

# Harmony Elementary School

October 11th, 12th, 13th

by: **Graham Fletcher**

email: [gfletchy@gmail.com](mailto:gfletchy@gmail.com)

twitter: [@gfletchy](https://twitter.com/gfletchy)

blog: [www.gfletchy.com](http://www.gfletchy.com)

## Broken Squares

- Designate shape keeper
- Share the shapes as evenly as possible between the group
- All the shapes will make 6 congruent squares with no shapes leftover
- Everyone is encouraged to **OFFER** a shape. No one may **TAKE** or **SNATCH**. You may RECEIVE a shape but only if it's **OFFERED**
- No “community square”



**NEXT TIME YOU'RE AFRAID  
TO SHARE IDEAS  
REMEMBER SOMEONE  
ONCE SAID IN A MEETING  
LET'S MAKE A FILM WITH A  
TORNADO FULL OF SHARKS**

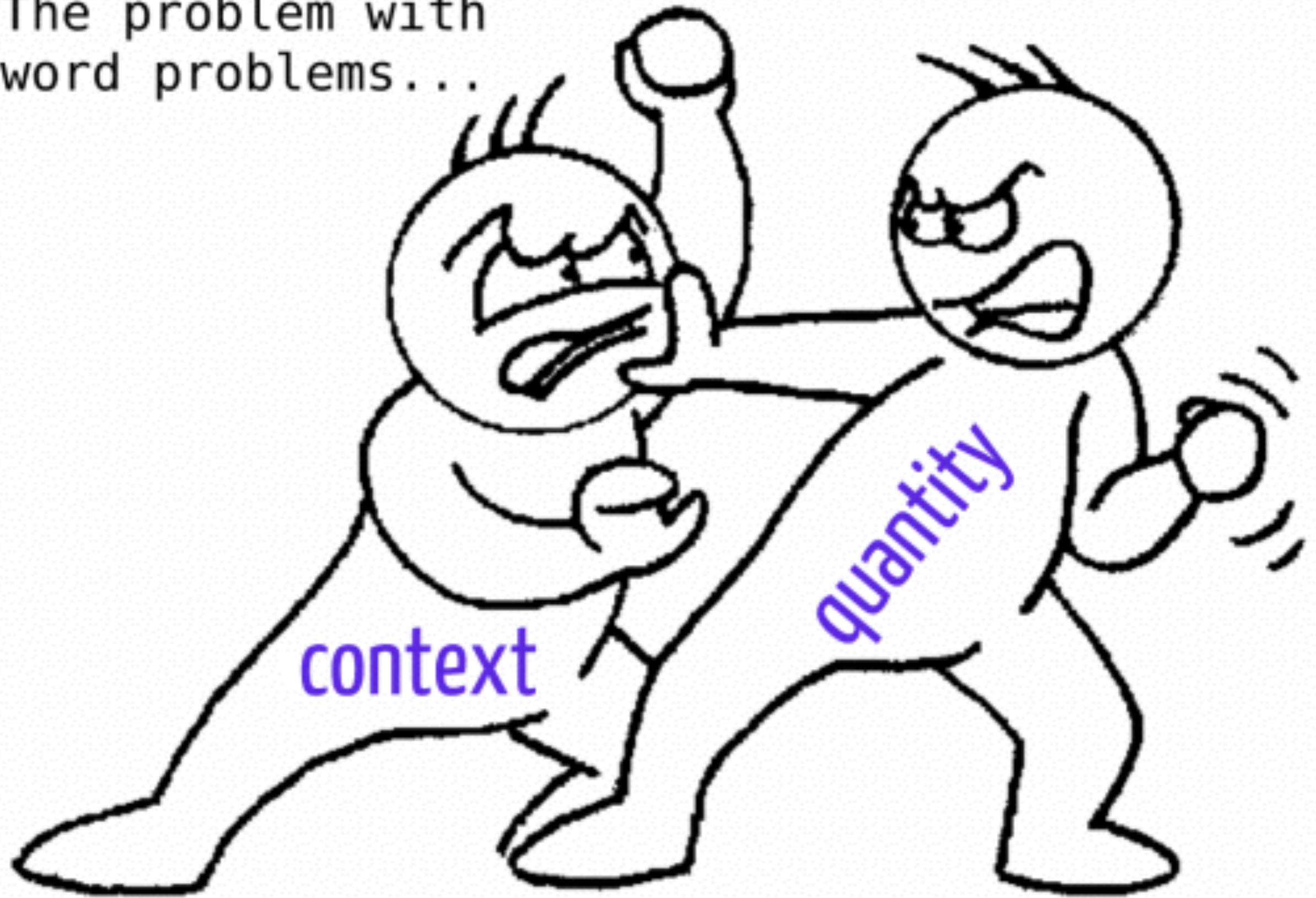


Joe had some playing cards in his bag. Ashley gave him 13 more cards. Joe now has 21 cards. How many cards did Joe have in his bag?

13

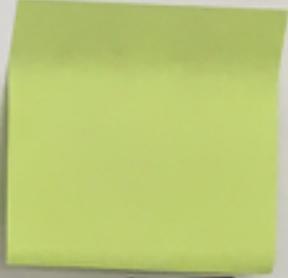
21

The problem with  
word problems...



 @gfletchy

Joe had some playing cards in his bag. Ashley gave him 13 more cards. Joe now has 21 cards. How many cards did Joe have in his bag?

Joe had some playing cards  
in his bag. Ashley gave him   
more cards. Joe now has   
cards. How many cards did Joe  
have in his bag?



How many cards did Joe have in his bag?

Name: \_\_\_\_\_

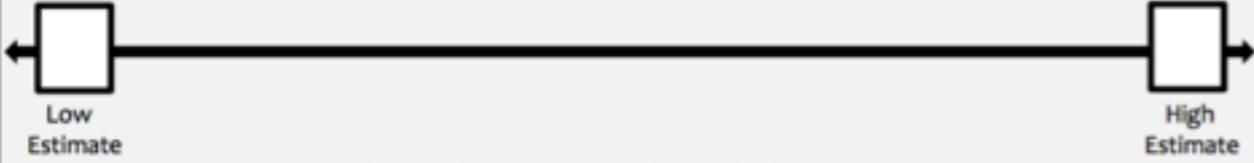
Date: \_\_\_\_\_

1. What did you notice?

2. What do you wonder?

3. Main Question:

4. Estimate



Low Estimate

High Estimate

*place your best estimate on the number line and label*

5. What information would you like to know?

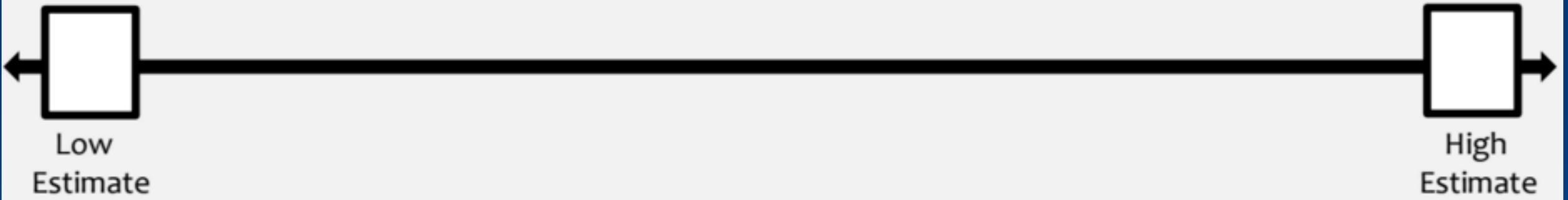
6. Answer



THIS PRODUCT IS NOT FOR SALE

Original  
**Skittles**

#### 4. Estimate



# Packages Used







The answer IS NOT 812!



Content Standards

VS

Practice Standards

# The 8 Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

# Missing the Modeling Mark

- The use of manipulatives does not ensure that modeling with mathematics is taking place.
- If the mathematics is not contextualized, modeling with mathematics cannot exist.
- Modeling with mathematics does not mean, “I do, we do, you do.”

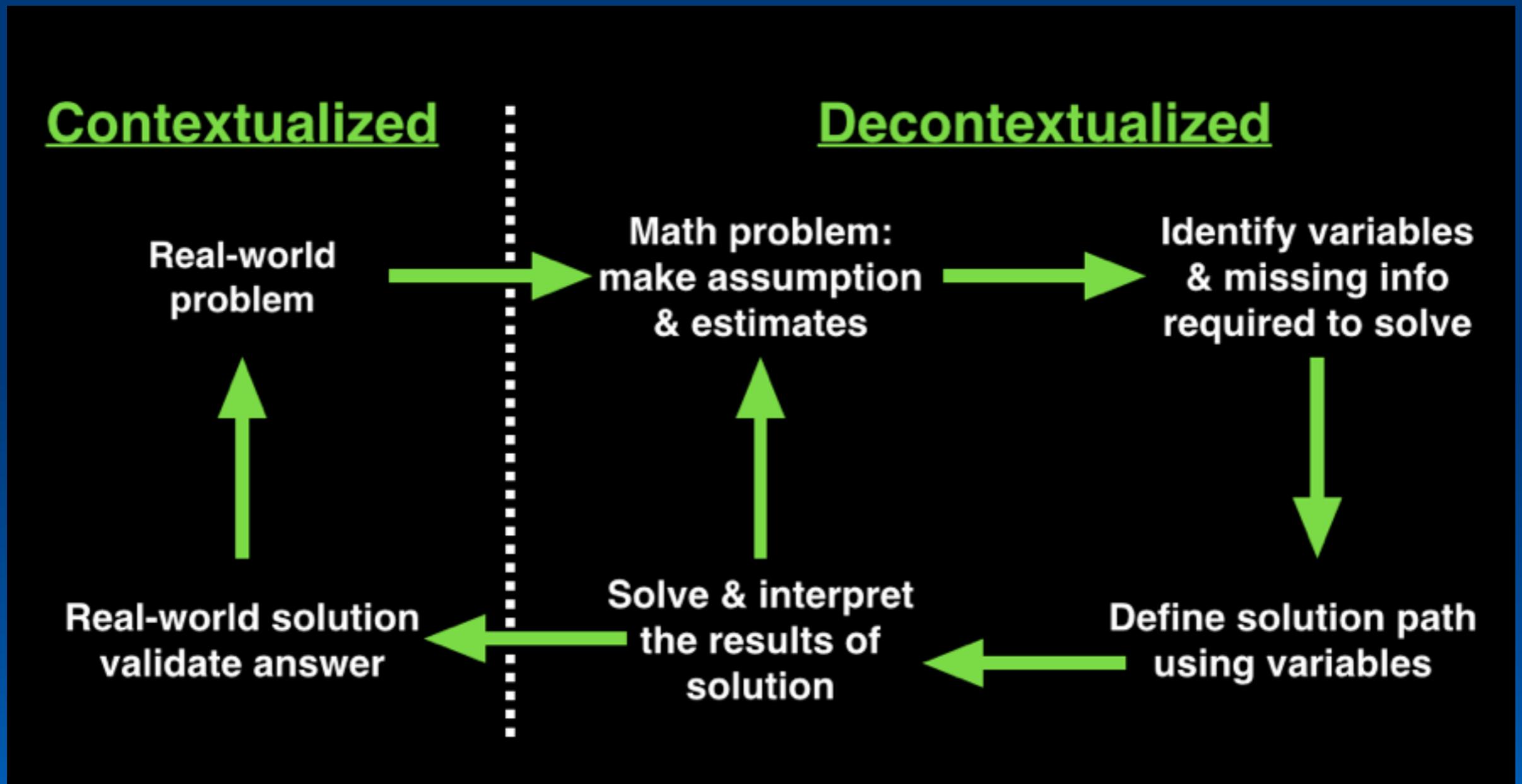
## Model with Mathematics

Mathematically proficient students can apply the mathematics they know to **solve problems arising in everyday life**, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. Mathematically proficient students who can apply what they know are comfortable **making assumptions and approximations** to simplify a complicated situation, realizing that these may need revision later. They are able to **identify important quantities** in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can **analyze those relationships** mathematically to draw conclusions. They routinely **interpret their mathematical results** in the context of the situation and **reflect on whether the results make sense**, possibly improving the model if it has not served its purpose.

## Model with Mathematics

Mathematically proficient students can apply the mathematics they know to **solve problems arising in everyday life**, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. Mathematically proficient students who can apply what they know are comfortable **making assumptions and approximations** to simplify a complicated situation, realizing that these may need revision later. They are able to **identify important quantities** in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can **analyze those relationships** mathematically to draw conclusions. They routinely **interpret their mathematical results** in the context of the situation and **reflect on whether the results make sense** possibly improving the model if it has not served its

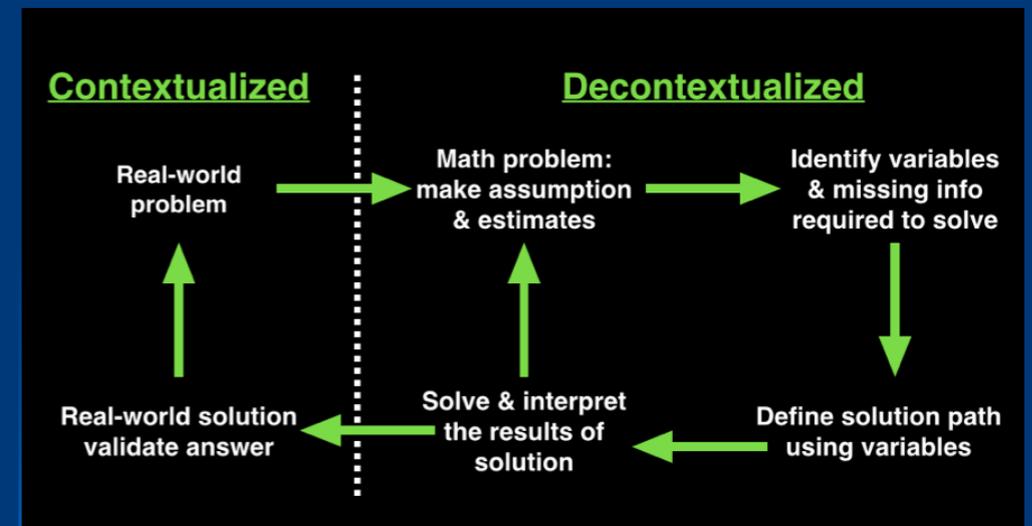
# #4 Model with Mathematics in the Elementary Grades



### **3 Things on the road to modeling...**

- Identify the problem, or pose a question.
- Make an estimate.
- Identify the variables needed to solve, and answer the problem or question posed.

# 3-Act Tasks



## Act 1:

- Real world problem or scenario presented
- What do you notice? What do you wonder?
- Make estimates

## Act 2:

- Identify missing variables and missing variables to solve
- Define solution path using variables

## Act 3:

- Solve and interpret results of the solution
- Validate answer

## Most asked questions:

- How often should we use 3-Act Tasks?
- When should we use 3-Act tasks? How do they fit into the scope of a unit?
- How long does one task usually take?
- What if we don't have the time?
- Any others?

# The 8 Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

# The 8 Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.



# 5 Practices for Orchestrating Productive Mathematical Discussions



## Orchestrating Discussions

*Five practices constitute a model for effectively using student responses in whole-class discussions that can potentially make teaching with high-level tasks more manageable for teachers.*

Margaret S. Smith, Elizabeth K. Hughes, Randi A. Engle, and Mary Kay Stein



**Margaret S. Smith**, [mgsm@pitt.edu](mailto:mgsm@pitt.edu), is an associate professor of mathematics education at the University of Pittsburgh. Over the past decade, she has been developing research-based materials for use in the professional development of mathematics teachers and studying what teachers learn from the professional development in which they engage. **Elizabeth K. Hughes**, [elizabeth.hughes@uni.edu](mailto:elizabeth.hughes@uni.edu), recently finished her doctorate in mathematics education at the University of Pittsburgh. Her areas of interest include preservice secondary mathematics teacher education and the use of practice-based materials in developing teachers' understanding of what it means to teach and learn mathematics. **Randi A. Engle**, [raengle@berkeley.edu](mailto:raengle@berkeley.edu), is an assistant professor of mathematics education and the social context of learning at the University of California Berkeley. She is interested in developing practical theories for how mathematics teachers can create discussion-based learning environments that promote strong student engagement, learning, and transfer. **Mary Kay Stein**, [mkslein@pitt.edu](mailto:mkslein@pitt.edu), is a professor of learning sciences and policy and the director of the Learning Policy Center at the University of Pittsburgh. Her research focuses on instructional practice and the organizational and policy conditions that shape it.

Discussions that focus on cognitively challenging mathematical tasks, namely, those that promote thinking, reasoning, and problem solving, are a primary mechanism for promoting conceptual understanding of mathematics (Hatano and Inagaki 1991; Michaels, O'Connor, and Resnick forthcoming). Such discussions give students opportunities to share ideas and clarify understandings, develop convincing arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things from other perspectives (NCTM 2000). Although discussions about high-level tasks provide important

548 MATHEMATICS TEACHING IN THE MIDDLE SCHOOL • Vol. 14, No. 9, May 2009

Copyright © 2009 The National Council of Teachers of Mathematics, Inc. www.nctm.org. All rights reserved. This material may not be copied or distributed electronically or in any other form without written permission from NCTM.

Vol. 14, No. 9, May 2009 • MATHEMATICS TEACHING IN THE MIDDLE SCHOOL 549

The **5** practices are:

1. **Anticipating** student responses to challenging mathematical tasks;

Anticipate student solutions for the Whopper Jar task.

Identify and record the types of strategies students will use to solve the task.

### Task Planning Document

<b>Task:</b>		
<b>Misconceptions:</b>		
Strategy	Who and What (highlight)	Order
Anticipate student solutions for the Skittles task. Identify and record the types of strategies students will use to solve the task.		

Anticipating → Monitoring → Selecting → Sequencing → Connecting

The **5** practices are:

1. **Anticipating** student responses to challenging mathematical tasks;
2. **Monitoring** students' work on and engagement with the tasks;
3. **Selecting** particular students to present their mathematical work;
4. **Sequencing** the student responses that will be displayed in a specific order and;
5. **Connecting** different students' responses and connecting the responses to key mathematical ideas.

## 5 Moves Planning

- Look at the student work samples
- Select the solutions you want to highlight in the closing
- Sequence how you'll share them
- Discuss the connection focus you're after

Sequence the following student work samples

58  $\times 14$   
 $\underline{58}$   
 $\times 580$   
 $\underline{2320}$

$\times 4$   
 $\underline{58}$   
 $4 + 200$   
 $\dots$   
 $\underline{232}$

①  $10 \times 58 = 580$  |  $232 = 812$

Part ①	Part ②	Part ③
$10 \times 58 = 580$	$4 \times 50 = 200$	$4 \times 8 = 32$

+  
812

S1

Packages = 58  
 SB per bag = 14

	50	8	
10	500	80	= 580
4	200	32	= 232

812

S2

58 packets  $\times 14$  SB per  
 Packets = 812 SB in the  
 Jar

$58 \times 10 = 580$  SB

$58 \times 11 = 638$  SB

$58 \times 12 = 696$  SB

$58 \times 13 = 754$  SB

$58 \times 14 = 812$  SB

S3

58 packs,  
 $\boxed{SB} = 14$

$50 \times 10 = 500$   
 $8 \times 4 = 32$   
 $\underline{532}$



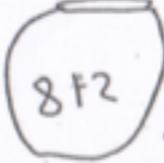
S4

58 bags  
 14 skittles

$\frac{58}{14}$

$50 \times 10 = 500$   
 $8 \times 4 = 32$   
 $\underline{532}$

812



S5

58 packs

$14 \times 58$

812

$8 \times 14 = 112$  |  $50 \times 14 = 700$

50 100 150 200 250 300  
 350 400 450 500 550 600 650  
 700

8 16 24 32 40 48 56  
 64 72 80 88 96 104 112

S6

58  $\times 14$

$58 \times 10 = 580$   
 $58 \times 4 = 232$   
 $\underline{812}$

S7

# Our next step as the teacher is.....?

- Identify student misconceptions
- Identify common strategy
- Are you surprised that a strategy wasn't used?
- Discuss your next moves as a teacher

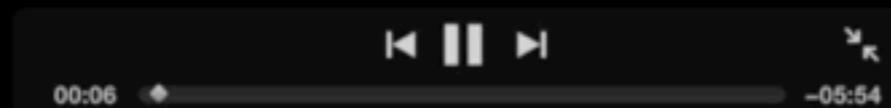
# Making Sense Series

the progression of multiplication  
and  
the standard traditional algorithm

created by Graham Fletcher



@gfletchy





**BIGPAD**

*Dream your big idea.*

**NOTAS GIGANTES**

*Sueña tu gran idea.*

**GRAND BLOC**

*Imaginéz de grandes ideas.*

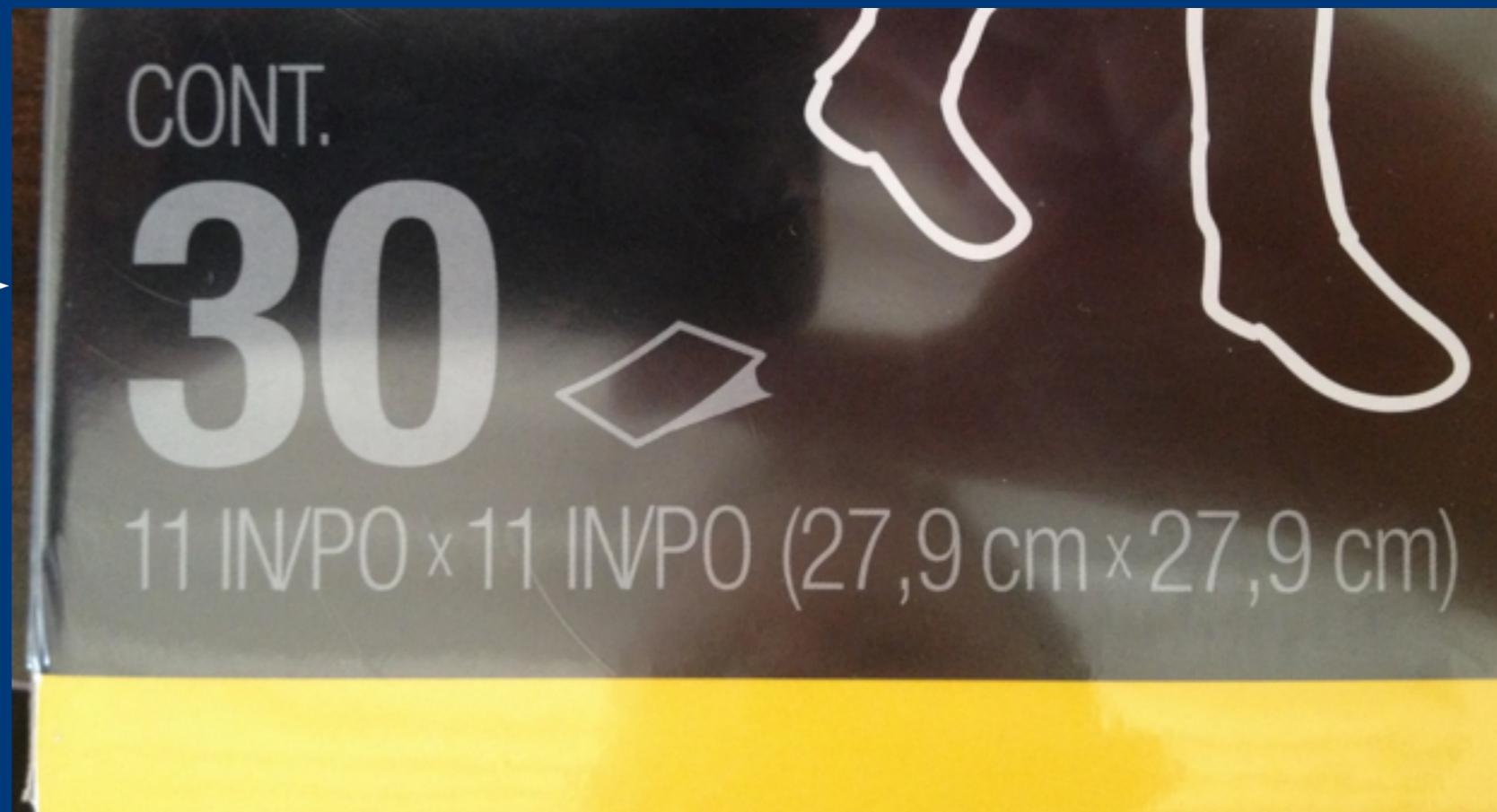


CONT.



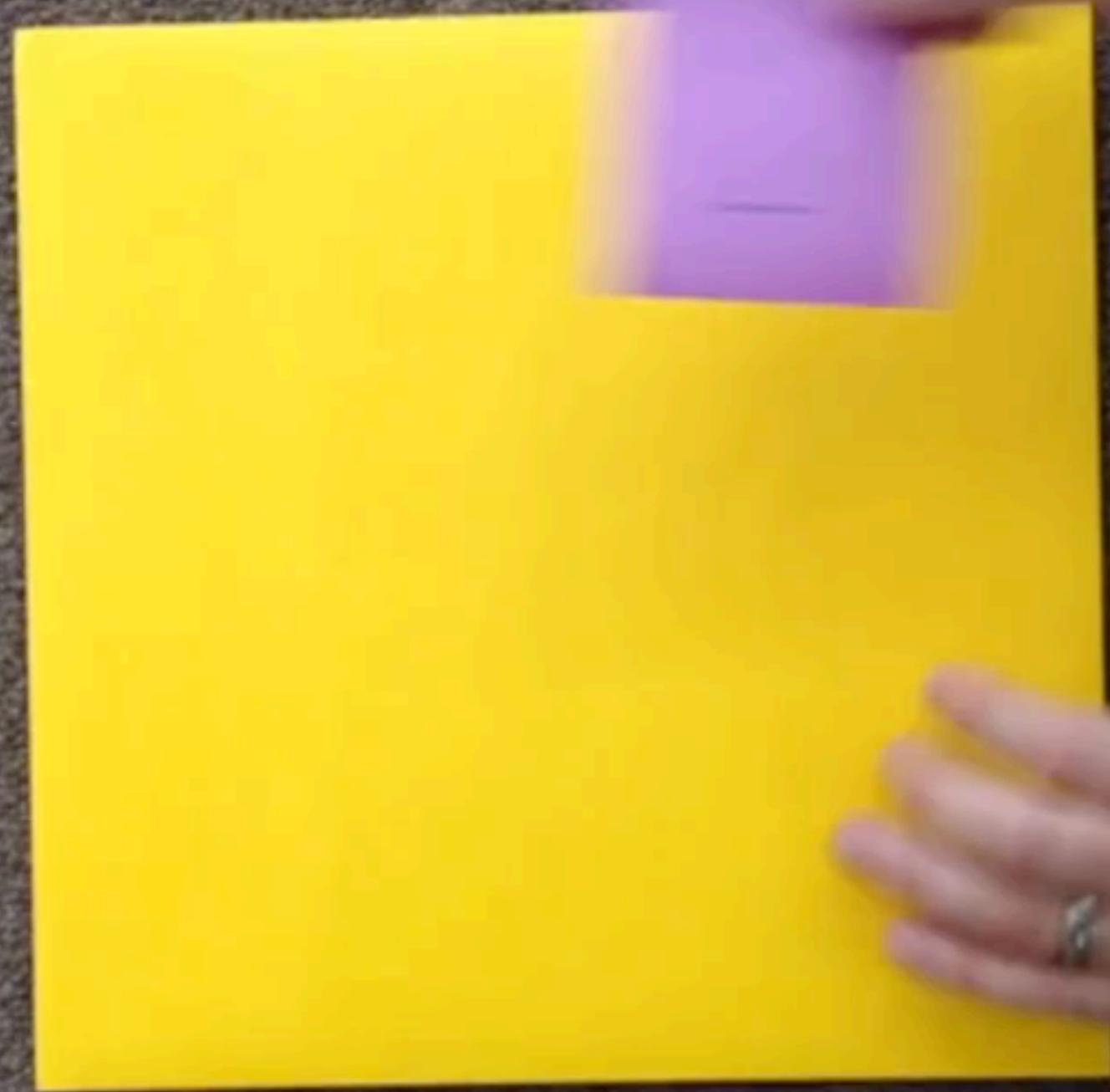


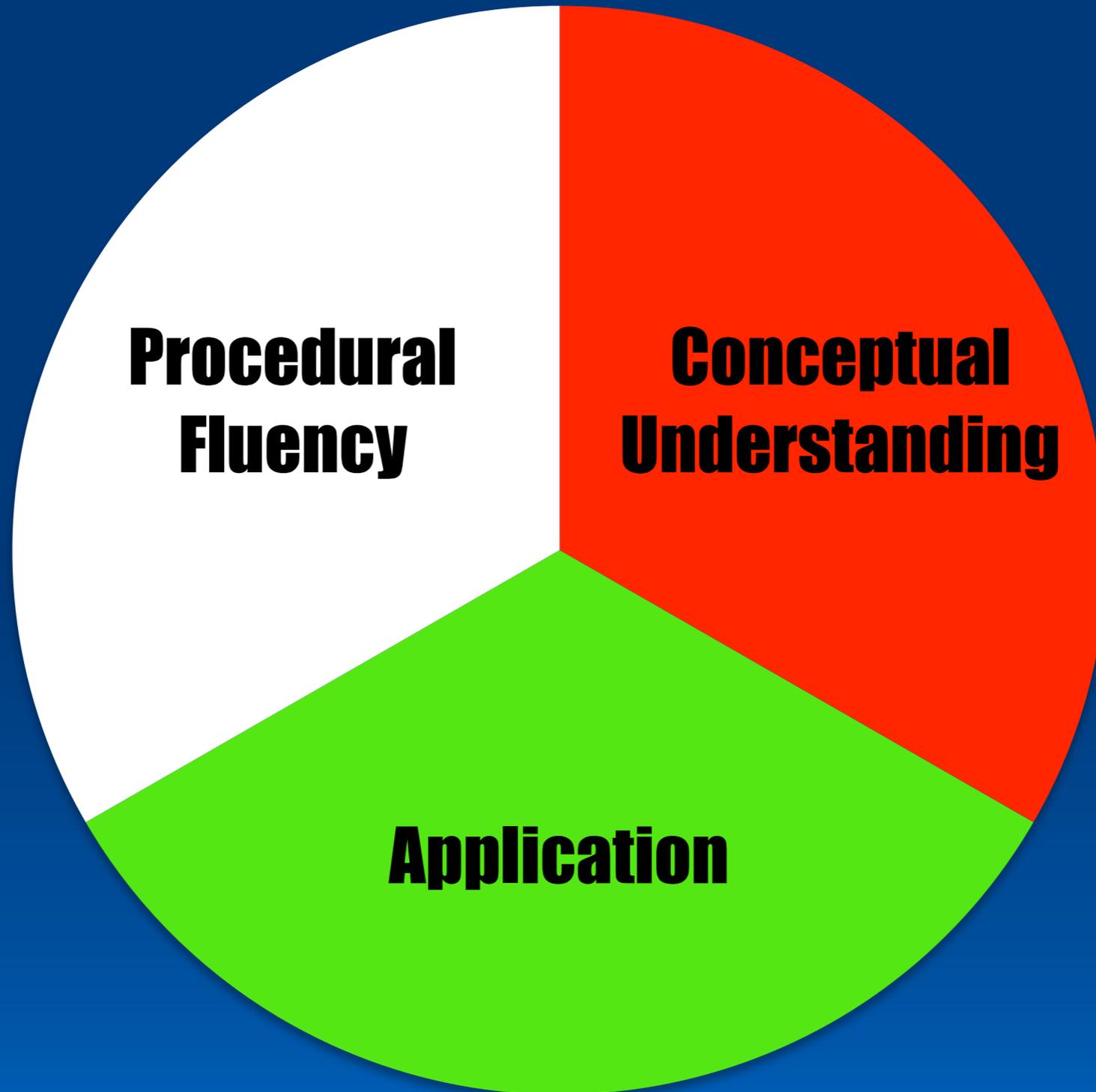
The Big Pad



The Post-it

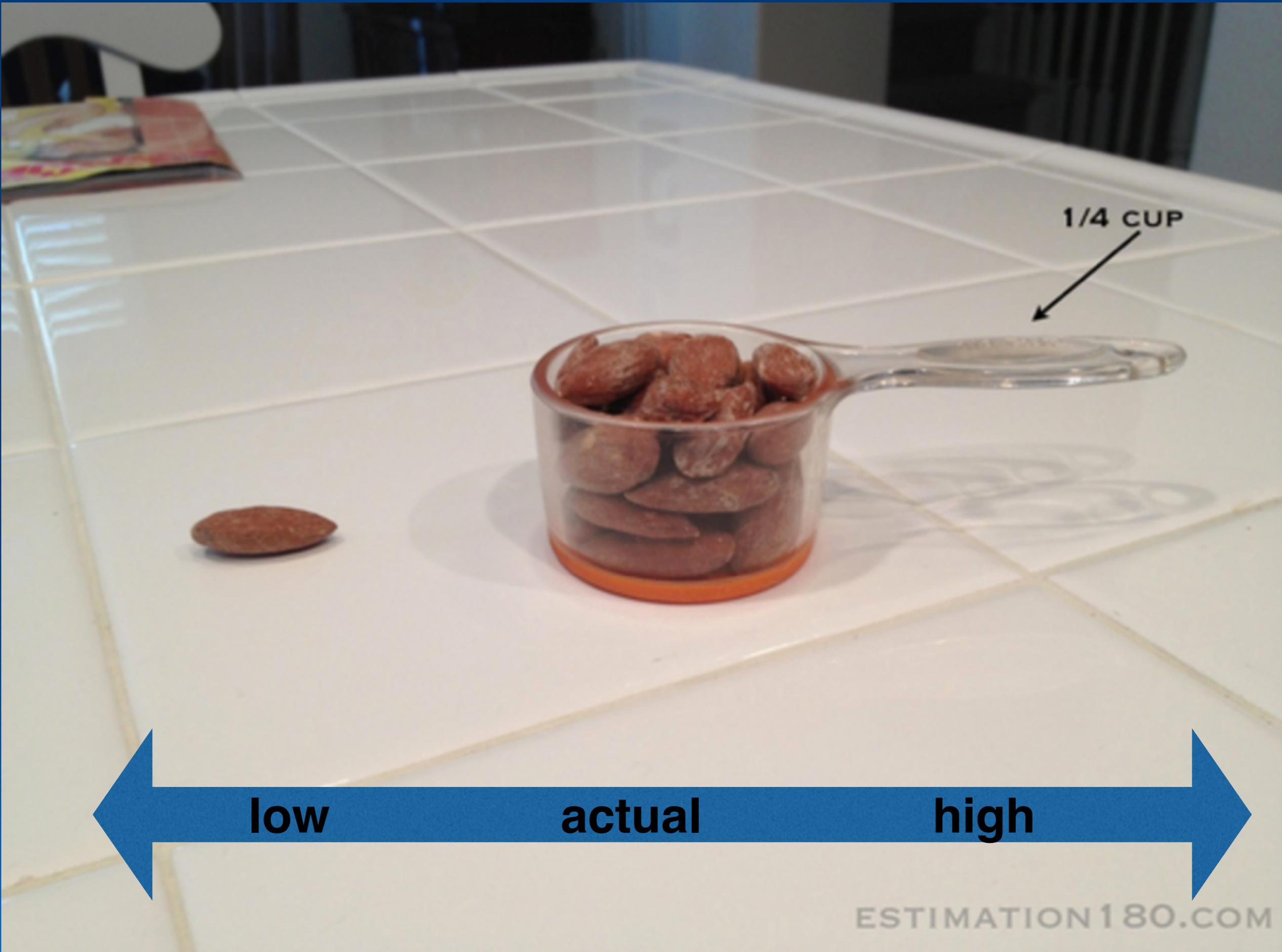




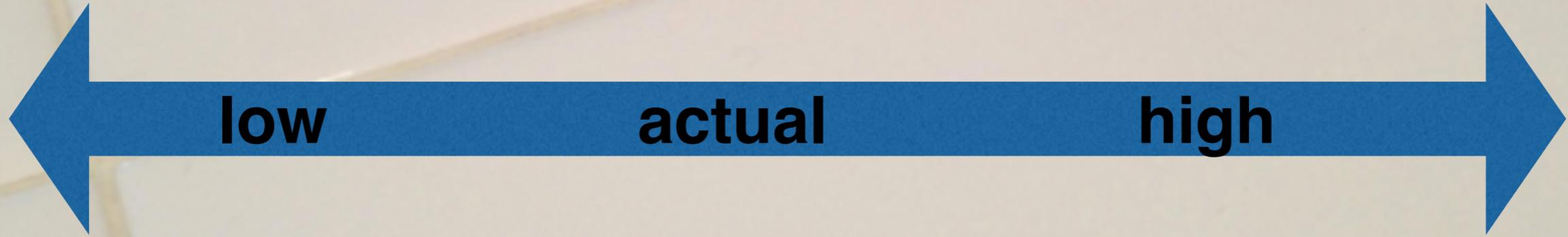
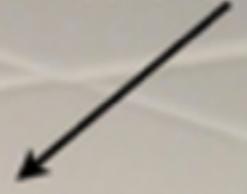


Becoming a good estimator takes practice

What are you currently doing to promote number sense through estimation in your classroom?



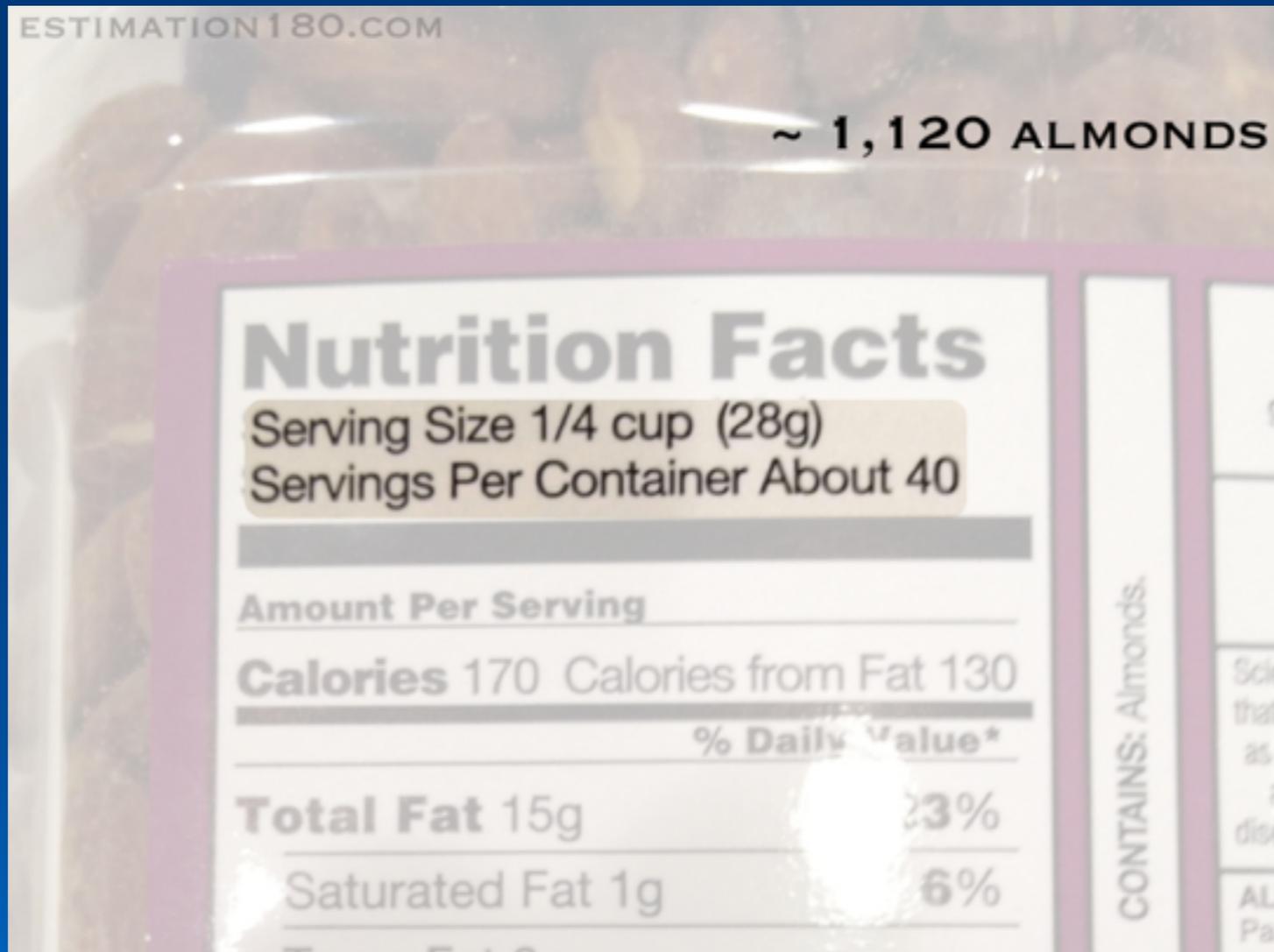
1/4 CUP







Building number sense one day at a time.



[www.esteemation.com](http://www.esteemation.com)

# Estimation Station



ANISHA  
21

Emma  
20

Kirsten  
14

Reginald  
1002

LAWSON  
16

Jimiyah  
20

20

Blake  
14

Thaddeus  
200

Evan  
20

Delia  
20

28

Jay  
20

Trianna  
1100

Arman tee  
100

RDBE  
100

malayah  
20

Hunter  
15

Sammy  
30

Abdul



estimation jar

Jan  
Larsen  
ADLE  
20  
Del in  
20  
Rejank  
1002  
50  
Kirsten  
14  
Blake  
19  
Thaddeus  
200  
Jimish  
20  
32

16 100 20 20 1002 50 14 19 200 20 32

Sammy

30

ARISM

21

Emma

20

E. Jain

20

91K

20

malayah

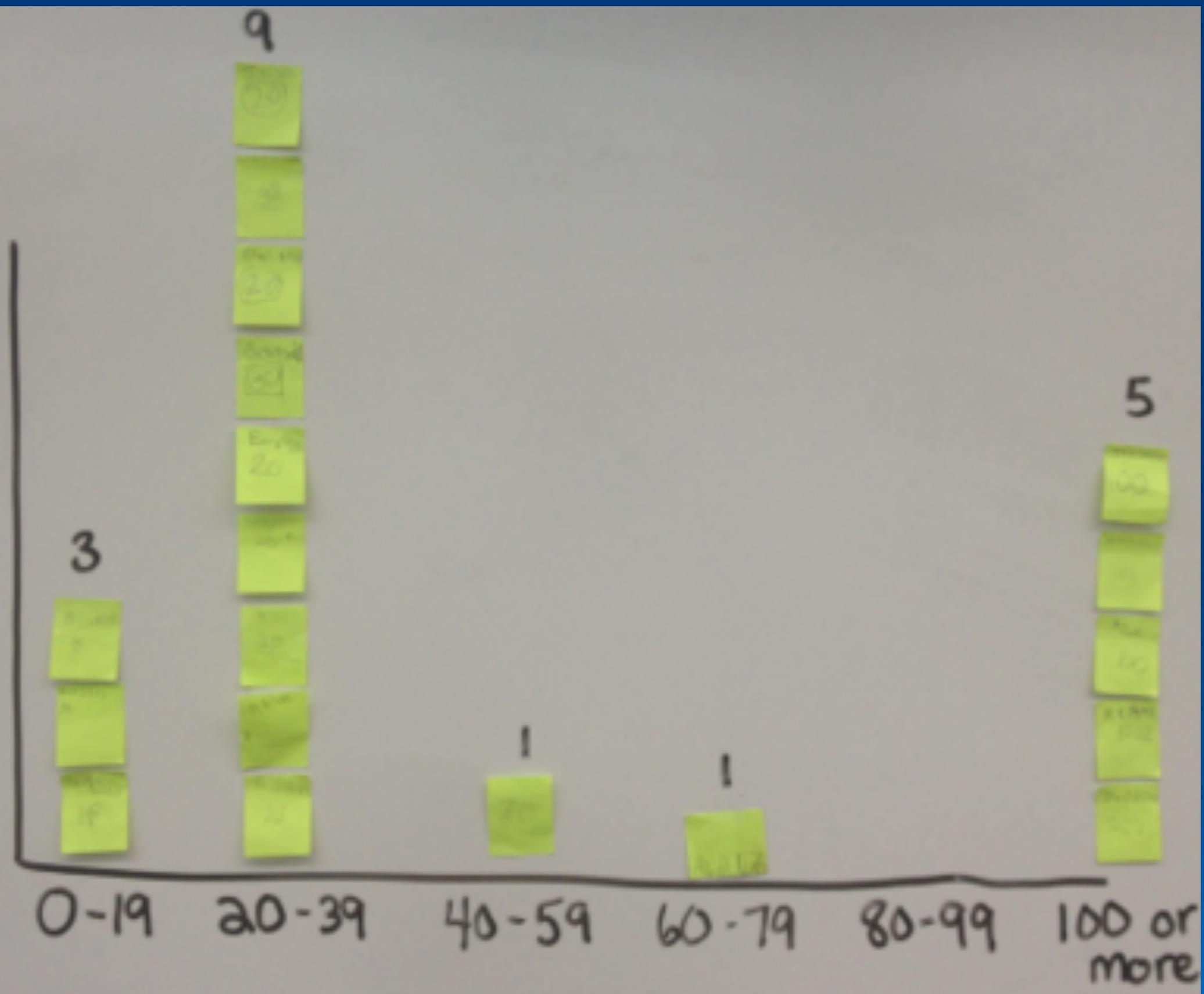
20

Delicon

20

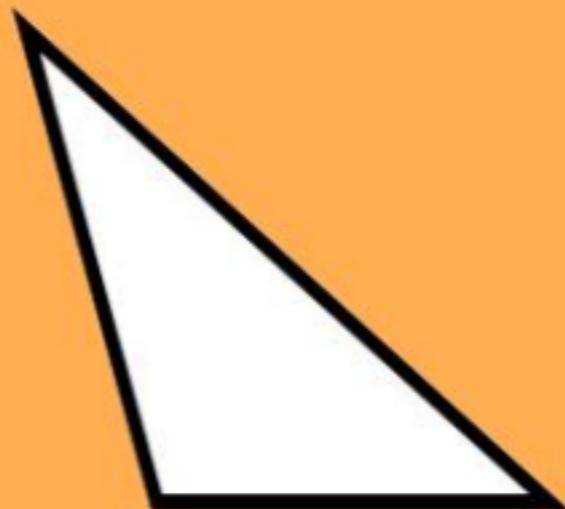
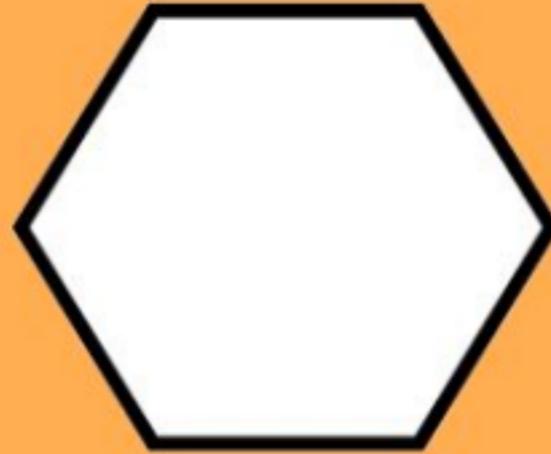
Jimitah

20





# The Spinning Dancer



9

16

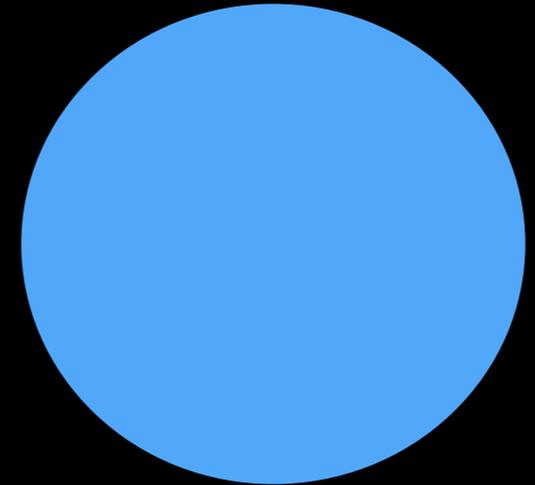
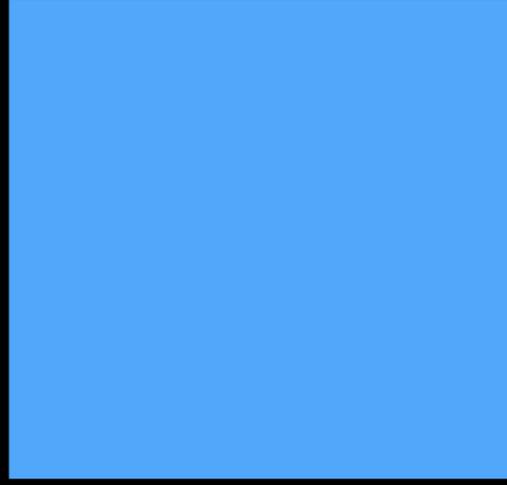
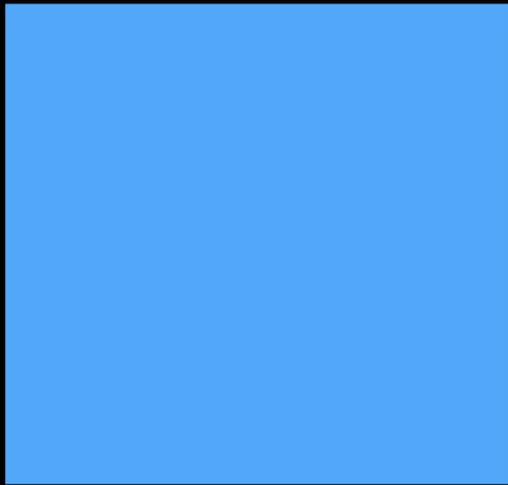
25

43



# Geometric Subitizing

**Ready...**



**How many shapes did you see?**

**What shapes did you see?**

**How many sides did you see?**

**4 sides**

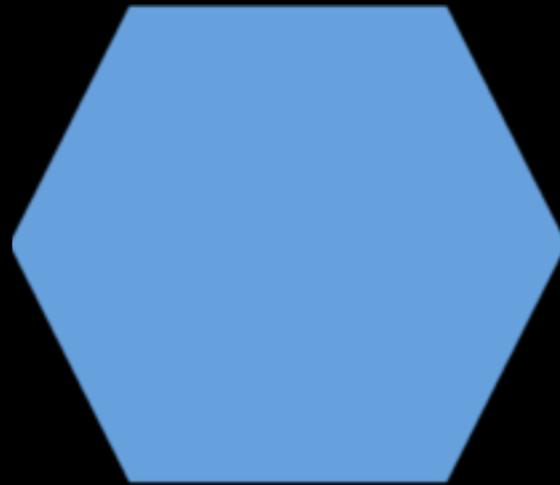
**4 sides**

**0 sides**

**How many shapes did you see? 3 shapes**

**How many sides did you see? 8 sides**

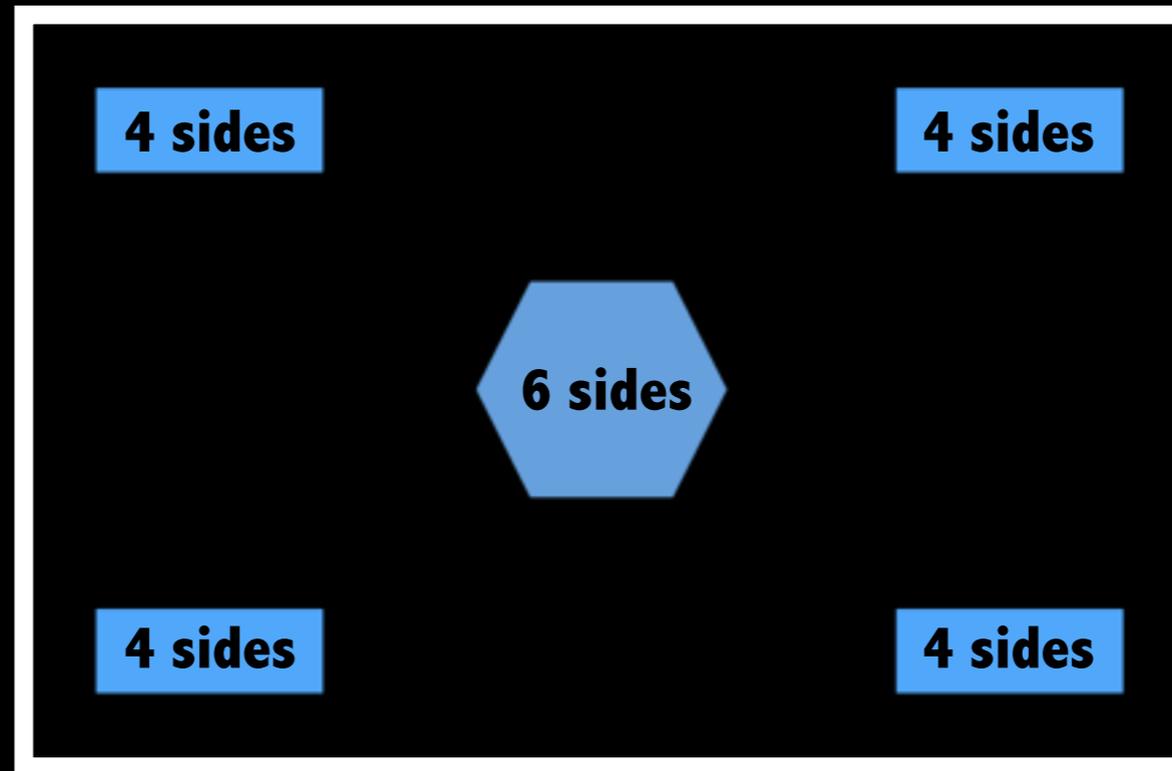
**Ready...**



**How many shapes did you see?**

**What shapes did you see?**

**How many sides did you see?**

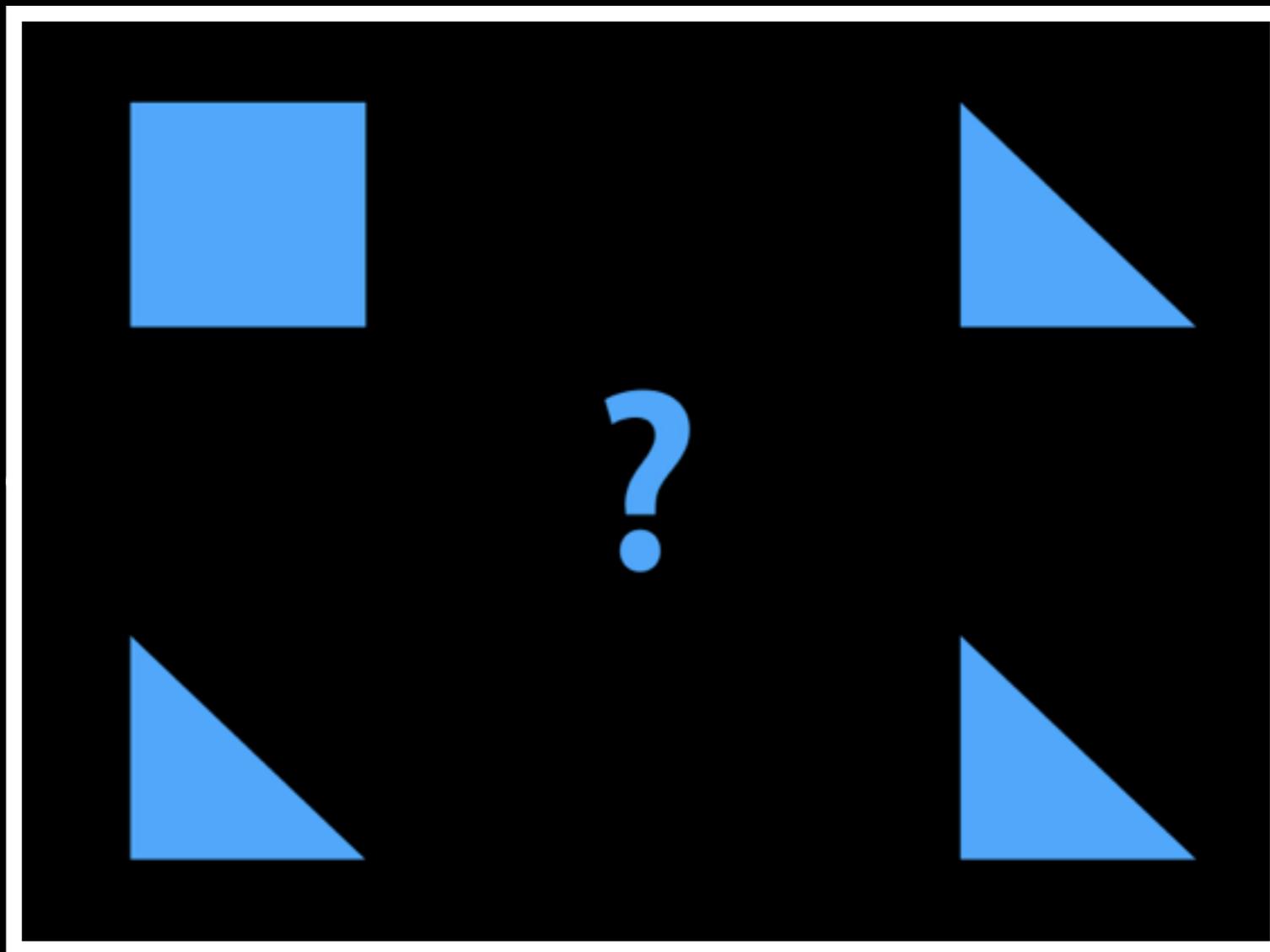


**How many shapes did you see? 5 shapes**

**How many sides did you see?**

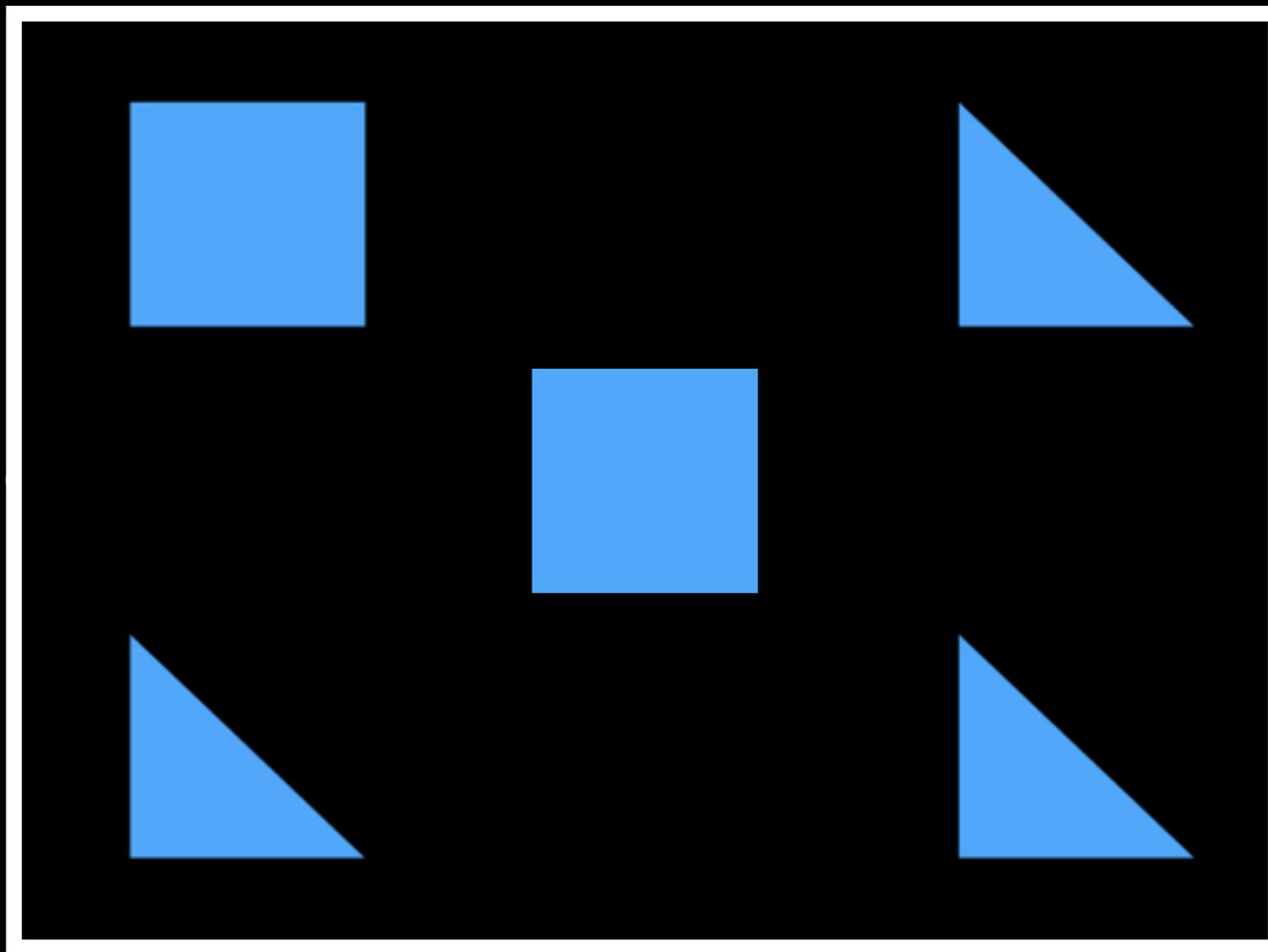
$$4 \times 4 + 6 = 22 \text{ sides}$$

**Ready...**



**There are 17 sides on this card.**

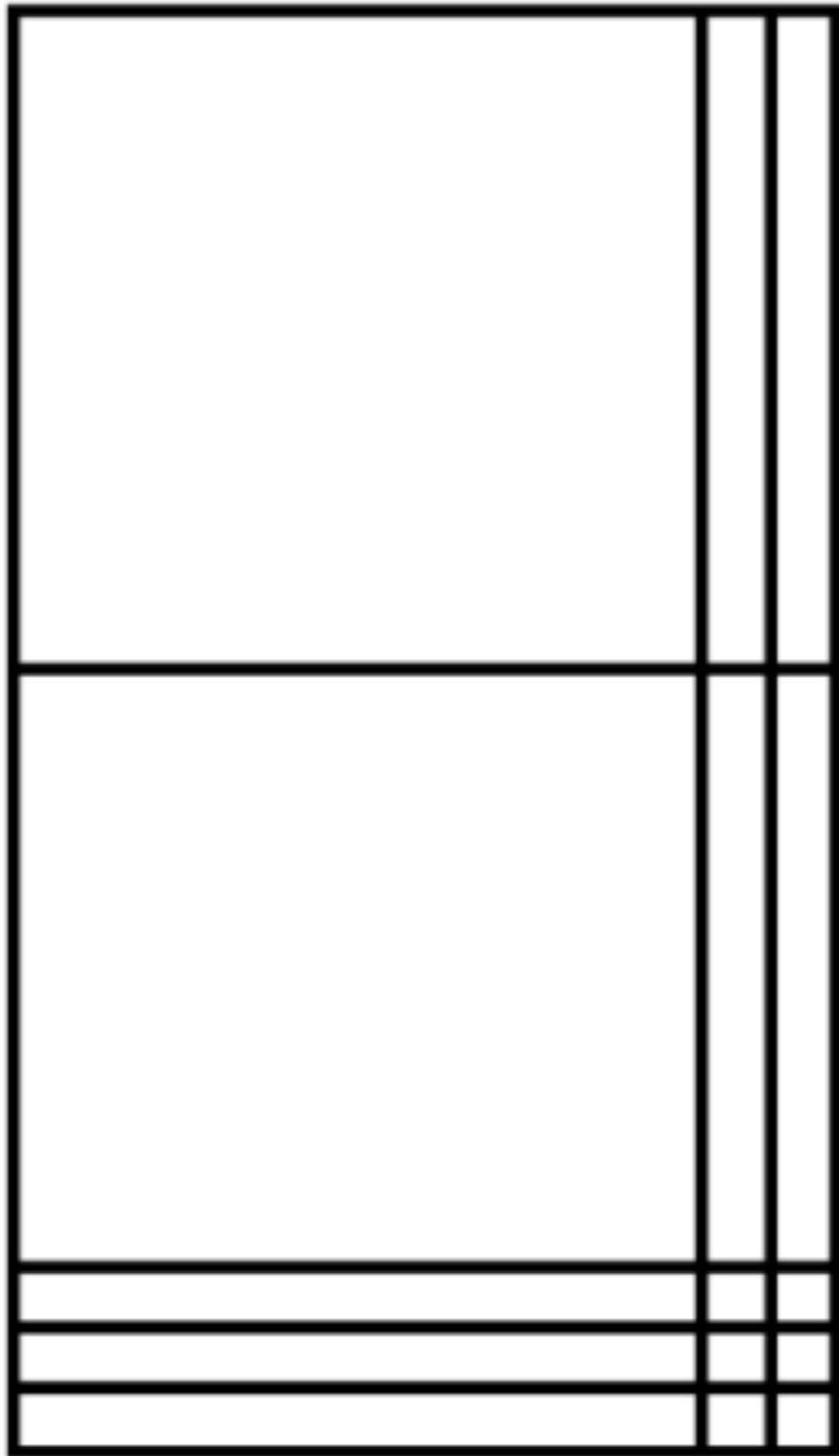
**What's the mystery shape?**



**There are 17 sides on this card.**

$$3(3) + 4 + ? = 17$$

# **Kindergarten Class #1**



Write an expression  
that matches the model



# Strip Model Diagramming for Division



