

Hanover School Divison

by: **Graham Fletcher**

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- Designate a shape keeper
- 6 congruent squares
- No shapes left over
- Everyone is encouraged to OFFER. No one may TAKE. They may ACCEPT a puzzle piece to use if OFFERED.



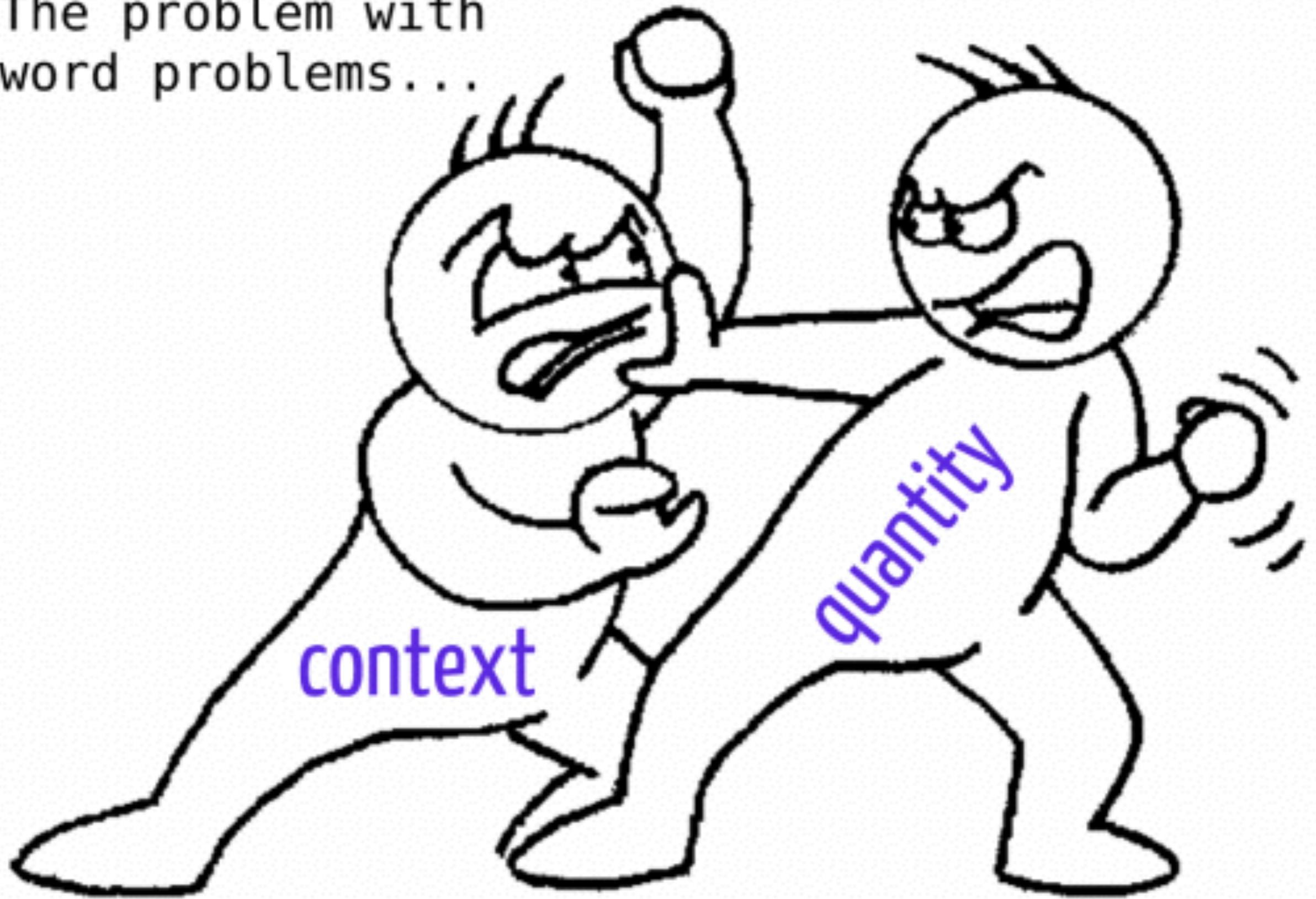


Joe had some playing cards in his bag. Ashley gave him 13 more cards. Joe now has 21 cards. How many cards did Joe have in his bag?

13

21

The problem with
word problems...



 @gfletchy

Joe had some playing cards in his bag. Ashley gave him 13 more cards. Joe now has 21 cards. How many cards did Joe have in his bag?

Joe had some playing cards
in his bag. Ashley gave him
more cards. Joe now has
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3-ACT Tasks

DATE

PROD.CO.

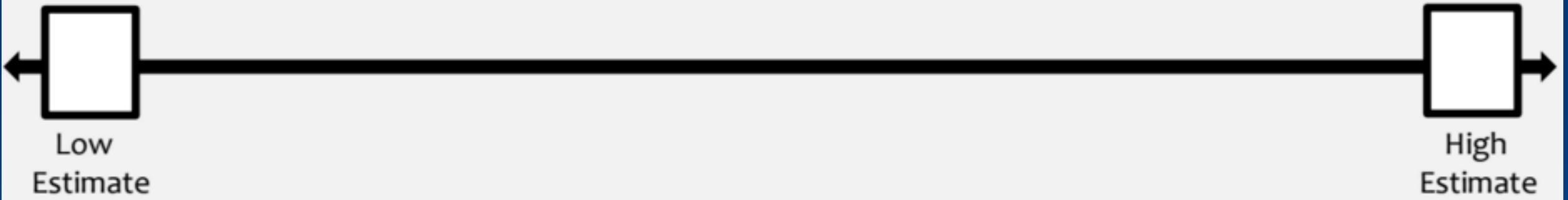
DIRECTOR



What do you notice?

What do you wonder?

4. Estimate





Questions

Packages Used



58 packages



THIS UNIT NOT LABELED FOR RETAIL SALE

ORIGINAL
Skittles

Questions? Comments?
Call 1-800-WRIGLEY-11-11
©2012 Wm. Wrigley

3 minutes

individual work time

The answer is not 812!



Mathematical Modeling



Modeling with Mathematics

What ISN'T mathematical modeling

- The use of manipulatives does not ensure that modeling with mathematics is taking place.
- If the mathematics is not contextualized, modeling with mathematics cannot exist.
- Modeling with mathematics does not mean, “I do, we do, you do.”

Model with Mathematics

Mathematically proficient students can apply the mathematics they know to **solve problems arising in everyday life**, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. Mathematically proficient students who can apply what they know are comfortable **making assumptions and approximations** to simplify a complicated situation, realizing that these may need revision later. They are able to **identify important quantities** in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can **analyze those relationships** mathematically to draw conclusions. They routinely **interpret their mathematical results** in the context of the situation and **reflect on whether the results make sense**, possibly improving the model if it has not served its purpose.

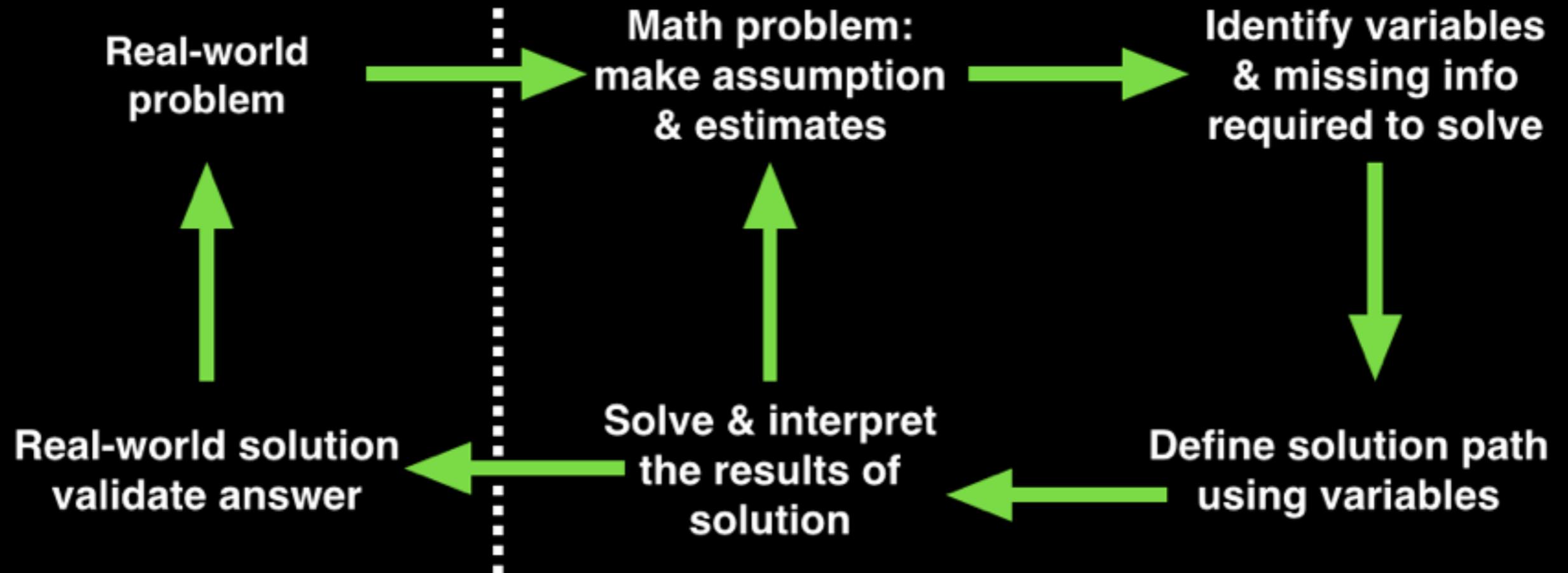
Model with Mathematics

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Mathematical Modeling

Contextualized

Decontextualized



3 Things on the road to modeling...

- Identify the problem, or pose a question.
- Make an estimate.
- Identify the variables needed to solve, and answer the problem or question posed.

Mathematical Processes



COMMUNICATION (C)

Reading about, representing, viewing, writing about, listening to, and discussing mathematical ideas allows students to create links among their own language and ideas, the language and ideas of others, and the formal language and symbols of mathematics. Communication enables students to reflect upon, to validate, and to clarify their thinking. Expression of mathematical meaning and ideas can be accomplished orally or in written representations such as journals and learning logs.



CONNECTIONS (CN)

Mathematics becomes more meaningful when it is contextualized and linked to students' experiences across disciplines. Furthermore, mathematics should be viewed as an integrated whole, rather than as the study of separate strands or units. Within a particular topic, students should see the connections between concrete, pictorial, and symbolic modes of representation. When mathematical ideas are connected to each other or to real-world phenomena, students begin to view mathematics as useful, relevant, and integrated.



MENTAL MATH AND ESTIMATION (ME)

Mental mathematics is a combination of strategies that enhances flexible thinking and number sense. Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Estimation is often used to make mathematical judgments and to develop useful, efficient strategies for dealing with situations in daily life. Strategies in mental mathematics and estimation enable students to calculate mentally without the use of external aids. In the process, they improve their computational fluency—developing efficiency, accuracy, and flexibility.



PROBLEM SOLVING (PS)

Students develop understanding of mathematical concepts and procedures when they apply their mathematical knowledge to solve problems in new ways and meaningful contexts. When students encounter new situations and respond to questions of the type *How would you...?* or *How could you...?*, the problem-solving approach is being modelled. Problems are often open-ended, so students may arrive at multiple solutions in different and creative ways. Rich problems allow students in the class to demonstrate their knowledge, skill, or understanding at a level appropriate to them. Learning through problem solving should be the focus of mathematics at all grade levels and should be embedded throughout all topics.



REASONING (R)

Mathematical reasoning involves generalizing from patterns, conjecturing, validating, and proving. Students need to develop confidence in their abilities to reason and to justify their mathematical thinking. Good reasoning is as important as finding correct answers. The thinking skills developed by a focus on reasoning can be used in life in a wide variety of contexts and disciplines.



TECHNOLOGY (T)

Technology contributes to and supports the learning of a wide range of mathematical concepts and can increase the focus on conceptual understanding by decreasing the time spent on repetitive procedures. It enables students to explore and create patterns, organize and display data, examine relationships, model situations, generate and test conjectures, solve problems, and reinforce the learning of basic facts. Technology can help to satisfy the curiosity of students and lead to rich mathematical discoveries at all grade levels. The use of technology can enhance, although it should not replace, conceptual understanding, procedural thinking, and problem solving.



VISUALIZATION (V)

Visual images and visual reasoning are important to a sense of number, space, and measurement. Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations. Visualization can help students gain a concrete understanding of abstract concepts.

5 Practices for Orchestrating Productive Mathematical Discussions



Orchestrating Discussions

Five practices constitute a model for effectively using student responses in whole-class discussions that can potentially make teaching with high-level tasks more manageable for teachers.

Margaret S. Smith, Elizabeth K. Hughes, Randi A. Engle, and Mary Kay Stein



Margaret S. Smith, mgs@pitt.edu, is an associate professor of mathematics education at the University of Pittsburgh. Over the past decade, she has been developing research-based materials for use in the professional development of mathematics teachers and studying what teachers learn from the professional development in which they engage. **Elizabeth K. Hughes**, elizabeth.hughes@uni.edu, recently finished her doctorate in mathematics education at the University of Pittsburgh. Her areas of interest include preservice secondary mathematics teacher education and the use of practice-based materials in developing teachers' understanding of what it means to teach and learn mathematics. **Randi A. Engle**, raengle@berkeley.edu, is an assistant professor of mathematics education and the social context of learning at the University of California Berkeley. She is interested in developing practical theories for how mathematics teachers can create discussion-based learning environments that promote strong student engagement, learning, and transfer. **Mary Kay Stein**, mks@pitt.edu, is a professor of learning sciences and policy and the director of the Learning Policy Center at the University of Pittsburgh. Her research focuses on instructional practice and the organizational and policy conditions that shape it.

Discussions that focus on cognitively challenging mathematical tasks, namely, those that promote thinking, reasoning, and problem solving, are a primary mechanism for promoting conceptual understanding of mathematics (Hatano and Inagaki 1991; Michaels, O'Connor, and Resnick forthcoming). Such discussions give students opportunities to share ideas and clarify understandings, develop convincing arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things from other perspectives (NCTM 2000). Although discussions about high-level tasks provide important

548 MATHEMATICS TEACHING IN THE MIDDLE SCHOOL • Vol. 14, No. 5, May 2009

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The **5** practices are:

1. **Anticipating** student responses to challenging mathematical tasks;

Task Planning Document

Task:		
Misconceptions:		
Strategy	Who and What (highlight)	Order

Anticipating → Monitoring → Selecting → Sequencing → Connecting

The **5** practices are:

1. **Anticipating** student responses to challenging mathematical tasks;
2. **Monitoring** students' work on and engagement with the tasks;
3. **Selecting** particular students to present their mathematical work;
4. **Sequencing** the student responses that will be displayed in a specific order and;
5. **Connecting** different students' responses and connecting the responses to key mathematical ideas.

5 Moves Planning

- Look at the student work samples
- Select the solutions you want to highlight in the closing
- Sequence how you'll share them
- Discuss the connection focus you're after

Sequence the following student work samples

Show your work

$$\begin{array}{r} 58 \\ \times 14 \\ \hline 232 \\ \times 580 \\ \hline 2320 \end{array}$$

$$\begin{array}{r} 58 \\ \times 4 \\ \hline 232 \end{array}$$

$$\begin{array}{r} 58 \\ \times 4 \\ \hline 232 \end{array}$$

① $10 \times 58 = 580$ $4 \times 58 = 232 = 812$

Part ①	Part ②	Part ③
$10 \times 58 = 580$	$4 \times 58 = 232$	$4 \times 8 = 32$

+

812

Show your work

Packages 58
58 per bag = 14

	50	8	
10	500	80	= 580
4	200	32	= 232

812

58 14
812

Show your work

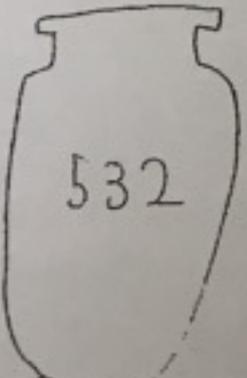
58 packets \times 14 85 per
Packets = 812 85 in the
Jar ^{Answer}

$58 \times 10 = 580.85$

$58 \times 11 = 638.85$
 $58 \times 12 = 696.85$
 $58 \times 13 = 754.85$
 $58 \times 14 = 812.85$

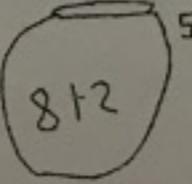
Show your work

58 packs,
58 = 14

$$\begin{array}{r} 58 \\ \times 10 \\ \hline 580 \\ \times 4 \\ \hline 232 \\ \hline 812 \end{array}$$


Show your work

58 bags
14 skittles

$$\begin{array}{r} 58 \\ \times 14 \\ \hline 812 \end{array}$$


Show your work

58 packs

$14 \times 58 = 812$

$8 \times 14 = 112$ $50 \times 14 = 700$

50 100 150 200 250 300
350 400 450 500 550 600 650
700

8 16 24 32 40 48 56
64 72 80 88 96 104 112

Show your work

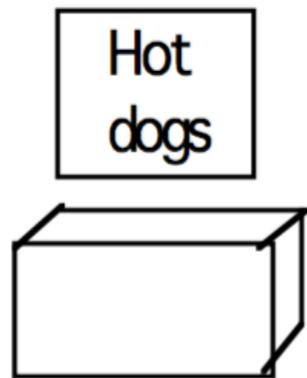
$$\begin{array}{r} 58 \\ \times 14 \\ \hline 232 \\ \times 580 \\ \hline 812 \end{array}$$

Five Representations of Functions

Language

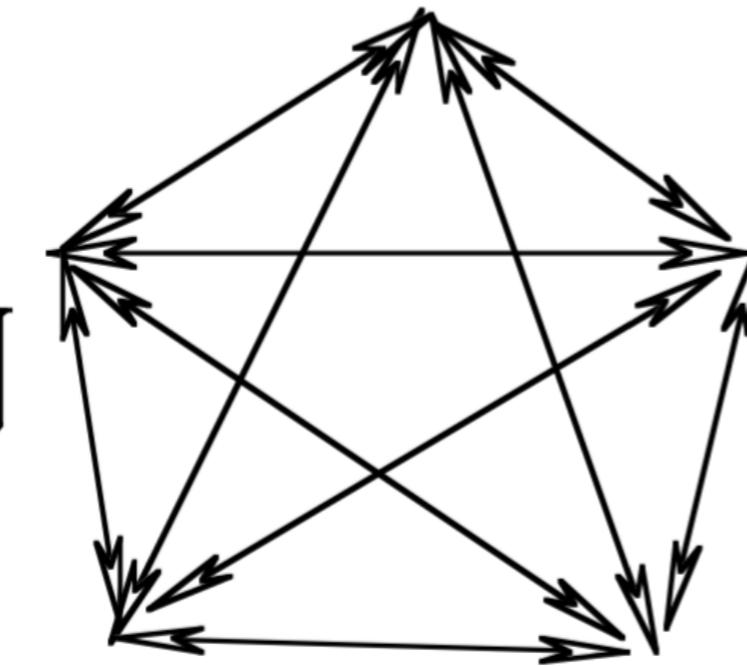
The amount of profit that can be made selling hot dogs is a function of the number of hot dogs that are sold.

Context

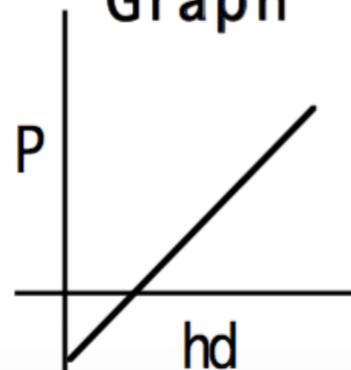


Table

hd	P
0	-35
100	30
200	



Graph

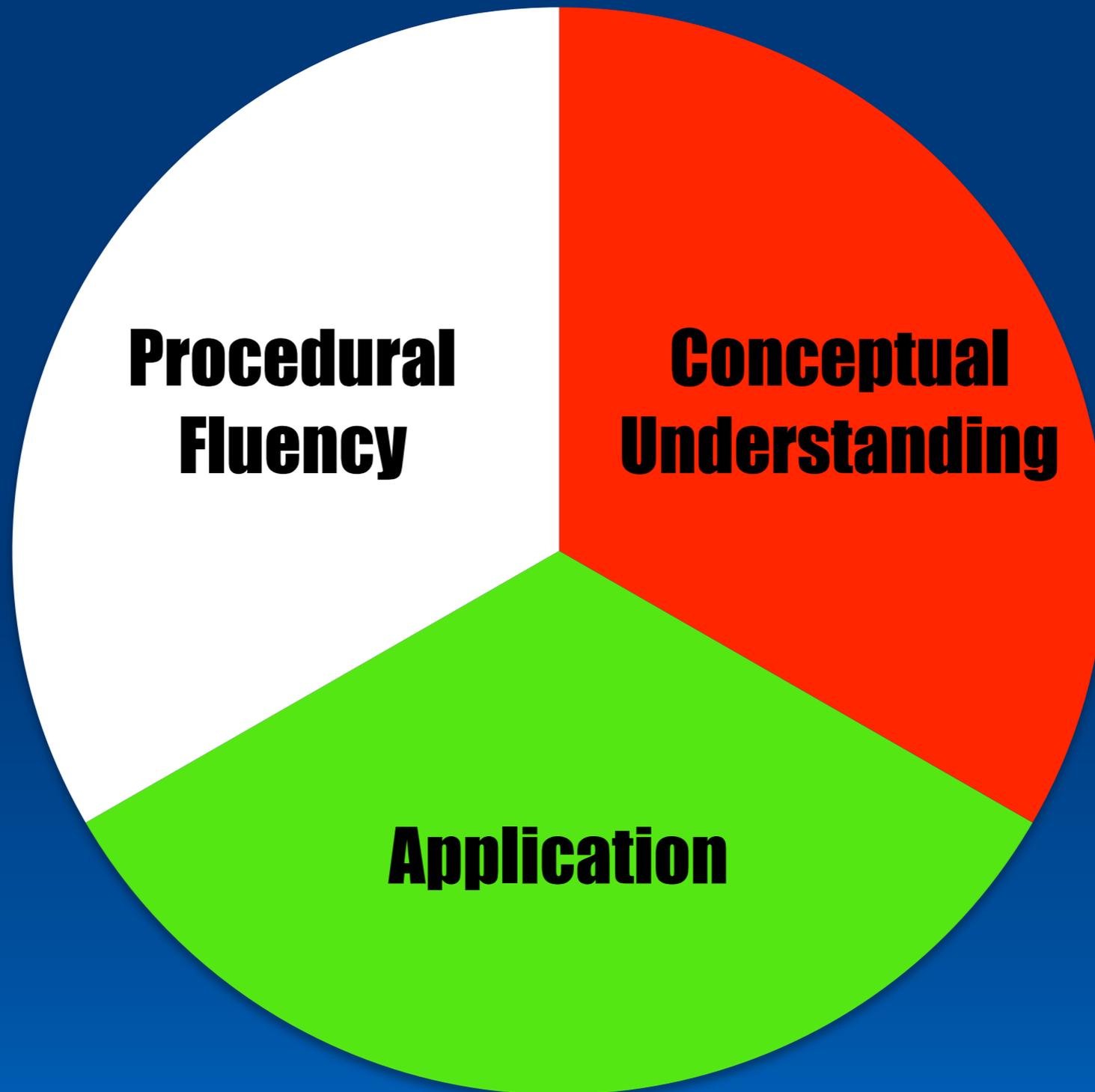


Equation

$$P = .65H - 35$$

Most asked questions:

- How often should we use 3-Act Tasks?
- When should we use 3-Act tasks? How do they fit into the scope of a unit?
- How long does one task usually take?
- What if we don't have the time?
- Any others?



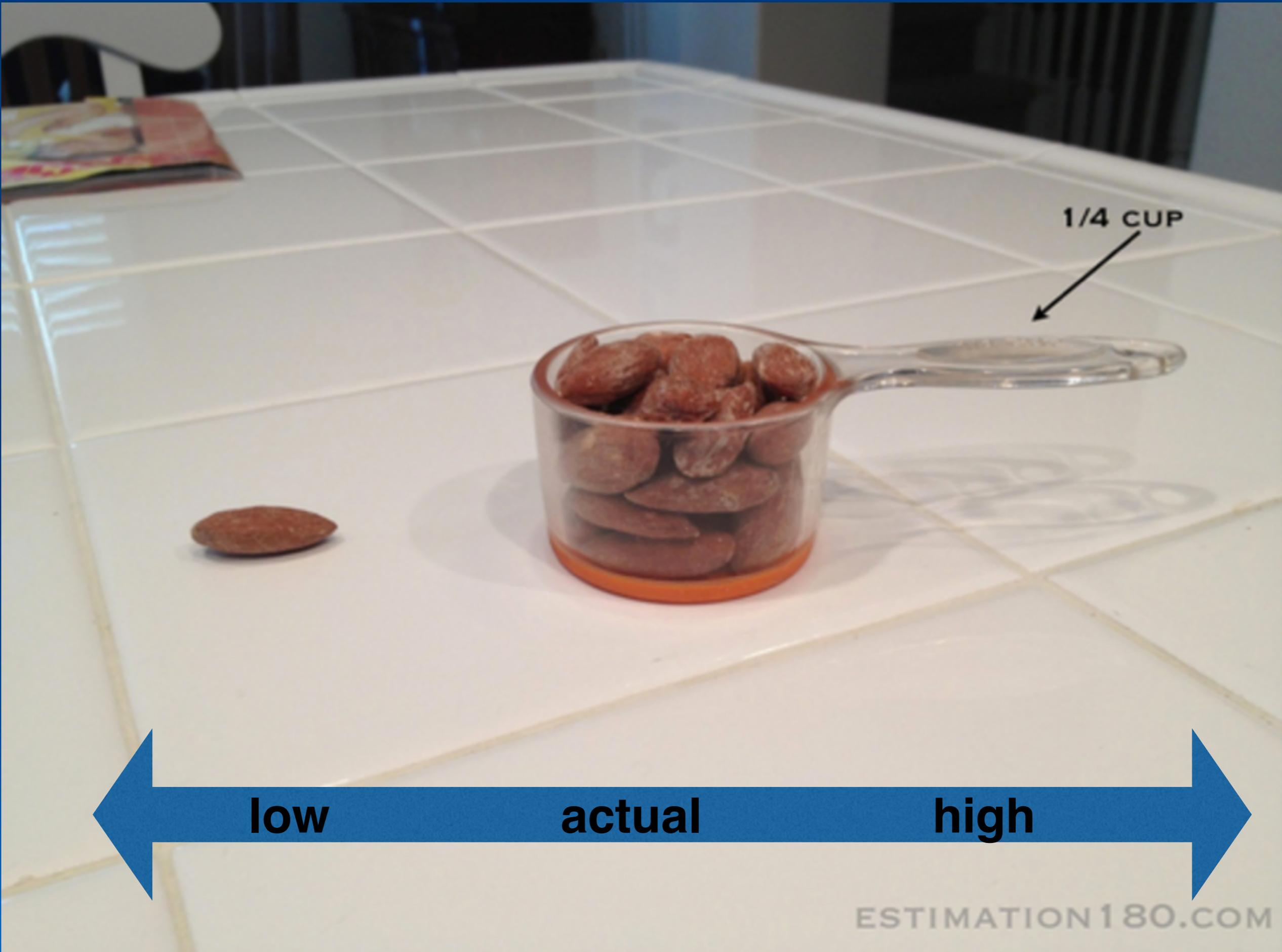
**Procedural
Fluency**

**Conceptual
Understanding**

Application

$$\begin{array}{r} 50 \square 6 \\ - \square 48 \square \\ \hline 16 \square 8 \end{array}$$





1/4 CUP

low

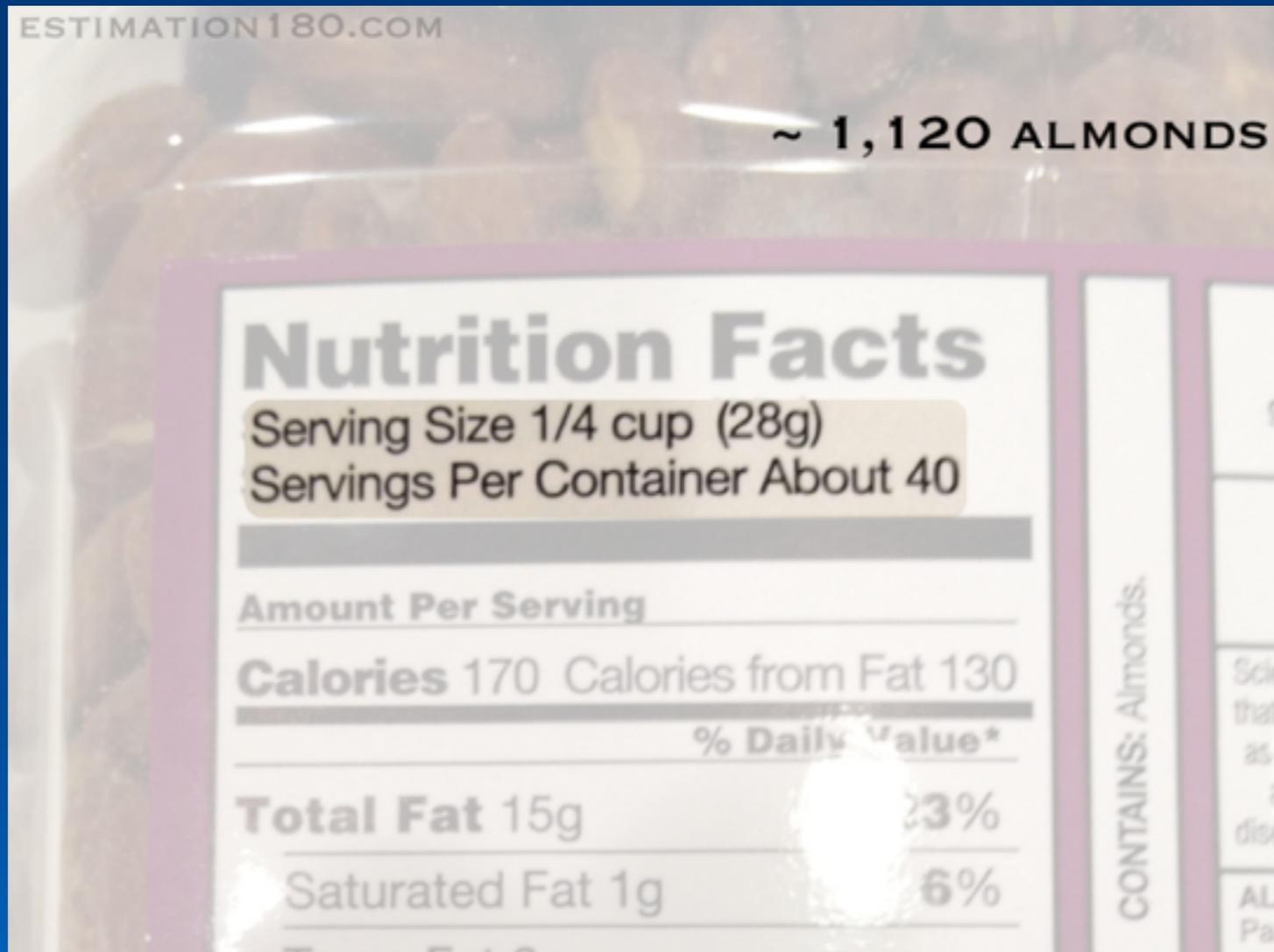
actual

high





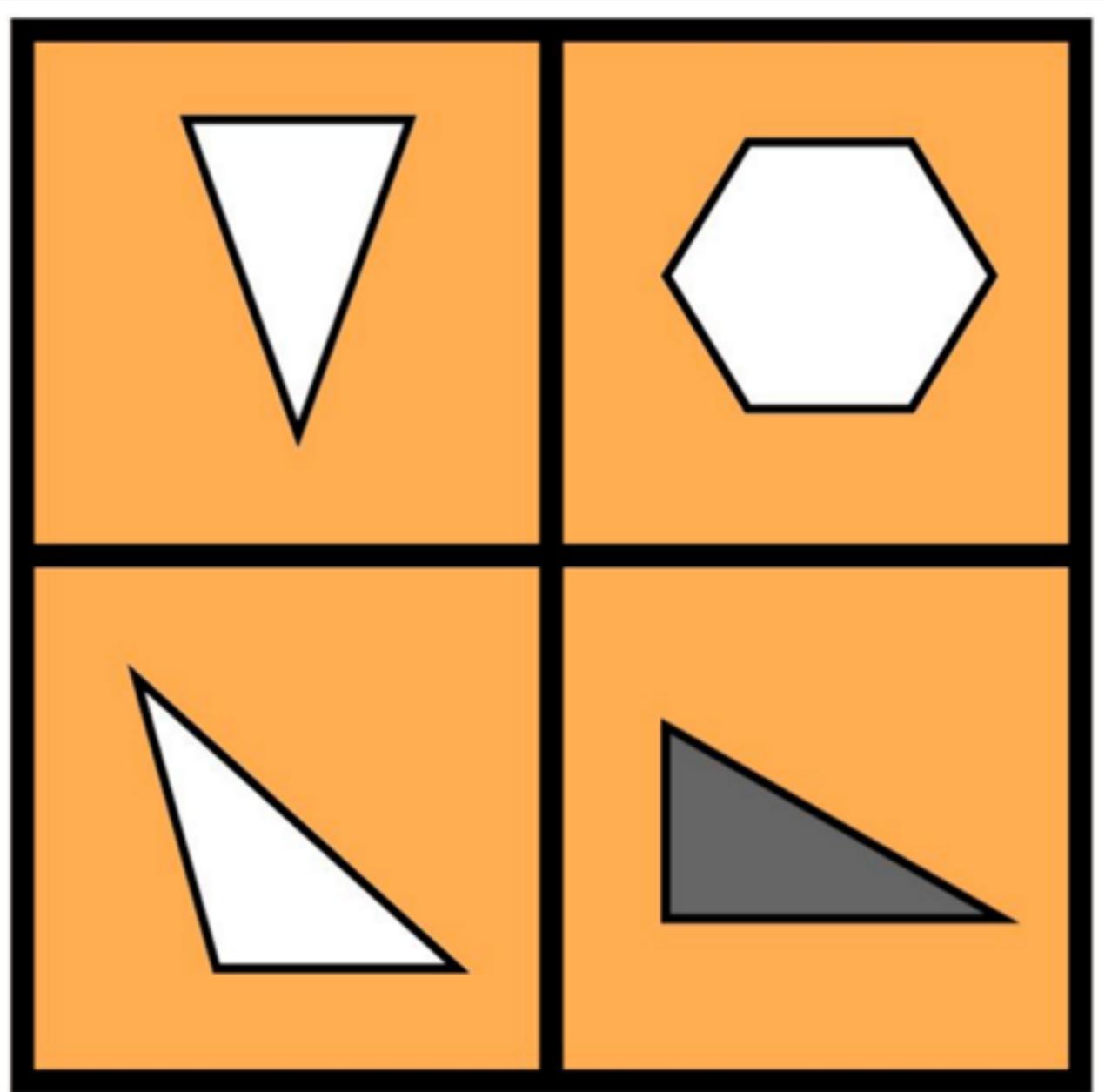
Building number sense one day at a time.



www.esteemation.com

Becoming a good estimator takes practice

What are you currently doing to promote number sense through estimation in your classroom?



SHAPE 1

from Mary Bourassa

9	16
25	43

NUMBER 1

from Pam Wilson

Open Middle

- Use the digits 0–9 and create four 2-digit numbers.
- Each digit can only be used once.
- Place the numbers in order from smallest to greatest.
- Create the smallest possible difference between the first and last number (range).

, , ,

Open Middle

Challenging math problems worth solving

SUMS TO 100

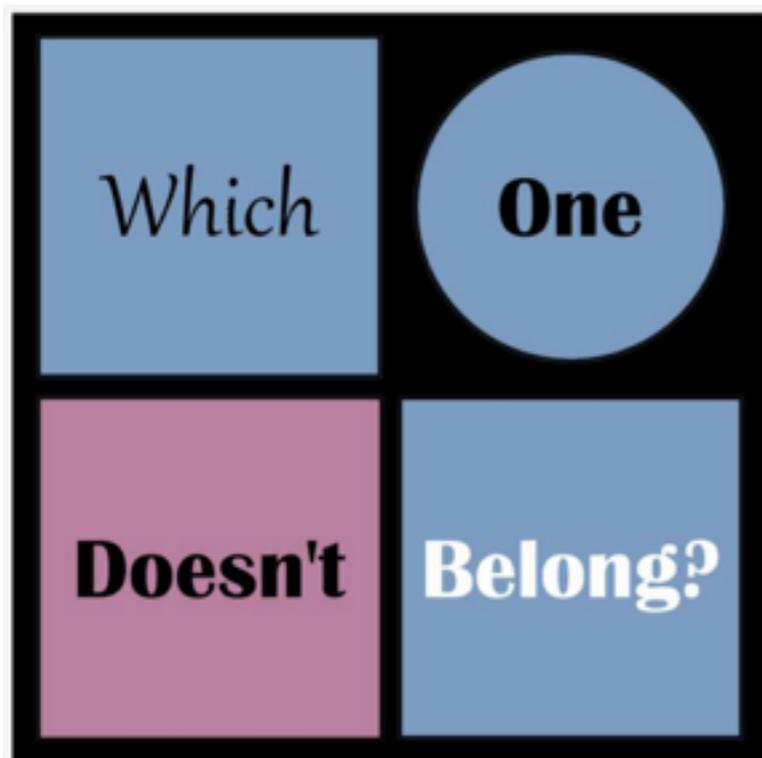
Directions: Create the closest possible sum to 100 by filling in the boxes using the whole numbers 1-9 no more than one time each.

<input type="text"/>	<input type="text"/>	+	<input type="text"/>	<input type="text"/>	+	<input type="text"/>	<input type="text"/>
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Tools that strengthen your modeling skills

Open Middle

Challenging math problems worth solving



FRACTION TALKS



The Math Forum @ Drexel
PEOPLE LEARNING MATH TOGETHER

Notice and Wonder



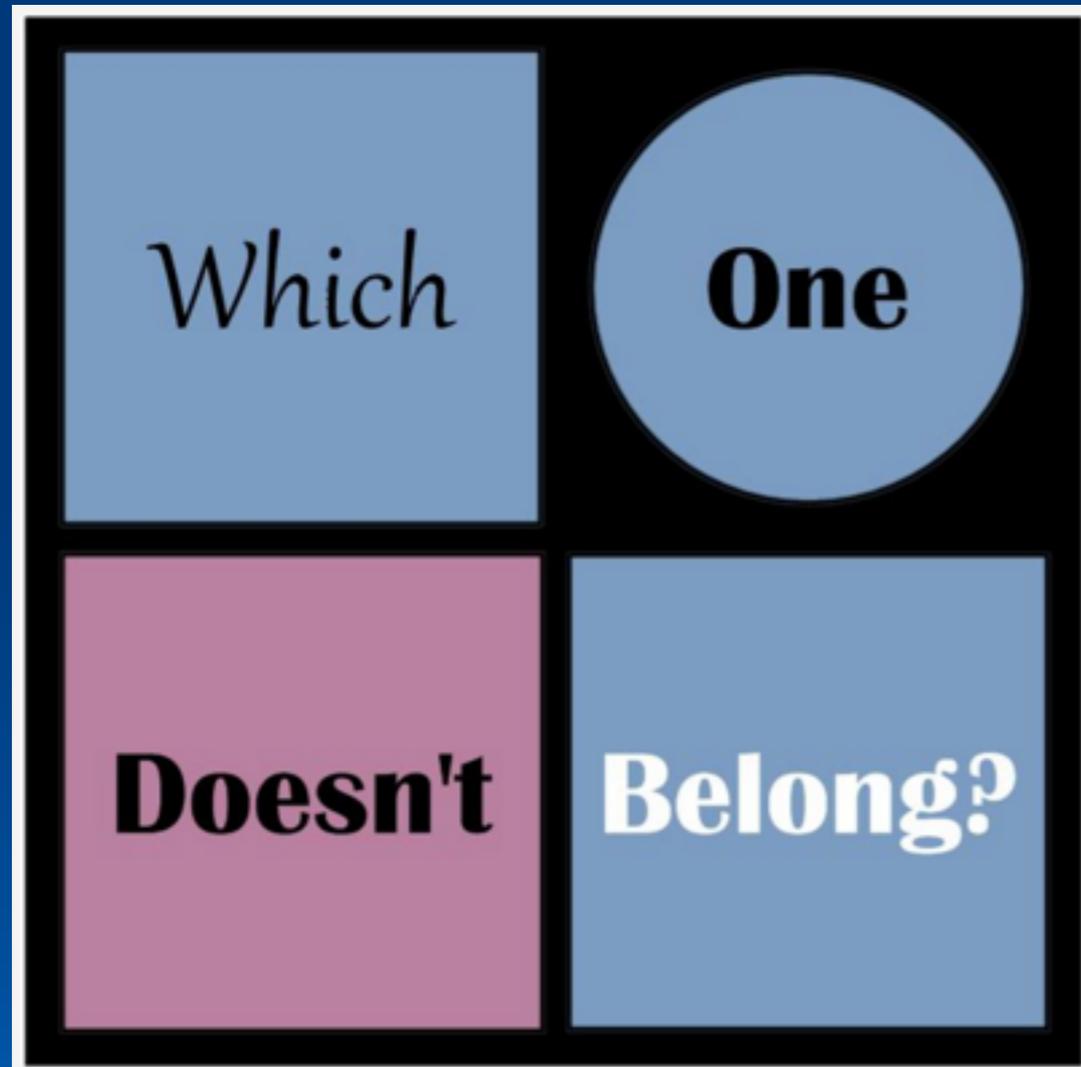
Would You Rather?



ASKING STUDENTS TO CHOOSE THEIR OWN PATH AND JUSTIFY IT

Building number sense one day at a time.

Create Your Own



Open Middle

<https://www.random.org/integers/>

the Big Pad



BIGPAD

Dream your big idea.

NOTAS GIGANTES

Sueña tu gran idea.

GRAND BLOC

Imaginéz de grandes ideas.



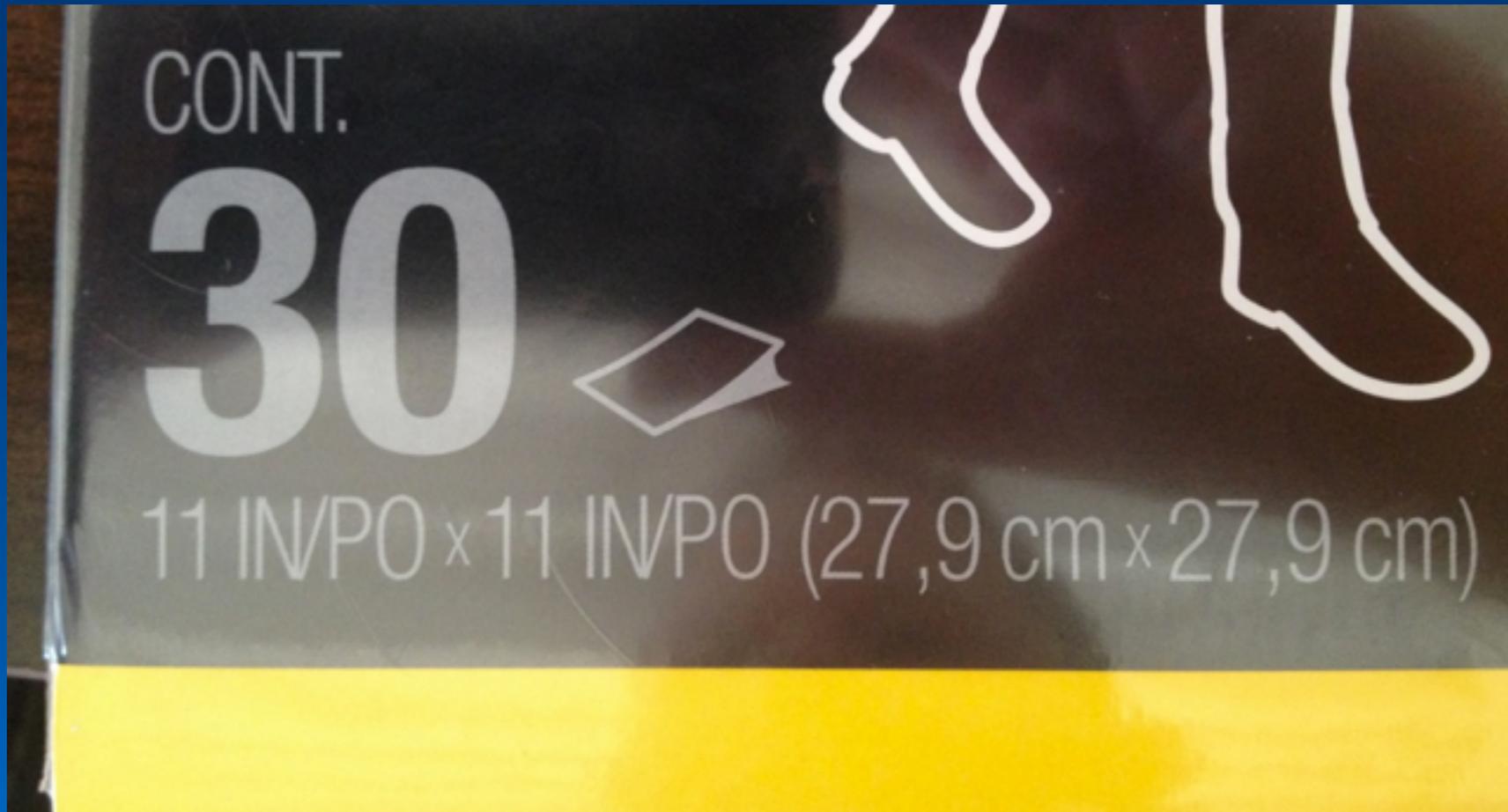
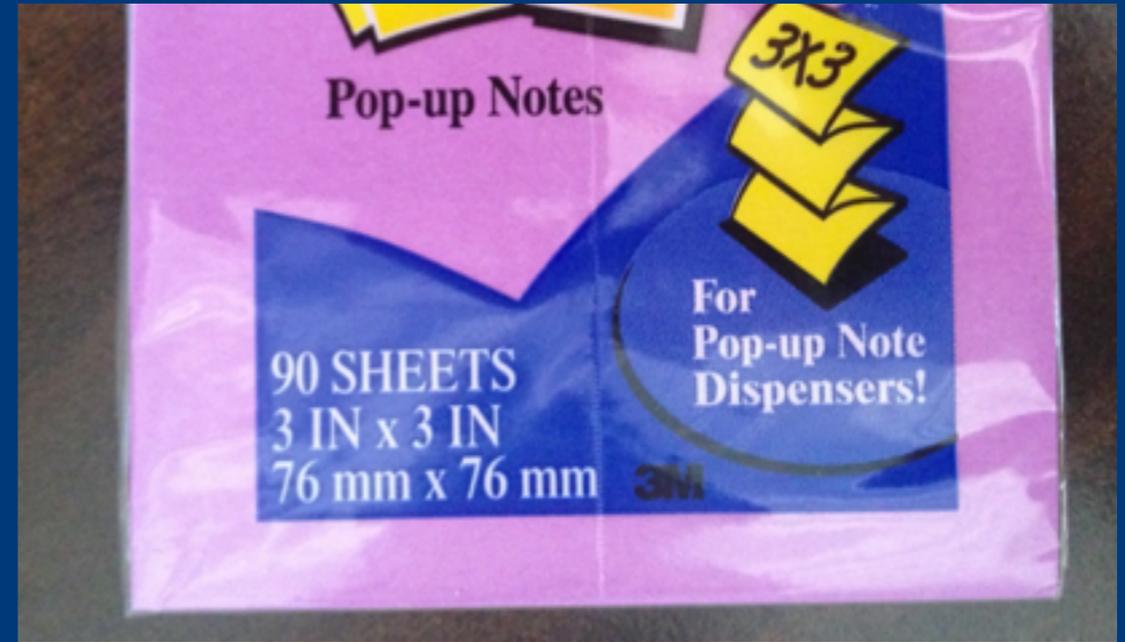
CONT.

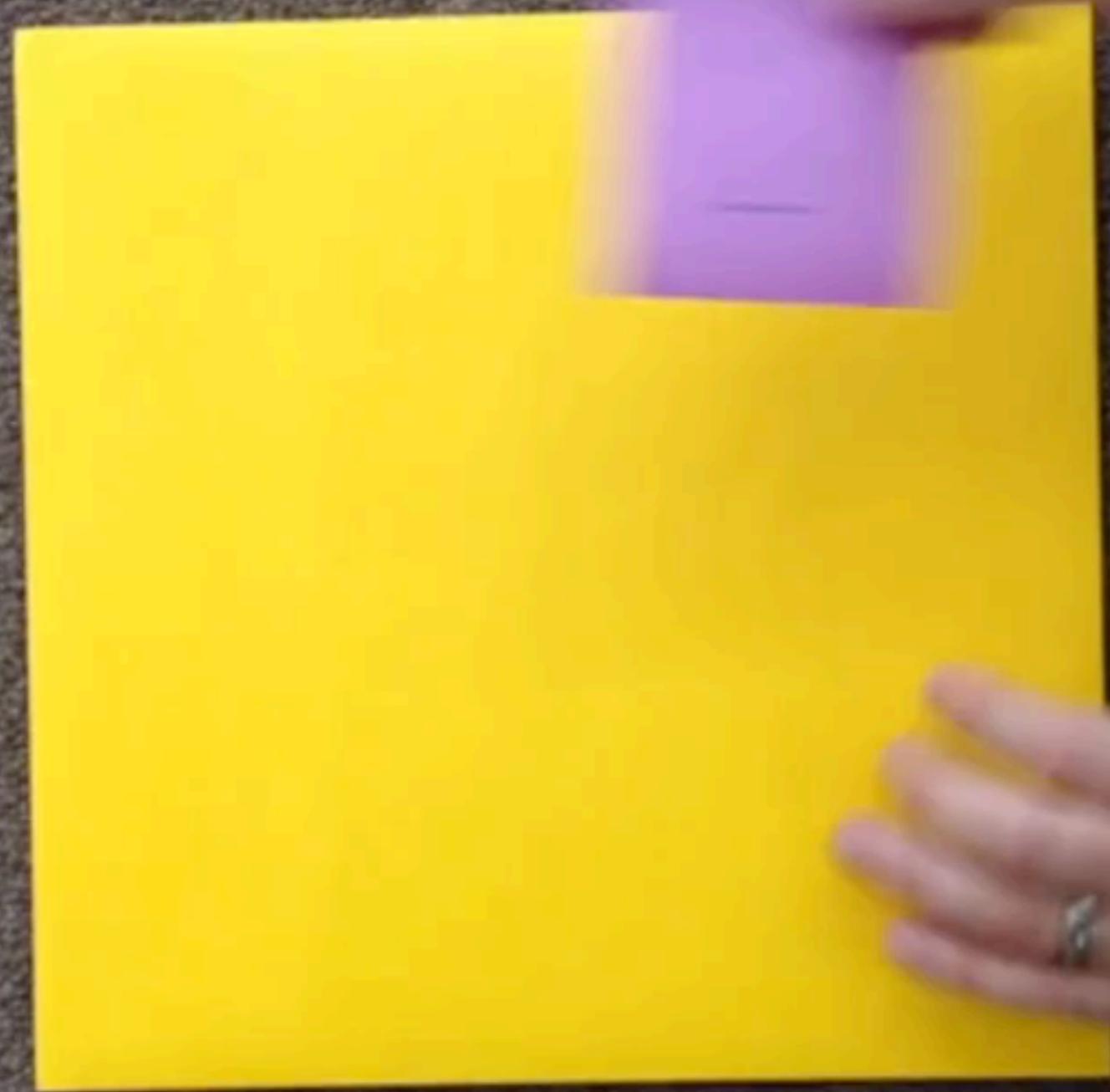


What do you notice?

What do you wonder?

need more?





Parking Lot

+
positive

▲
change

?
**not sure
going
forward**



ah-ha!

Questioning My Metacognition

Trying to be a better teacher



3-Acts Lessons

Check out [Dan's Blog](#) or watch [Math Class Needs a Makeover](#)

Date Added	Lesson Title	Standard 1	Standard 2
4/17/2014	Peas in a Pod	K.NBT.1	K.CC.4
4/25/2014	Dotty	K.CC.1,2,3	K.CC.4,5
1/16/2015	Counting Squares	K.NBT.1	K.CC.4,5
1/16/2015	Stage 5 Series	K.NBT.1	K.CC.4,5
3/24/2015	Shark Bait	K.NBT.1	K.CC.4,5
3/4/2014	Lil' Sister	K.MD.2	K.CC.6
9/1/2015	Bag-O-Chips	K.OA.4	K.OA.5

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My Addiction Feed

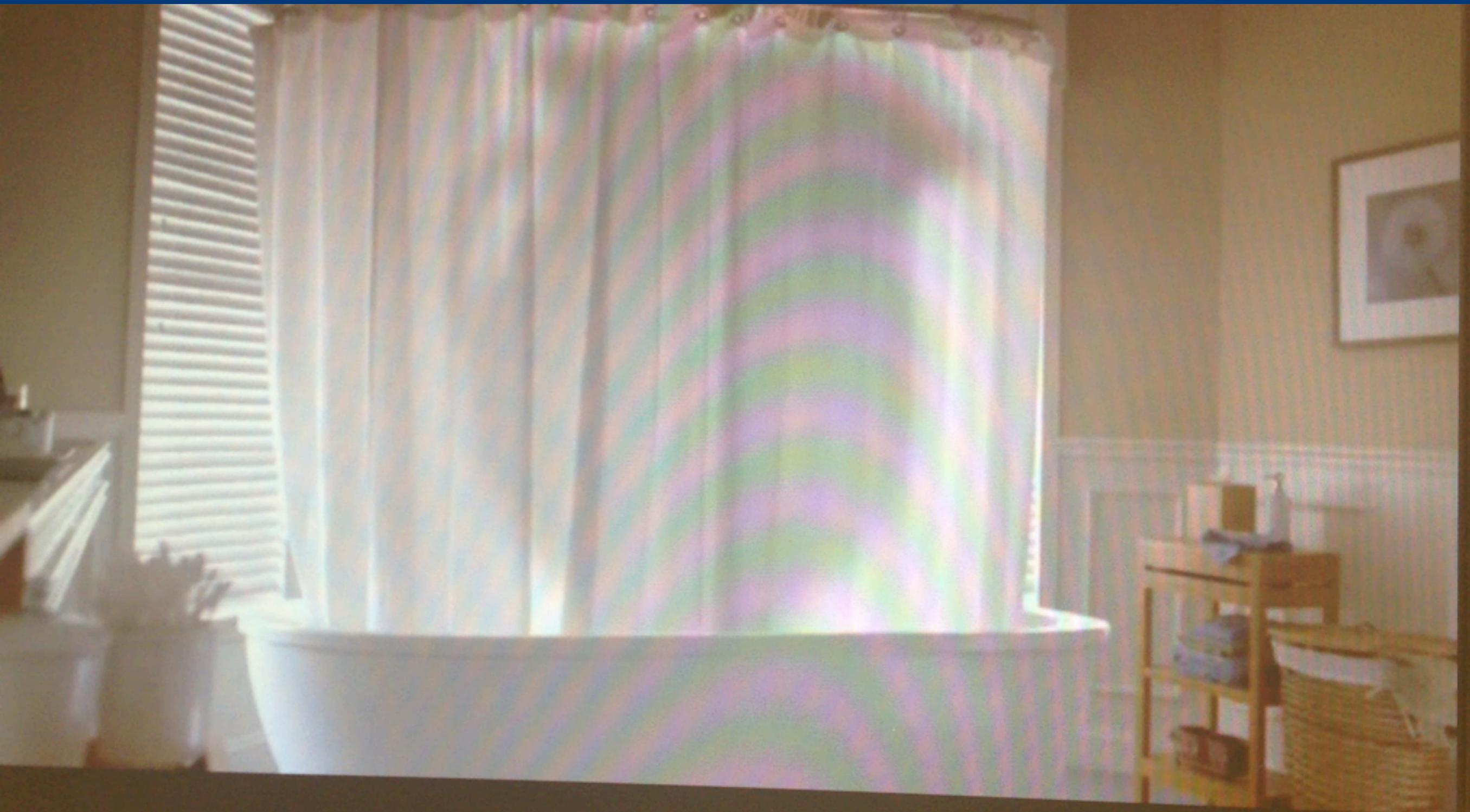
1. [Traditionalist Becoming Non-Traditional](#)
2. [Becoming the Math Teacher You Wish You'd Had](#)
3. [Following Learning](#)
4. [Continuous Everywhere but Differentiable Nowhere](#)
5. [Tools for the Common Core Standards](#)
6. [dy/dan](#)
7. [Math Minds](#)
8. [Elementary Teacher in Middle School](#)
9. [Exit 10A](#)
10. [Bridging The Gap](#)
11. [Robert Kaplinsky - Glenrock](#)



get plugged in

#MTBoS

#ElemMathChat





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WEBSITE: <https://gfletchy.com/nctm-2016/>