The Trajectories for the Effective Learning and Teaching of Mathematics in the Primary Grades (k-2)

"Children follow natural developmental progressions in learning and development. As a simple example, children first learn to crawl, which is followed by walking, running, skipping, and jumping with increased speed and dexterity. Similarly, they follow natural developmental progressions in learning math; they learn mathematical ideas and skills in their own way. When educators understand these developmental progressions, and sequence activities based on them, they can build mathematically enriched learning environments that are developmentally appropriate and effective. These developmental paths are a main component of a learning trajectory."
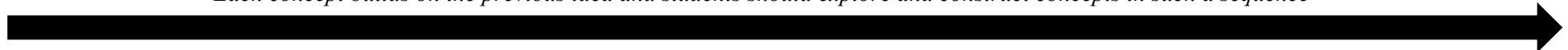(Clements & Sarama, 2009)

"The fundamental core of effective teaching of mathematics combines an understanding of how children learn and promote that language through problem solving, and how to plan for and assess that learning on a daily basis"
(Van de Walle & Lovin, 2006)

Number Sense Trajectory –*Putting It All Together*

	Subitizing	Comparison	Counting	One-to-One Correspondence	Cardinality	Hierarchical Inclusion	Number Conservation
Trajectory	Being able to visually recognize a quantity of 5 or less.	Being able to compare quantities by identifying which has more and which has less.	Rote procedure of counting. The meaning attached to counting is developed through one-to-one correspondence.	Students can connect one number with one object and then count them with understanding.	Tells how many things are in a set. When counting a set of objects, the last word in the counting sequence names the quantity for that set.	Numbers are nested inside of each other and that the number grows by one each count. 9 is inside 10 or 10 is the same as 9 + 1.	The number of objects remains the same when they are rearranged spatially. 5 is 4&1 OR 3&2.

Each concept builds on the previous idea and students should explore and construct concepts in such a sequence

→

	Spatial Relationship Patterned Set Recognition	One and Two-More or Less	Understanding Anchors	Part-Part-Whole Relationship
Number Relationships	Students can learn to recognize sets of objects in patterned arrangements and tell how many without counting.	Students need to understand the relationship of number as it relates to +/- one or two. Here students should begin to see that 5 is 1 more than 4 and that it is also 2 less than 7.	Students need to see the relationship between numbers and how they relate to 5s and 10s. 3 is 2 away from 5 and 7 away from 10.	Students begin to conceptualize a number as being made up from two or more parts.

Developing Number Sense: The Big Picture
This trajectory is designed to show number sense development through the early years of elementary school. Although the graphic organizer flows horizontally left to right, it also aligns vertically. Each concept builds from the previous stage and is the foundation to developing the number sense required of all students.

Addition and Subtraction Strategies

One/Two More/Less	Make a Ten	Near Doubles
These facts are a direct application of the One/Two More/ Less than relationships	Use a quantity from one addend to give to another to make a ten then add the remainder. $9 + 7 = 10 + 6$	Using the doubles anchor and combining it with 1 and 2 more/less.
Facts with Zero Need to be introduced so that students don't overgeneralize that answers to addition are always bigger.	**Doubles** Many times students will use doubles as an anchor when adding and subtracting.	

Clements, D. H. & Sarama, J. (2009). Learning trajectories in early mathematics – sequences of acquisition and teaching. *Encyclopedia of Language and Literacy Development* (pp. 1-7). London, ON: Canadian Language and Literacy Research Network. Retrieved [insert date] from http://literacyencyclopedia.ca/pdfs/topic.php?topId=270

Van de Walle, J., A., & Lovin, L., H. (2006). *Teaching student-centered mathematics grades K-3.* Boston: Person Education.

Curated by @gfletchy