5 Practices for Orchestrating Productive Mathematics Discussions*



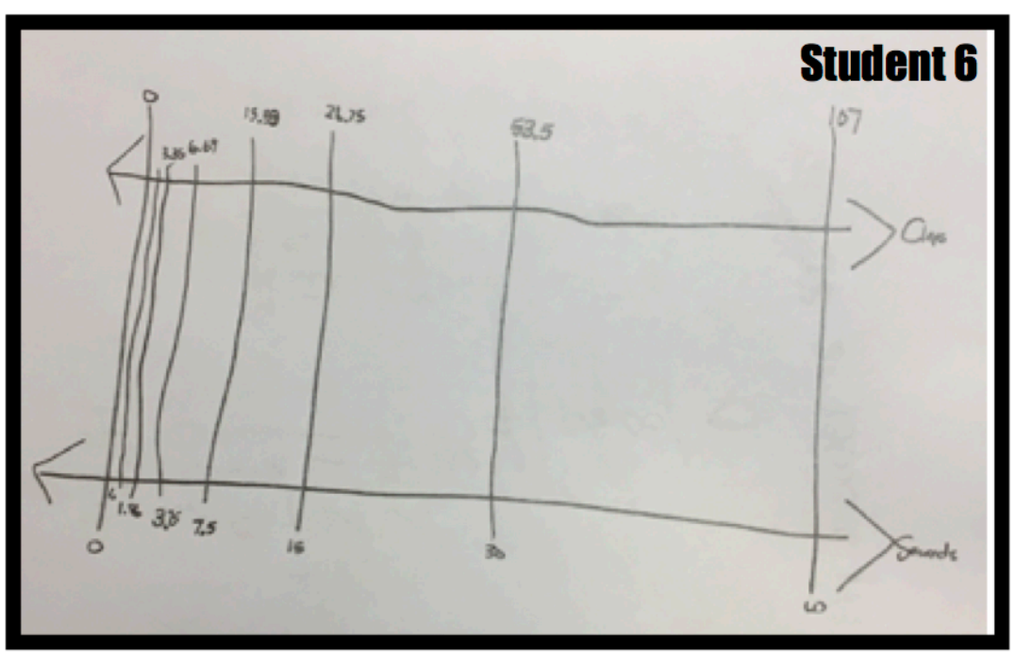



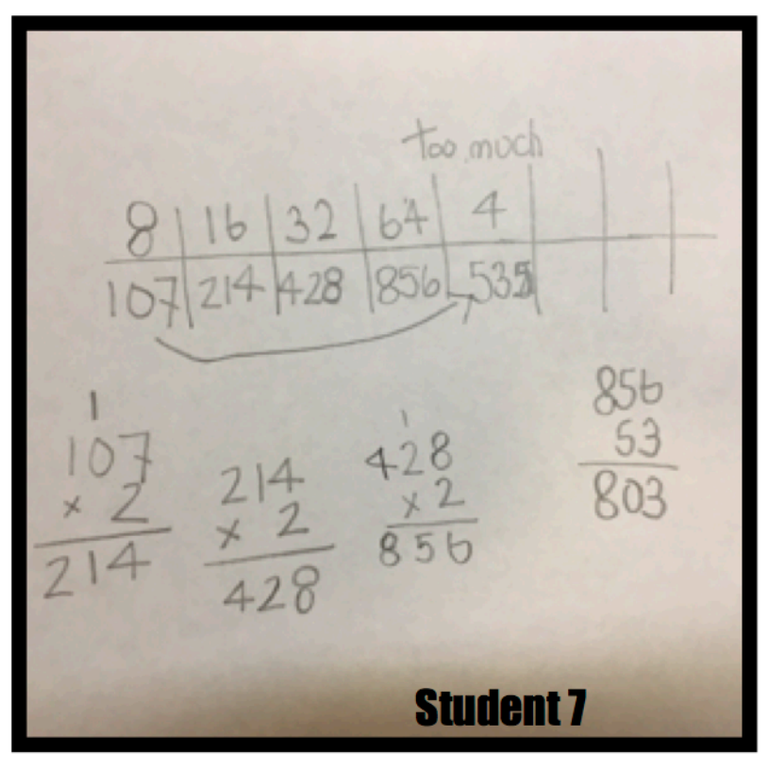



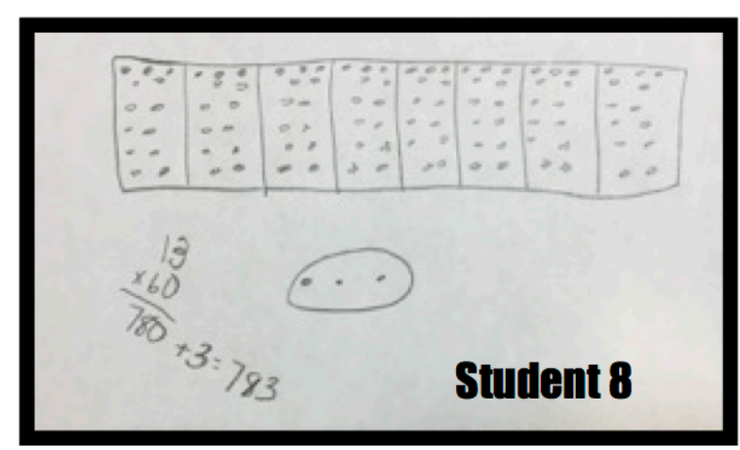




Select and sequence student work in the order of least efficient to most efficient



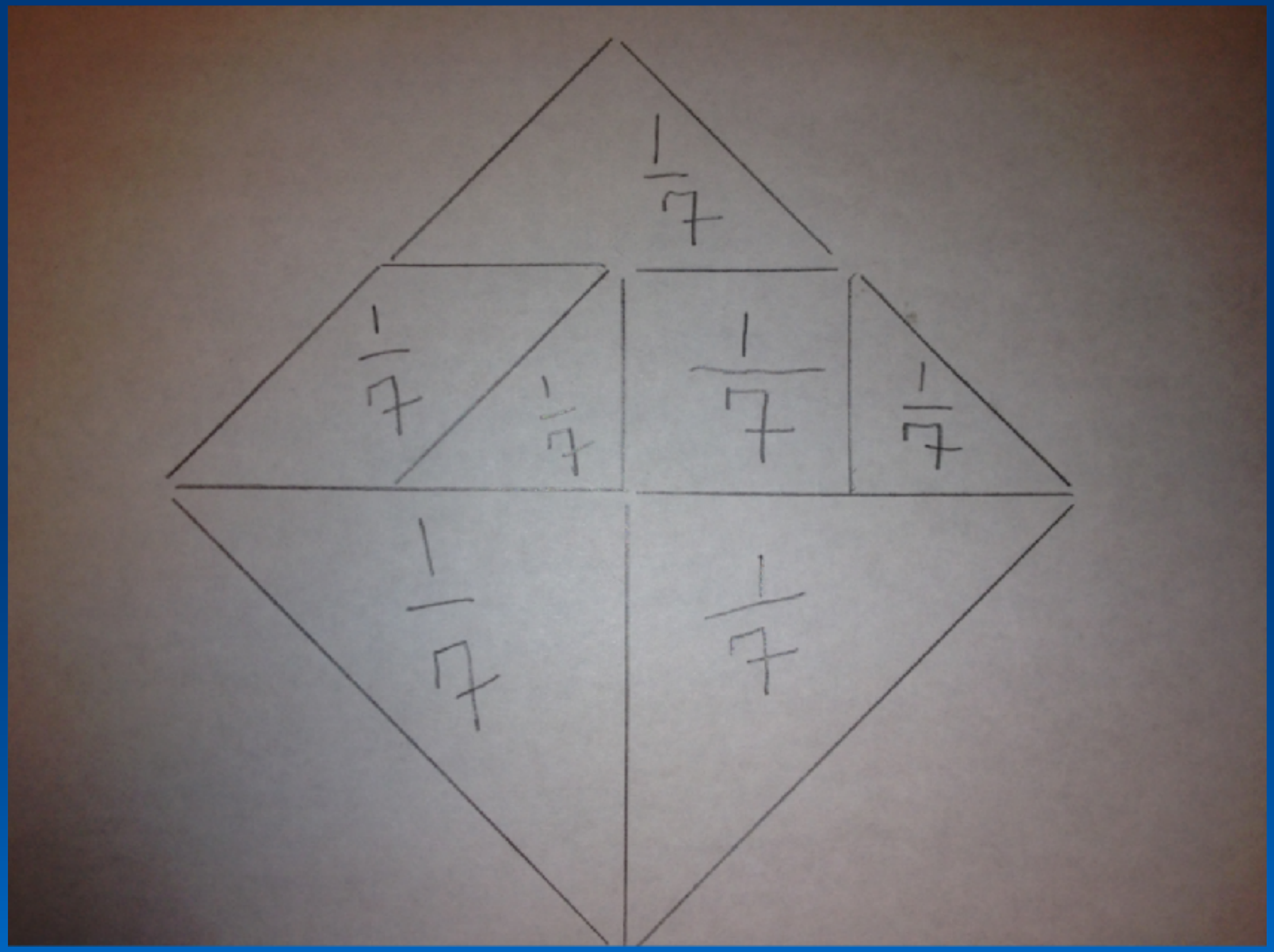



# The Condominium Problem

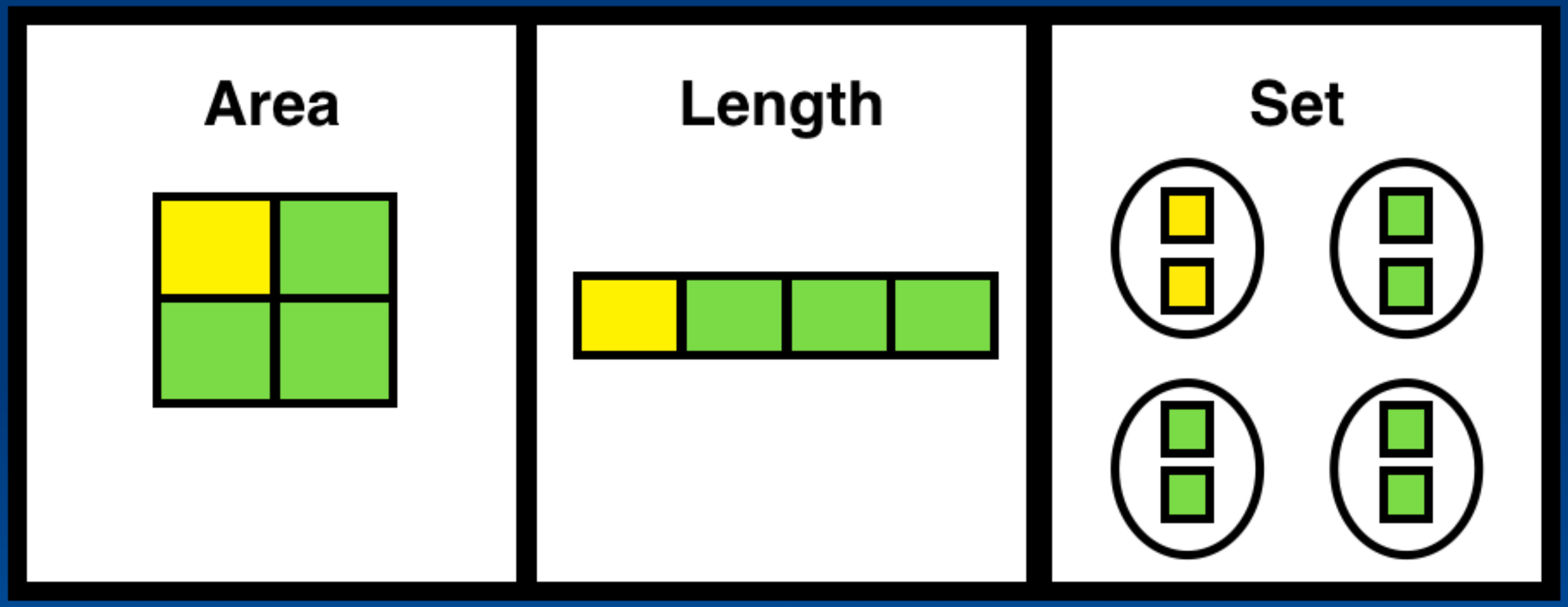
In a particular condominium community 2/3 of all of the men are married to 3/5 of all of the women.

What fraction of the entire condominium community are married?

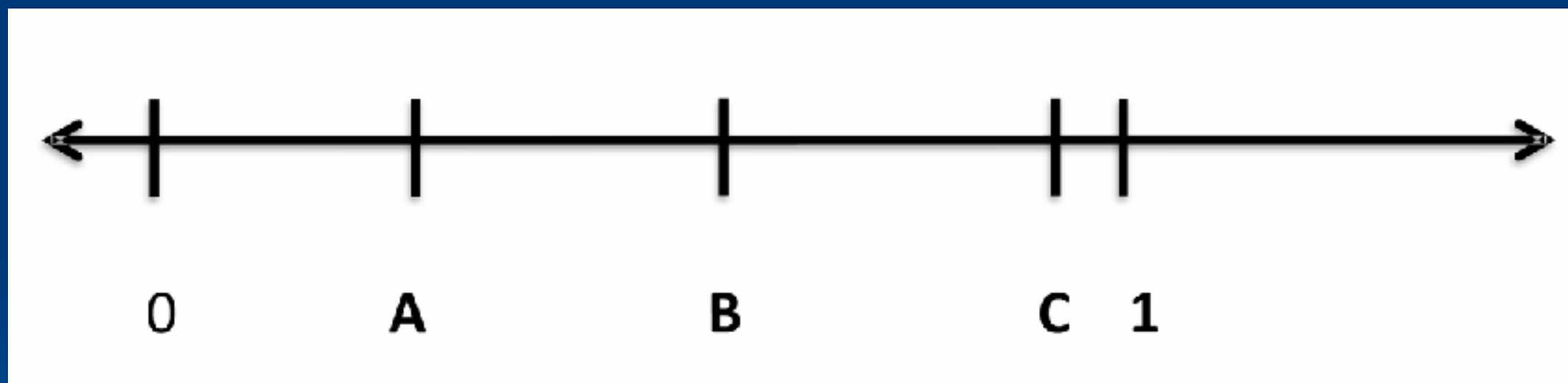
# Unit Fractions


1/7
1/7
1/7
1/7
1/7
1/7
1/7

# Representation of a Fraction




unit fraction $\frac{1}{a}$

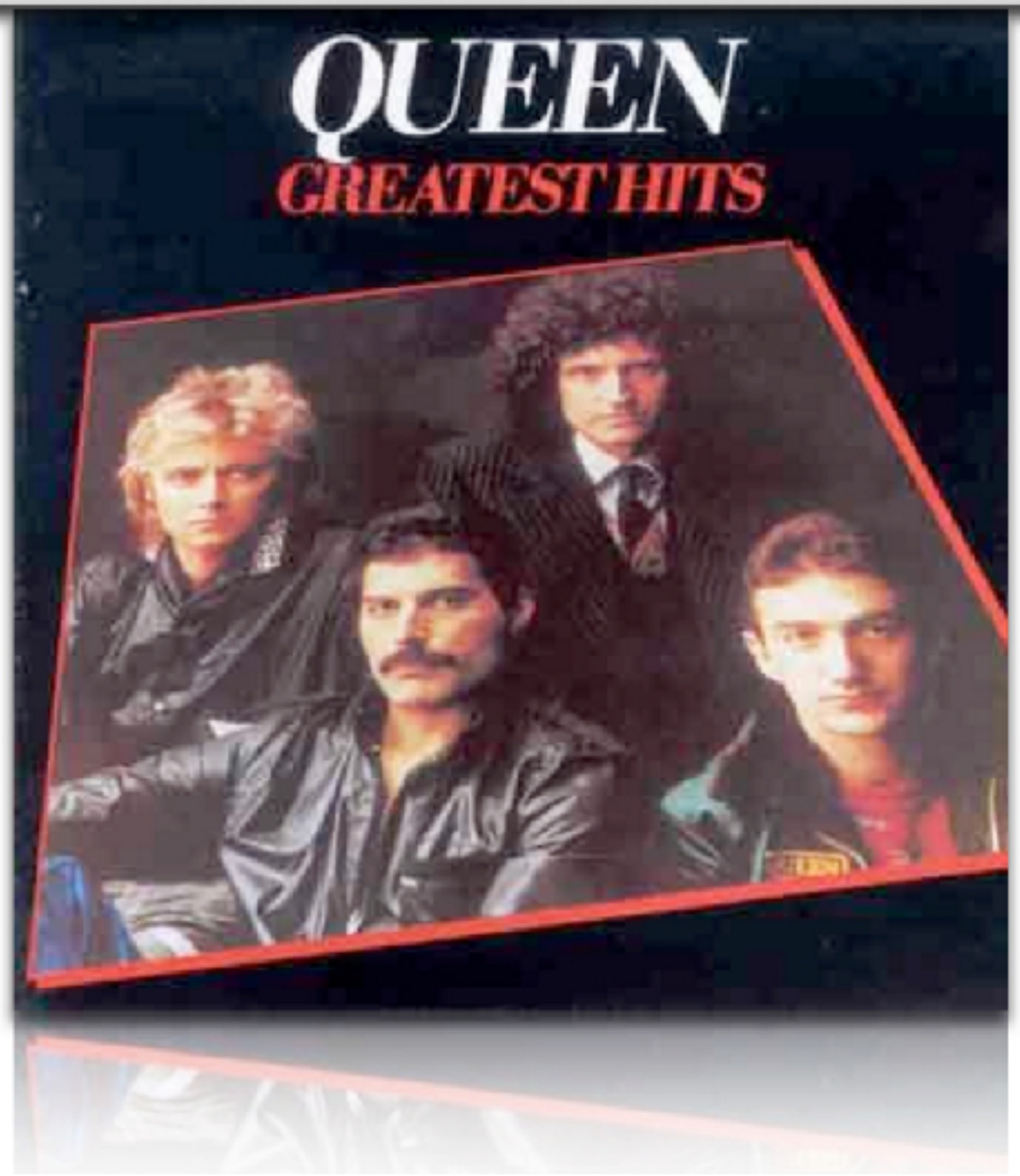

0
A
B
C
1


½
A
2
B

We Will Rock You

Queen — Greatest Hits I

0:21




ESTIMATION 180

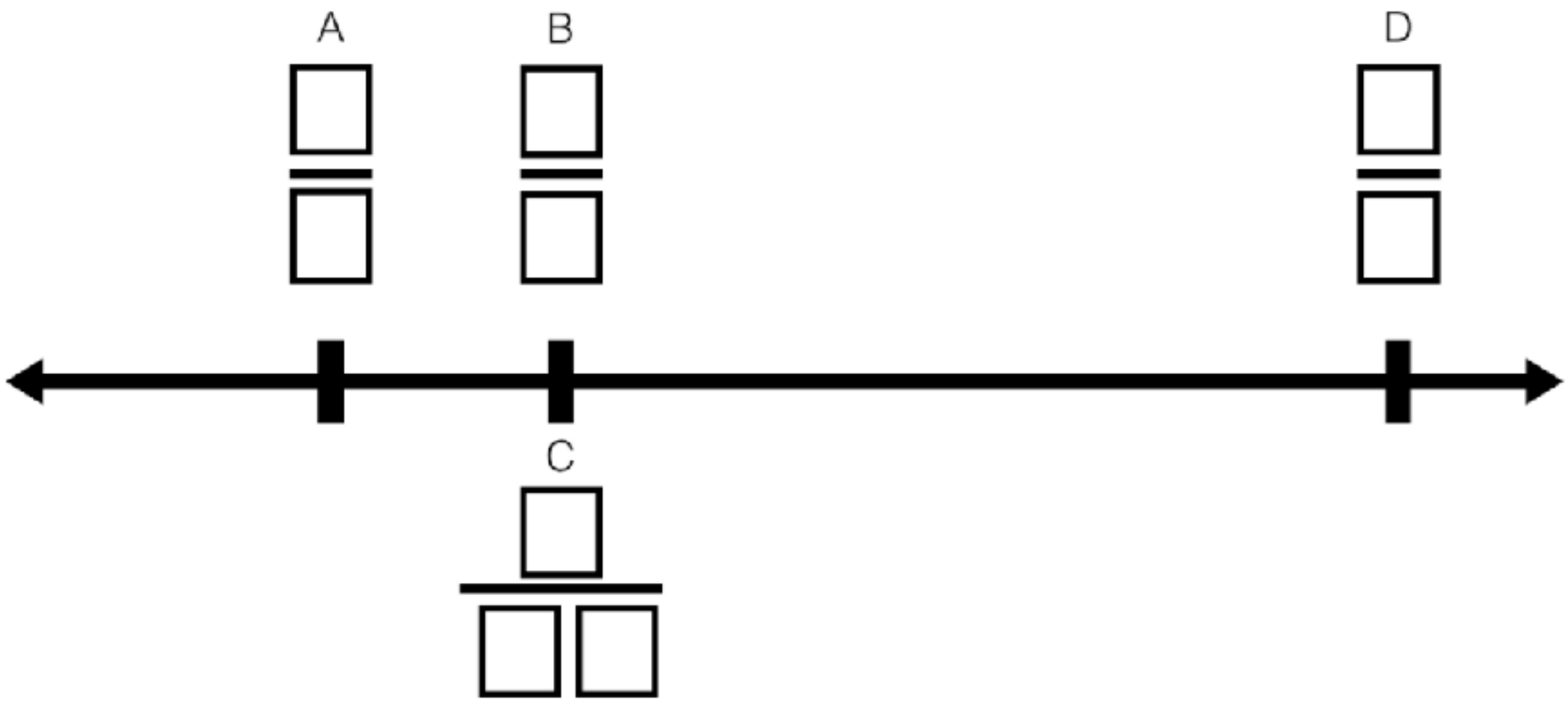
We Will Rock You

Queen — Greatest Hits I

0:00

# COMPARING AND IDENTIFYING FRACTIONS ON A NUMBER LINE

Directions: Using the whole numbers 1-9 once each, create and place 4 fractions on the number line in the correct order. (fractions B & C are equal)

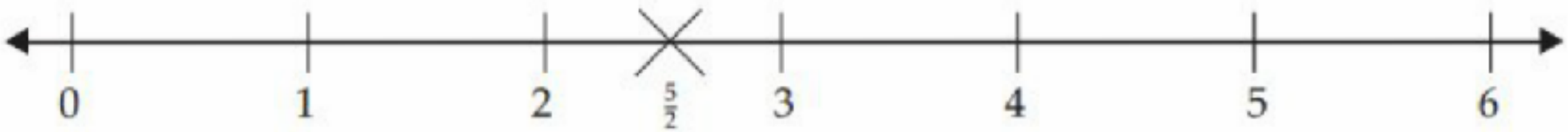



Open Middle

Challenging math problems worth solving

## Dotty Pairs Game

The students play in pairs. One student takes dots, the other takes crosses. Place the cards (cards 1–6, two lots, see Material Master 4-1) face down in a pile. The players take turns turning over two cards. The numbers are used to form a fraction, e.g., 2 and 5 are turned over, so $\frac{5}{2}$ or $\frac{2}{5}$ can be made. One fraction is chosen, made with the fraction pieces, if necessary, and marked on a 0–6 number line with the player's identifying mark (dot or cross).




Players take turns. The aim of the game is to get three of their marks uninterrupted by their opponent's marks on the number line. If a player chooses a fraction that is equivalent to a mark that is already there, they miss that turn.

NB: A fraction such as $\frac{4}{1}$ can be made using the cards. Students may not be familiar with fractions in this form and the meaning of the numerator and denominator will need to be explored with the fraction circles.

https://www.random.org/integers/

# 4-in-a-Row Equivalence

**Materials:**

- A game board, three 6-sided dice, two color counters

**Directions:**

- Roll three dice at the same time and select two dice to create an equivalent fraction to a fraction on the game board.
- Represent the 2 selected dice as a fraction. (Discard #3 die)
- Use a color marker to cover an equivalent fraction on the game board.
- The winner is the first player to cover 4 squares in a row.

Dice 1

Dice 2

| | | | | | |
|---|---|---|---|---|---|
| $\frac{2}{12}$ | $\frac{8}{10}$ | $\frac{2}{4}$ | $\frac{1}{1}$ | $\frac{2}{3}$ | $\frac{2}{8}$ |
| $\frac{4}{2}$ | $\frac{1}{2}$ | $\frac{8}{12}$ | $\frac{12}{4}$ | $\frac{2}{10}$ | $\frac{10}{5}$ |
| $\frac{4}{8}$ | $\frac{10}{12}$ | $\frac{4}{6}$ | $\frac{2}{1}$ | $\frac{6}{12}$ | $\frac{5}{10}$ |
| $\frac{6}{10}$ | $\frac{6}{3}$ | $\frac{3}{12}$ | $\frac{1}{3}$ | $\frac{8}{4}$ | $\frac{4}{8}$ |
| $\frac{3}{9}$ | $\frac{3}{1}$ | $\frac{4}{10}$ | $\frac{3}{6}$ | $\frac{8}{2}$ | $\frac{3}{12}$ |
| $\frac{9}{12}$ | $\frac{4}{12}$ | $\frac{1}{4}$ | $\frac{6}{8}$ | $\frac{6}{9}$ | $\frac{2}{2}$ |

# Making Sense Series

The Progression of Fractions
Meaning, Equivalence, & Comparison

created by Graham Fletcher
@gfletchy
www.gfletchy.com

## Task Instruction

For each part of the problem, start with a square sheet of paper and make folds to construct a new shape. Then, explain how you know the shape you constructed has the specified area.

1. Construct a square with exactly ¼ the area of the original square.
2. Construct a triangle with exactly ¼ the area of the original square.
3. Construct another triangle, also with ¼ the area, that is not congruent to the first one you constructed.
4. Construct a square with exactly ½ the area of the original square.

# Begin with Fair Share Problems

Take 2 minutes and solve question #1 independently.

Try to solve using only drawings.

Numbers can only be used to label drawings.

Share your solution with the group or with a partner.

Notice the different representations for how to solve the question. Are there any connections between the models?

# Fair Share

1. There are 11 yards of ribbon for 4 people to share. How many yards of ribbon can each person get if they share the ribbon equally?

2. Macey and Bryson have 13 cookies. If they share the cookies equally, how many cookies would each person get?

3. 12 children in art class have to share 8 packages of clay so that everyone gets the same amount. How much clay can each child have?

Taken from Epson and Levi's book *Extending Children's Mathematics: Fractions and Decimals*

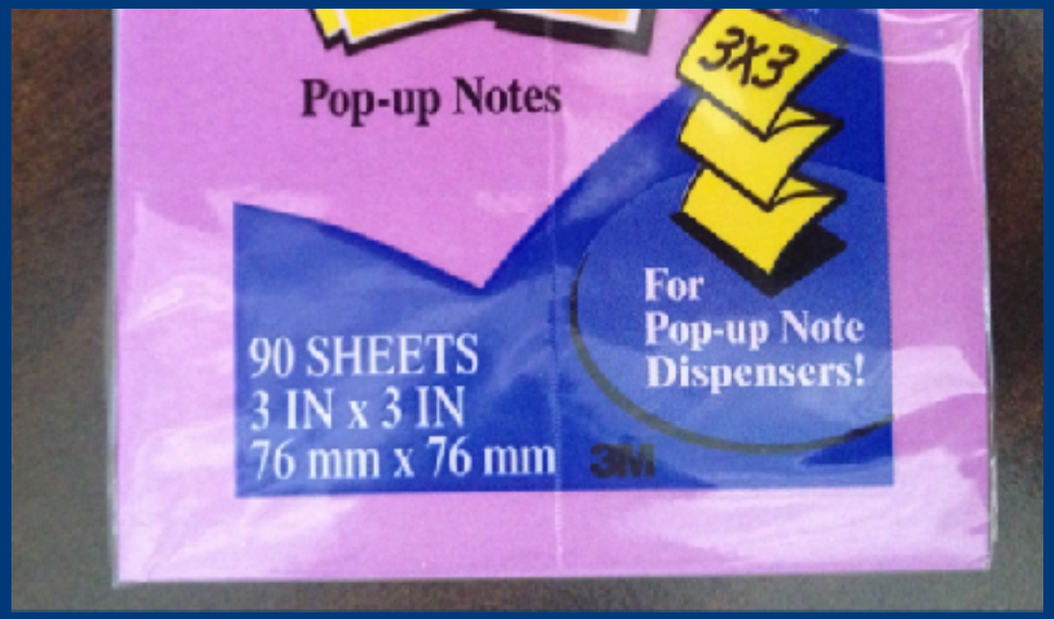
# the Big Pad

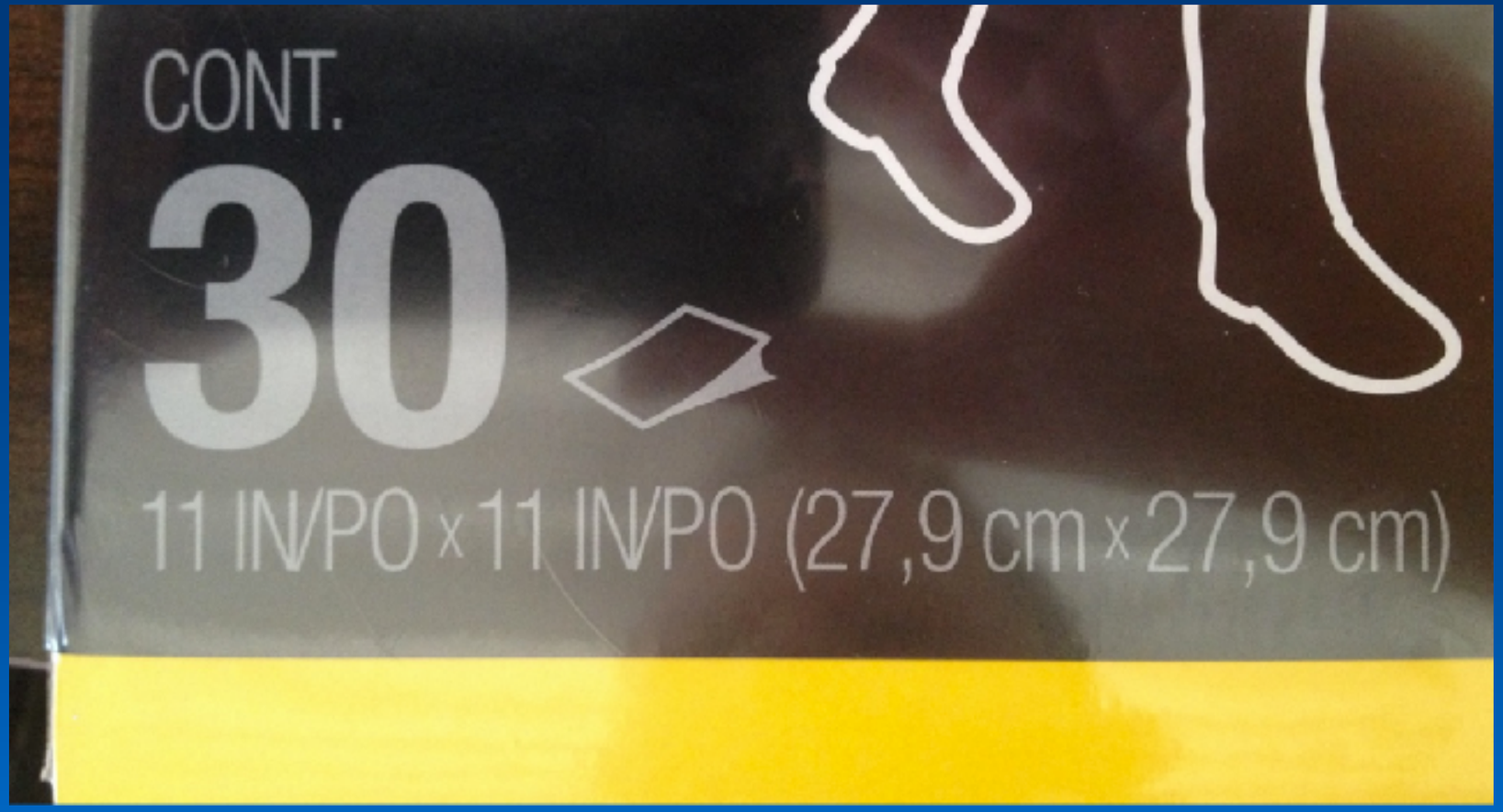

Post-it
BIG PAD
NOTAS GIGANTES
GRAND BLOC
CONT.

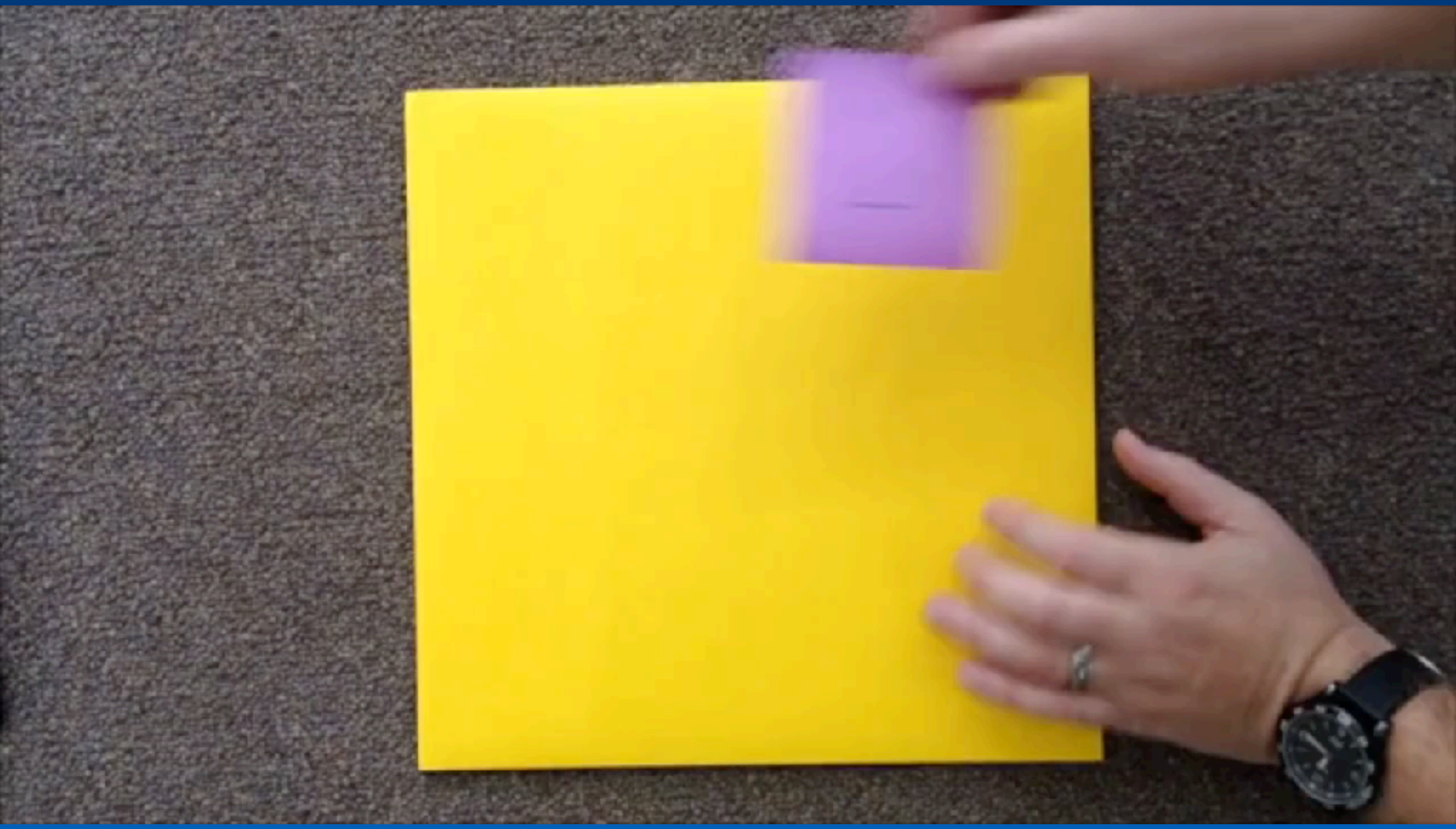
What do you notice?

What do you wonder?

need more?


Pop-up Notes
3X3
90 SHEETS
3 IN x 3 IN
76 mm x 76 mm
3M
For
Pop-up Note
Dispensers!


CONT.
30
11 IN/PO x 11 IN/PO (27,9 cm x 27,9 cm)



Where does the concept of area model and partial products with whole numbers extend from elementary school?

$$3\frac{1}{2} \text{ x } 3\frac{1}{3}$$

$$4(x + 3)$$

$$(x + 8)(x + 7)$$

$$x^2 + 6x - 3 = 0$$

$$3\frac{1}{2} \times 3\frac{1}{3}$$

| | $3$ | $+\ \frac{1}{2}$ |
|---|---|---|
| $2$ | $6$ | $1$ |
| $+\ \frac{1}{3}$ | $1$ | $\frac{1}{6}$ |

$$4(x + 3)$$

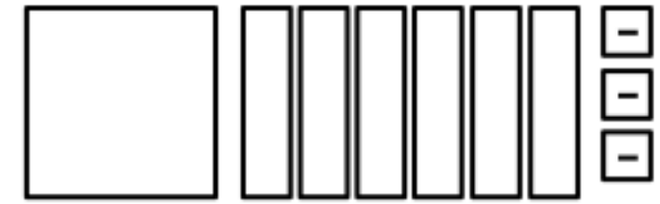
| | $x$ | $+\ 3$ |
|---|---|---|
| $4$ | $4x$ | $12$ |

$$(x+8)(x+7)$$

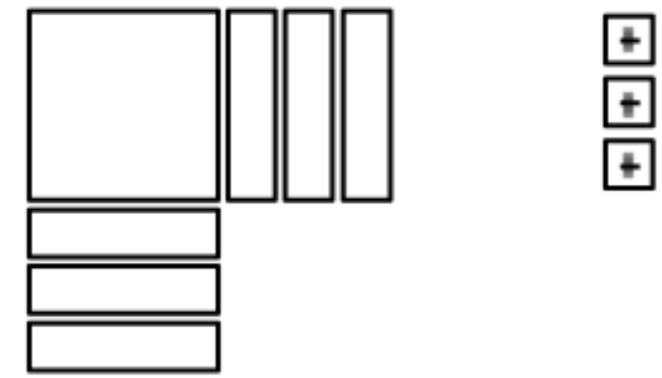
| | $x$ | $+\ 7$ |
|---|---|---|
| $x$ | $x^2$ | $7x$ |
| $+\ 8$ | $8x$ | $56$ |

$$x^2 + 6x - 3 = 0$$

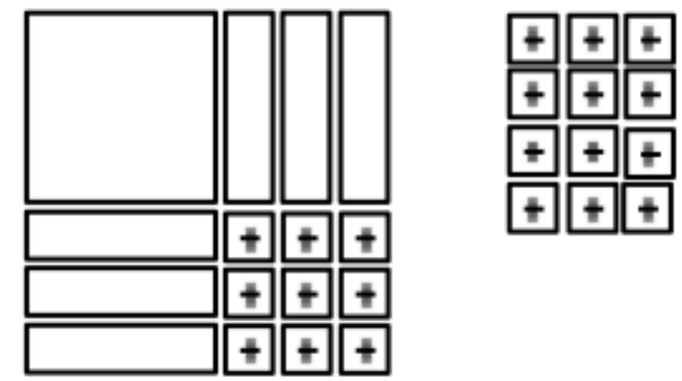
$$x^2 + 6x - 3 = 0$$



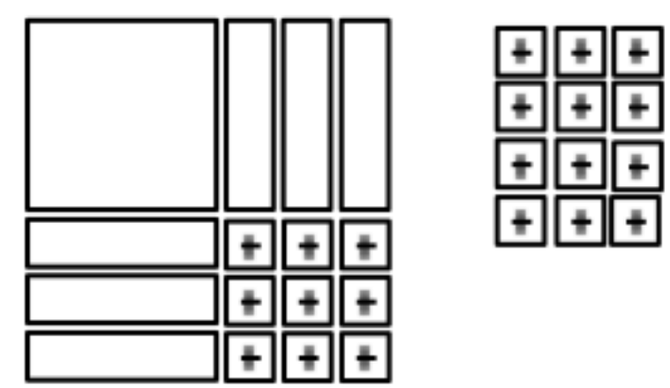
$$x^2 + 6x + \quad = 3$$



$$x^2 + 6x + 9 = 12$$



$$(x + 3)^2 = 12$$



# Breakout #4

**Step 1:** Open the task folder and complete the task as a group

- Act 1
- Act 2
- Act 3

**Step 2:** Anticipate the ways students will solve this problem

**Step 3:** Sequence and order the student work from the student work folder

**Step 4:** Where is the math from and where is going?

# Multiple Group Problems

1. Eve's gecko eats 2/7 jar of baby food a day. She has 10 jars of baby food. How many days can she feed her gecko with this food?

2. Emma drinks 2/3 cups of water for every mile she hikes. Her water bottle holds 4 cups of water. How many miles can she hike before her water runs out?

3. It takes 3/5 yard of ribbon to make a bow. How many bows could you make with 7.5 yards of ribbon?

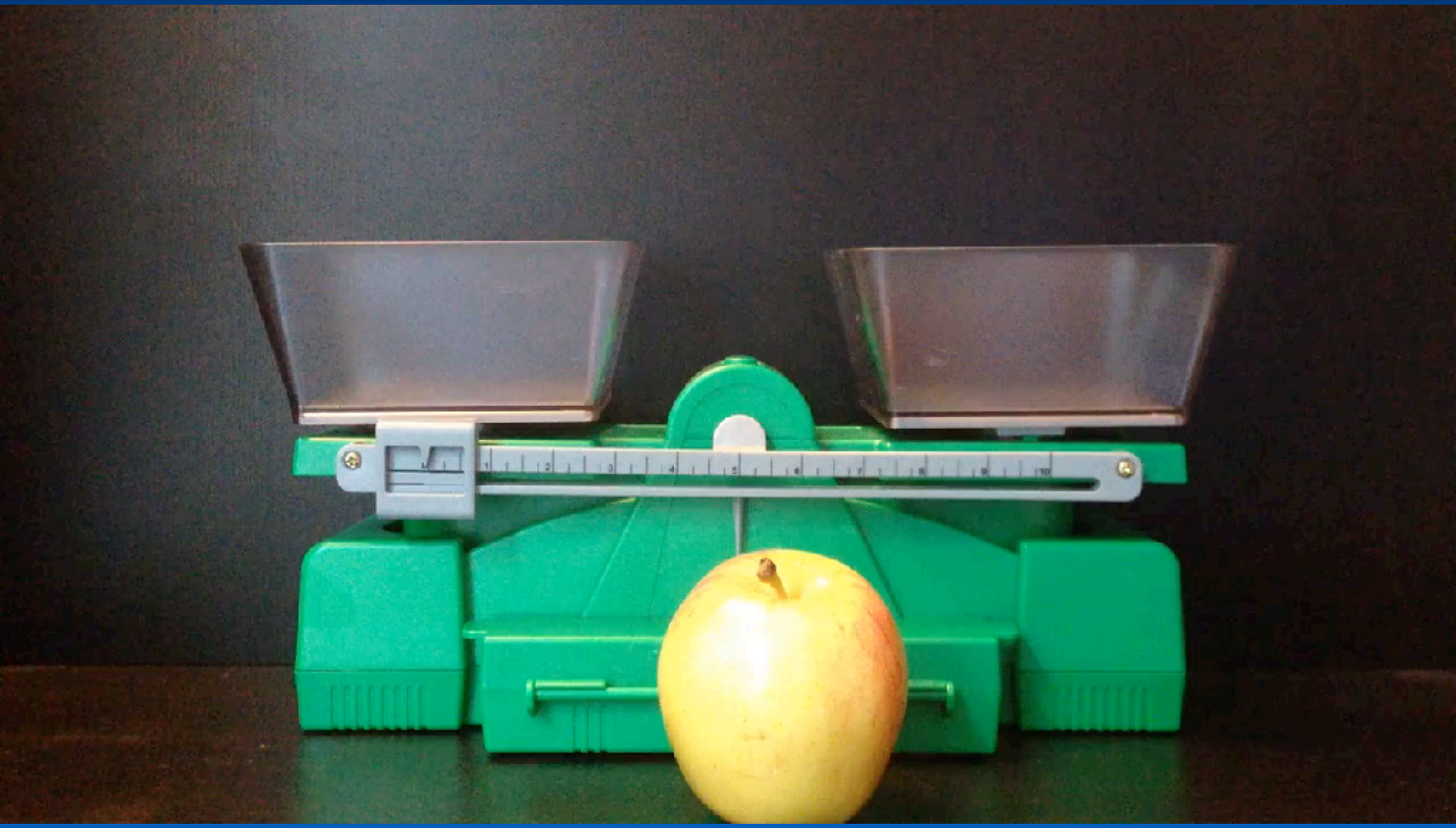
Taken from Epson and Levi's book *Extending Children's Mathematics: Fractions and Decimals*

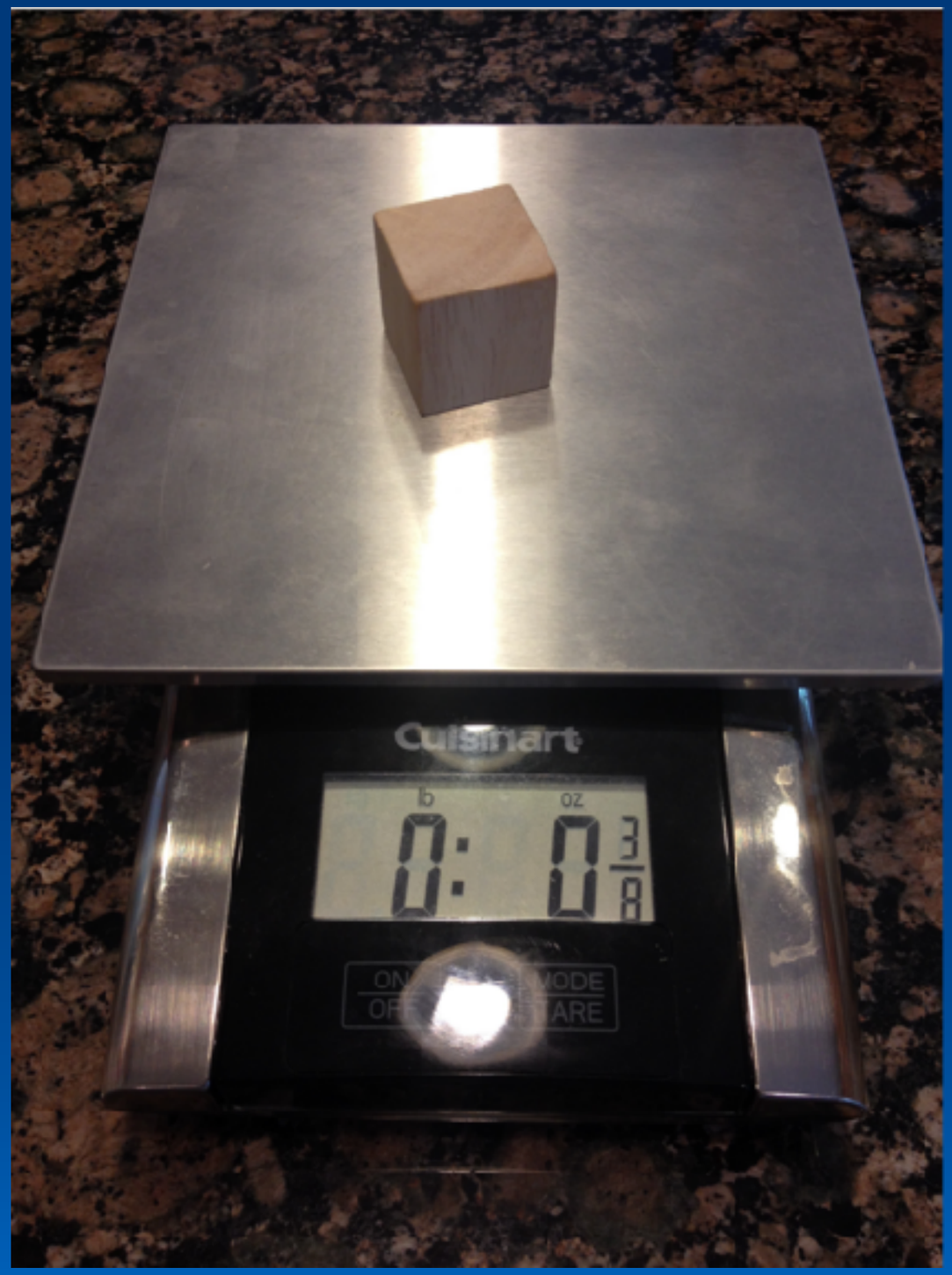
model with manipulatives or drawings

NO NUMBERS ALLOWED

be prepared to explain and justify your model

$$\frac{3}{4} \div \frac{2}{3}$$

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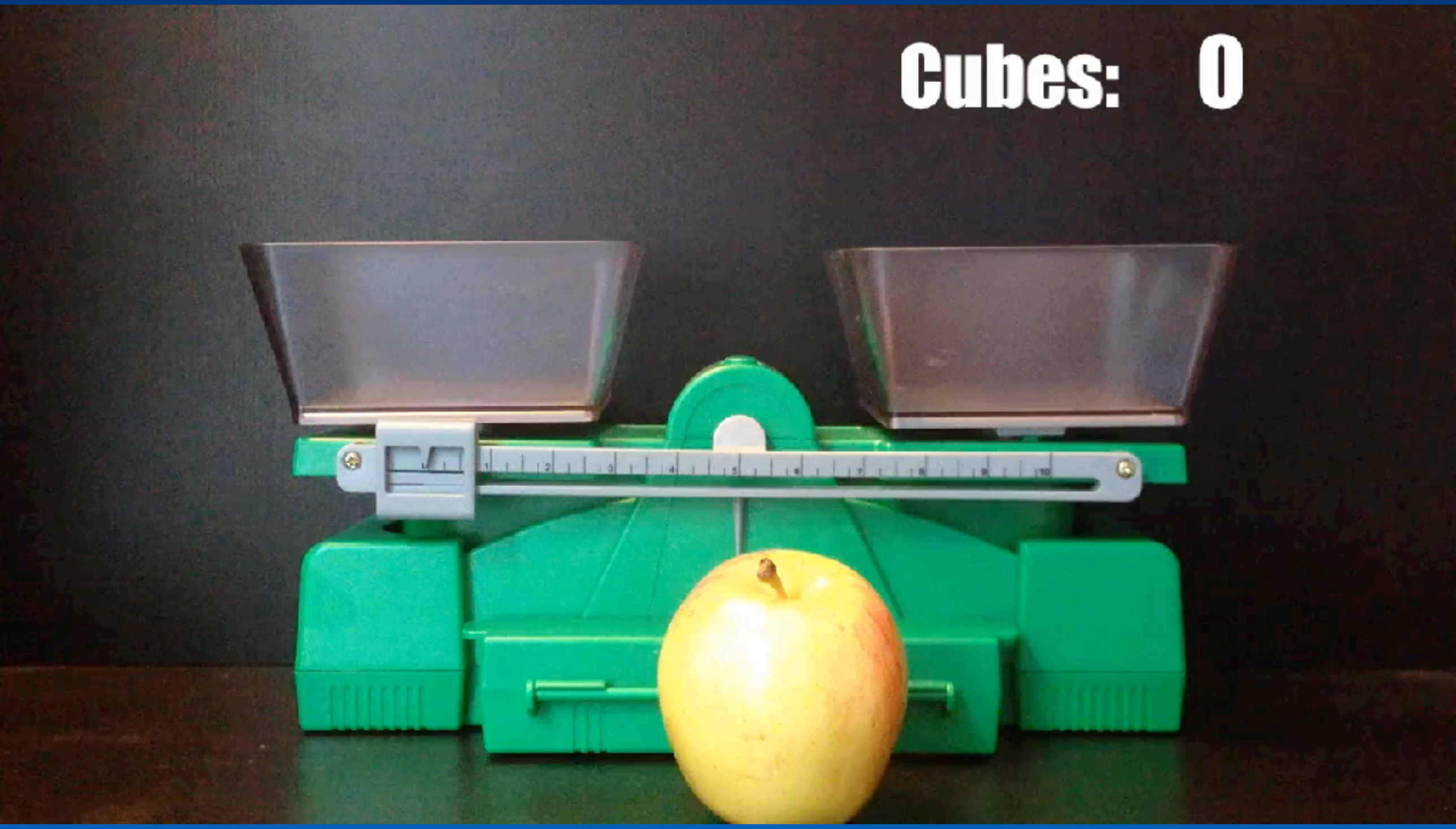

Cubes: 0


MATH
0:00 / 0:14

# Equivalencing Problems

1. David used exactly 24 cups of flour to make 8 loaves of bread. How many loaves of bread can he make with 6 cups of flour?

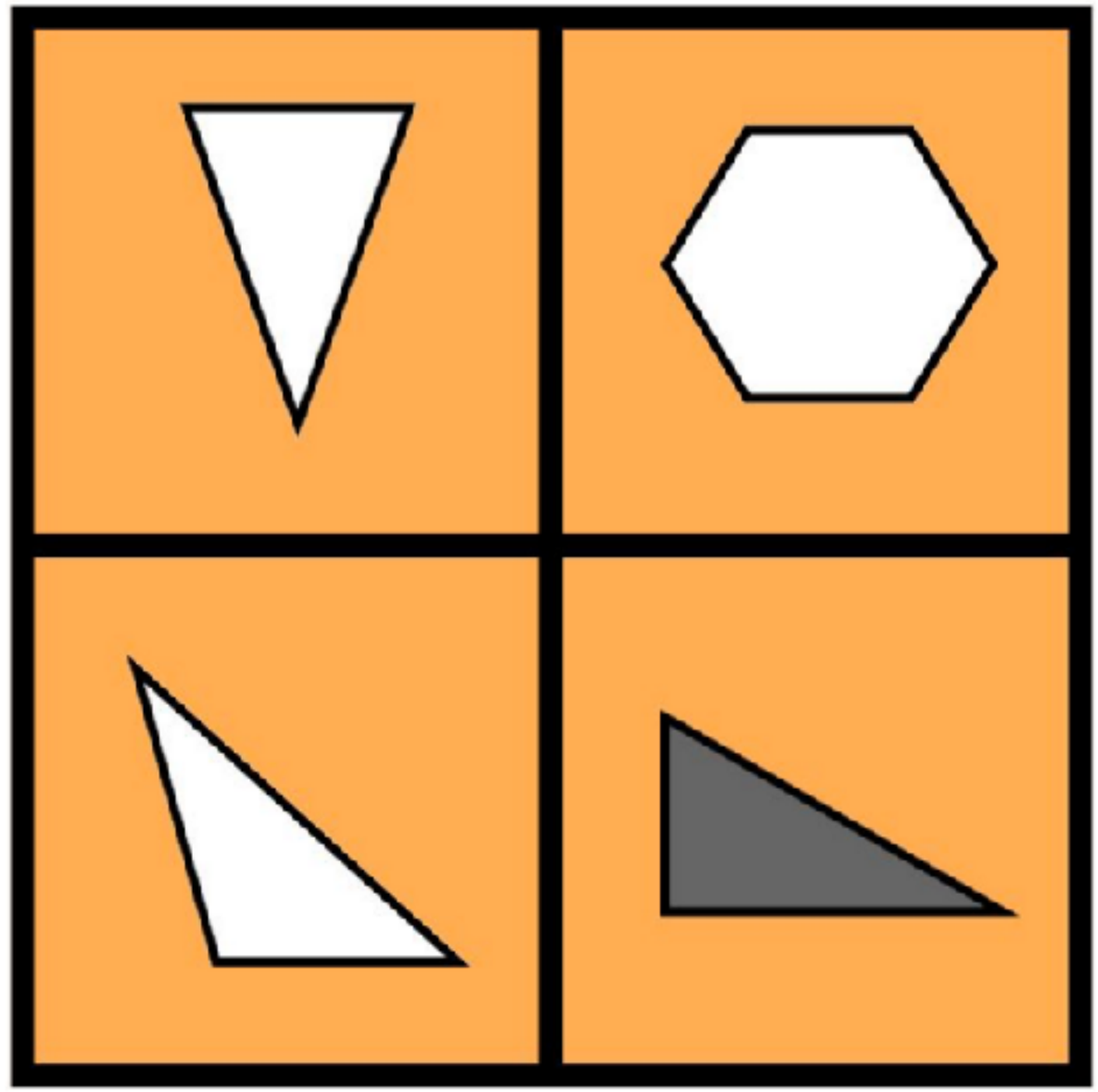
2. Socks are selling for $5 for 4 pairs. How much will 9 pairs of socks cost at this rate?

3. Who gets more popcorn? A person in a group where 3 people are sharing 2 small bags of popcorn or a person in a group where 6 children are sharing 4 small bags of popcorn?

Taken from Epson and Levi's book *Extending Children's Mathematics: Fractions and Decimals*

# Rope Jumper

# Modification
# &
# Accessibilty



**SHAPE 1**

from Mary Bourassa

| | |
|---|---|
| $\frac{1}{20}$ | $\frac{20}{25}$ |
| $\frac{2}{3}$ | $\frac{5}{4}$ |