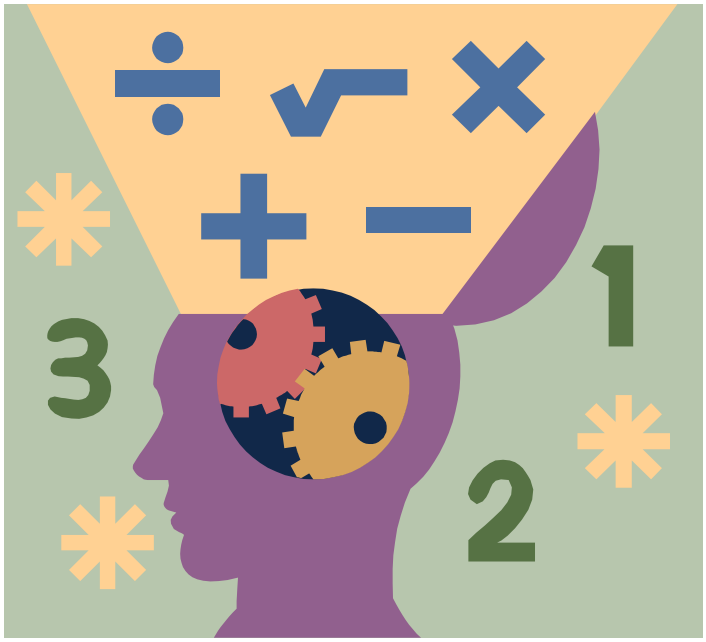
Effective Collaboration Framework



Teachers unlocking students' mathematical potential through purposeful collaboration

Created and prepared for CSRA-RESA

Preface

All students learn differently and there is no one “correct” way to teach. It is for this reason that there is not a silver bullet curriculum or textbook that can successfully meet the needs of all educators and students.

In order to truly meet the needs of students, educators must reference and employ multiple resources to design a curriculum that is specifically tailored to meet the needs of their students.

Table of Contents

Standards for Mathematical Practice

The 8 Standards for Mathematical Practice (SMP) should be at the heart of every lesson you teach and should be what drives the mathematics. **Math must not be about *answer-getting*.** Students must be given the opportunity to develop reasoning and the ability to communicate precisely.

In years past, the focus of mathematics was solely on content standards. The specific alignment of what the SMPs should look like and how teachers can integrate them into their everyday teaching practice can be found within each Grade Level Overview provided by the Georgia Department of Education.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

"The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years." (CCSSM, pg. 8)

Integration of the Standards for Mathematical Practice

The SMPs should drive instruction from kindergarten through high school. Realistically, it would be difficult to infuse all 8 SMPs into each lesson. However, there are 2 standards which must be evident in every single lesson. The following two standards are non-negotiable:

- *#1 Make sense of problems and persevere in solving them-* if the math is not problematic it should not be used. Often, teachers give students a task or problem which students are able to easily complete within 10 minutes. If this is the case, teachers may be settling for rote and "low-engagement" tasks.

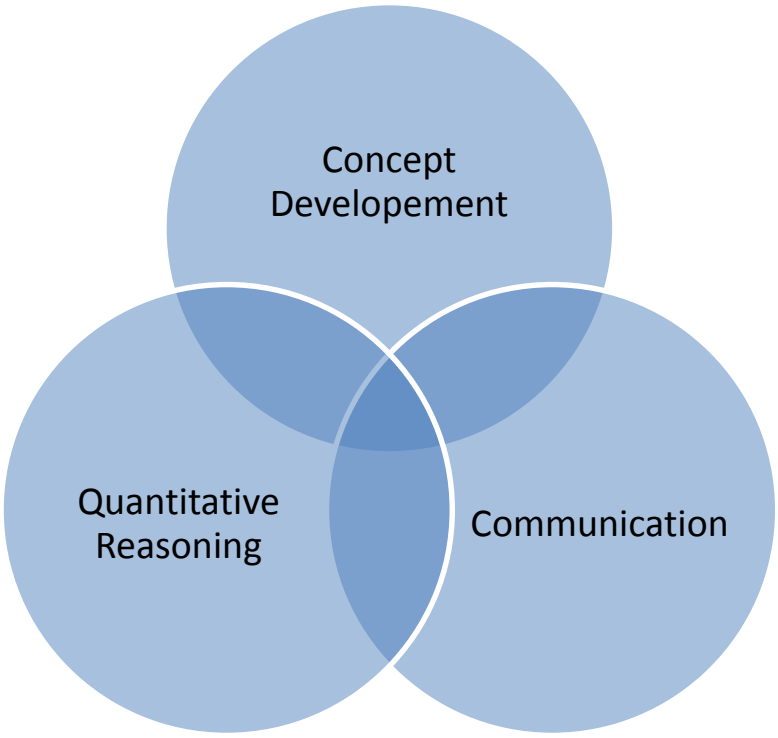
- *#6 Attend to precision-* whether communicating one's math reasoning verbally or through a written recording, it is imperative that teachers and students remain precise in their calculations and explanations.

*****Every teaching decision moving forward is made with a focus that remains on the Standards for Mathematical Practice. The following two pages show a teacher reflection tool modified from *Institute for Advanced Study/Park City Mathematics Institute*. The rubric is provided to guide teachers in their decision making when planning units and tasks*****

	NEEDS IMPROVEMENT	EMERGING	PROFICIENT	EXEMPLARY
#1 MAKES SENSE OF PROBLEMS AND PERSERVER IN SOLVING THEM	**Task:** □ Is strictly procedural. □ Does not require students to check solutions for errors. **Teacher:** □ Does not allow for wait time; asks leading questions to rush through task. □ Does not encourage students to individually process the tasks. □ Is focused solely on answers rather than processes and reasoning.	**Task:** □ Is overly scaffolded or procedurally "obvious". □ Requires students to check answers by plugging in numbers. **Teacher:** □ Allots too much or too little time to complete task. □ Encourages students to individually complete tasks, but does not ask them to evaluate the processes used. □ Explains the reasons behind procedural steps. □ Does not check errors publicly.	**Task:** □ Is cognitively demanding. □ Has multiple entry points. □ Requires a balance of procedural fluency and conceptual understanding. □ Requires students to check solutions for errors using one other solution path. **Teacher:** □ Allows ample time for all students to struggle with task. □ Expects students to evaluate processes implicitly. □ Models making sense of the task (given situation) and the proposed solution.	**Task:** □ Has multiple entry points and solution paths. □ Requires students to defend and justify their solution by comparing multiply solution paths. **Teacher:** □ Differentiates to keep advanced students challenged □ Integrates time for explicit meta-cognition. □ Expects students to make sense of the task and the proposed solution.
#2 REASON ABSTARCTLY AND QUANTITITIVLEY	**Task:** □ Lacks context. □ Does not make use of multiple representations or solution paths. **Teacher:** □ Does not expect students to interpret representations. □ Expects students to memorize procedures with no connection to meaning.	**Task:** □ Is embedded in a contrived context. **Teacher:** □ Expects students to model and interpret tasks using a single representation. □ Explains connections between procedures and meaning.	**Task:** □ Has realistic context. □ Requires students to frame solutions in a context. □ Has solutions that can be expressed with multiple representations. **Teacher:** □ Expects students to interpret and model using multiple representations. □ Provides structure for students to connect algebraic procedures to contextual meaning. □ Links mathematical solution with a question's answer.	**Task:** □ Has relevant realistic context. **Teacher:** □ Expects students to interpret, model, and connect multiple representations. □ Prompts students to articulate connections between algebraic procedures and contextual meaning.
#3 CONSTRUCT VIABLE ARGUMENT	**Task:** □ Is either ambiguously stated or lacks evidence and/or reasoning. **Teacher:** □ Does not ask students to present arguments or solutions. □ Expects students to follow a given solution path without opportunities to make conjectures.	**Task:** □ Is not at the appropriate level. **Teacher:** □ Does not help students differentiate between assumptions and logical conjectures. □ Asks students to present arguments but not to evaluate them. □ Allows students to make conjectures without justification.	**Task:** □ Avoids single steps or routine algorithms. **Teacher:** □ Identifies students' assumptions. □ Models evaluation of student arguments. □ Asks students to explain their conjectures.	**Teacher:** □ Helps students differentiate between assumptions and logical conjectures. □ Prompts students to evaluate peer arguments. □ Expects students to formally justify the validity of their conjectures.
#4 MODEL THE MATHEMATICS	**Task:** □ Requires students to identify variables and to perform necessary computations. **Teacher:** □ Identifies appropriate variables and procedures for students. □ Does not discuss appropriateness of model.	**Task:** □ Requires students to identify variables and to compute and interpret results. **Teacher:** □ Verifies that students have identified appropriate variables and procedures. □ Explains the appropriateness of model.	**Task:** □ Requires students to identify variables, compute and interpret results, and report findings using multiple representations. □ Illustrates the relevance of the mathematics involved. □ Requires students to identify extraneous/missing information. **Teacher:** □ Asks questions to help students identify appropriate variables and procedures. □ Facilitates discussions in evaluating the appropriateness of model.	**Task:** □ Requires students to identify variables, compute and interpret results, report findings, and justify the reasonableness of their results and procedures within context of the task. **Teacher:** □ Expects students to justify their choice of variables and procedures. □ Gives students opportunity to evaluate the appropriateness of model.

#5 USE APPROPRIATE TOOLS STRATEGICALLY	**Task:** □ Does not require additional learning tools. **Teacher:** □ Does not incorporate additional learning tools.	**Task:** □ Lends itself to a single learning tool. □ Does not involve mental computations or estimation. **Teacher:** □ Demonstrates use of appropriate learning tool.	**Task:** □ Lends itself to multiple learning tools. □ Gives students opportunity to develop fluency in mental computations. **Teacher:** □ Chooses appropriate learning tools for student use. □ Models error checking by estimation.	**Task:** □ Requires multiple learning tools (i.e., graph paper, calculator, manipulatives). □ Requires students to demonstrate fluency in mental computations. **Teacher:** □ Allows students to choose appropriate learning tools. □ Creatively finds appropriate alternatives where tools are not available.
#6 ATTEND TO PRECISION	**Task:** □ Gives imprecise and/or unclear instructions. **Teacher:** □ Does not intervene when students are being imprecise. □ Does not point out instances when students fail to address the question completely or directly.	**Task:** □ Has overly detailed or wordy instructions. **Teacher:** □ Inconsistently intervenes when students are imprecise. □ Identifies incomplete responses but does not require student to formulate further response.	**Task:** □ Has precise instructions. **Teacher:** □ Consistently demands precision in communication and in mathematical solutions. □ Identifies incomplete responses and asks student to revise their response.	**Task:** □ Includes assessment criteria for communication of ideas. **Teacher:** □ Demands and models precision in communication and in mathematical solutions. □ Encourages students to identify when others are not addressing the question completely.
#7 LOOK FOR AND MAKE USE OF STRUCTURE	**Task:** □ Requires students to automatically apply an algorithm to a task without evaluating its appropriateness. **Teacher:** □ Does not recognize students for developing efficient approaches to the task. □ Requires students to apply the same algorithm to a task although there may be other approaches.	**Task:** □ Requires students to analyze a task before automatically applying an algorithm. **Teacher:** □ Identifies individual students' efficient approaches, but does not expand understanding to the rest of the class. □ Demonstrates the same algorithm to all related tasks although there may be other more effective approaches.	**Task:** □ Requires students to analyze a task and identify more than one approach to the Problem. **Teacher:** □ Facilitates all students in developing reasonable and efficient ways to accurately perform basic operations. □ Continuously questions students about the reasonableness of their intermediate results.	**Task:** □ Requires students to identify the most efficient solution to the task. **Teacher:** □ Prompts students to identify mathematical structure of the task in order to identify the most effective solution path. □ Encourages students to justify their choice of algorithm or solution path.
#8 LOOK FOR AND EXPRESS REGULARITY IN REASONING	**Task:** □ Is disconnected from prior and future concepts. □ Has no logical progression that leads to pattern recognition. **Teacher:** □ Does not show evidence of understanding the hierarchy within concepts. □ Presents or examines task in isolation.	**Task:** □ Is overly repetitive or has gaps that do not allow for development of a pattern. **Teacher:** □ Hides or does not draw connections to prior or future concepts.	**Task:** □ Reviews prior knowledge and requires cumulative understanding. □ Lends itself to developing a pattern or structure. **Teacher:** □ Connects concept to prior and future concepts to help students develop an understanding of procedural shortcuts. □ Demonstrates connections between tasks.	**Task:** □ Addresses and connects to prior knowledge in a non-routine way. □ Requires recognition of pattern or structure to be completed. **Teacher:** □ Encourages students to connect task to prior concepts and tasks. □ Prompts students to generate exploratory questions based on current task. □ Encourages students to monitor each other's intermediate results.

Planning your Unit



In this section, you will find information about the 3 main components of every math lesson and how to purposefully plan to integrate the components in your units, lessons and instruction.

Concept Development- the focus of each unit and lesson should be explicitly connected to the critical areas in the CCSS introduction for each grade. The critical areas have also been provided by the Georgia Department of Education in the Grade Level Overviews. The development of the critical areas and key concepts must be orchestrated through a constructivist approach to learning. *SMP #1, 4 & 5*

Quantitative Reasoning- as students grapple with concept development they must be able to reason abstractly and quantitatively. This reasoning can happen both in and out of context. It entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. *SMP # 2, 4, 7 & 8*

Communication-as students begin to conceptually develop their understanding and reasoning they must be given an opportunity to articulate their thinking. This can be done using multiple teaching strategies but it is a MUST so that students can summarize their thinking. SMP # 3, 4, & 6

Planning for Concept Development

The CCGPS Frameworks provided by the Georgia Department of Education are an ideal place to start because they are created by Georgia educators, FOR Georgia educators. The CCGPS Mathematics Unit Frameworks were published on July 1, 2014 at the GeorgiaStandards.Org and Teacher Resource Link (TRL) sites to provide Georgia educators with improved resources for instructional purposes. The unit frameworks reflect the thoughtful collaboration and dedication of mathematics teachers, coaches, and supervisors from across the state of Georgia. The K-12 instructional frameworks can be found using the following links:

- K-5 Instructional Frameworks (includes Grade Level Overviews)
- 6-8 Instructional Frameworks (includes Grade Level Overviews)
- High School Instructional Frameworks (includes Grade Level Overviews)

There are many things to consider when collaborating and planning to teach a unit. Before diving into the first lesson, the teacher or grade level team should:

- Begin with the culminating task of the unit
- Discuss how to teach content through the SMPs
- Identify key standards and how they connect back to the critical areas(s)
- Identify potential misconceptions
- Discuss Formative Assessment – the "What" and the "How"
- Discuss recommended supplemental resources and materials
- Discuss vocabulary embedded within the task/unit

"To begin with the end in mind means to start with a clear understanding of your destination. It means to know where you're going so that you better understand where you are now so that the steps you take are always in the right direction."

Steven Covey

<u>Step 1:</u> Begin with the culminating task of the unit

- Many times, teachers assign tasks to students without ever having worked through the tasks themselves. This can cause problems within the lesson and often frustrates teachers and students. Solving the task with your team is an extremely important piece when collaborating and planning for a unit. Prior *<u>assignment of and teaching of tasks is not the same as working through a task.</u>*

- Many teachers become frustrated with a problem-based approach to teaching because they are not equipped with the appropriate questions to scaffold learning. By sitting down and working through the tasks you will gain insight and an understanding into the struggles your students encounter. By working through the tasks you will recognize that there are multiple entry points and multiple solution paths. This is a vital component. When teachers completely understand a task, they are able to effectively question students.

- After working through the culminating task, look through all of the tasks in the unit and choose those that help support the understanding of the culminating task. *Note that these choices may change after your formative assessment, but your plan is always malleable and can be adapted based upon the needs of your students.*

<u>Step 2:</u> Discuss how to teach content through the SMPs

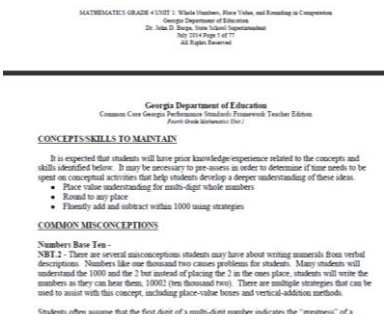
- Each task within the GaDOE frameworks has a list of the SMPs that are a focus for that particular task. Next to each SMP is also a description of how students may engage in it within the context of the task. Use the SMP Rubric previously discussed (page 5 & 6).

<u>Step 3:</u> Identify key standards and how they connect back to the critical areas(s)

- The grade level overviews unpack the standards and explain the expectation of each standard. If further explanation of a standard is required you can post a question to the appropriate Georgia Math Wikispace or search for the specific standard using <u>Learnzillion</u> and <u>Illustrative Mathematics</u>.

<u>Step 4:</u> Identify potential misconceptions

- Being able to identify and anticipate student misconceptions is at the heart of any good teaching practice. In each unit overview are the common misconceptions that occur when students are working with the standards addressed in the forthcoming unit. Within each of the lessons included in the frameworks are common misconceptions that pertain specifically to that lesson.


MATHEMATICS GRADE 4 UNIT 1: Whole Numbers, Place Value, and Rounding to Comparison
Georgia Department of Education
Dr. John D. Barge, State School Superintendent
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Georgia Department of Education
Common Core Georgia Performance Standard: Framework Teacher Edition
Fourth Grade Mathematics Unit 1

CONCEPTS/SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Place value understanding for multi-digit whole numbers
- Round to any place
- Fluently add and subtract within 1000 using strategies

COMMON MISCONCEPTIONS

Numbers Base Ten -
NBT.2 - There are several misconceptions students may have about writing numerals from verbal descriptions. Numbers like one thousand two causes problems for students. Many students will understand the 1000 and the 2 but instead of placing the 2 in the ones place, students will write the numbers as they can hear them, 10002 (ten thousand two). There are multiple strategies that can be used to assist with this concept, including place-value boxes and vertical-addition methods.

Students often assume that the first digit of a multi-digit number indicates the "greatness" of a

Step 5: Discuss Formative Assessment – the "What" and the "How"

- Formative Assessment Lessons (FALs) are designed for teachers to use in order to target specific strengths and weaknesses in their students' mathematical thinking in different areas. A Formative Assessment Lesson (FAL) includes a short task that is designed to target mathematical areas specific to a range of tasks from the unit. Teachers should give the task where suggested in the unit table of contents and use the information from the FAL to differentiate the material to fit the needs of the students. The FAL should not be graded. It is to be used to guide instruction. Each unit provides a minimum of one FAL which can be used to guide instruction during the unit.

- FALs fall under Formative Instructional Practices (FIP) which are intentional behaviors that teachers and students use to obtain information about learning so that decisions can be made about additional learning opportunities. Formative instructional practices are the formal and informal ways that teachers and students gather and respond to evidence of student learning. To learn more about how FALs and FIP are related, look on the *CCGPS Effective Instructional Practices Guide* (pg. 10) provided by the DOE.

Step 6: Discuss supplemental material and materials

- The math frameworks alone cannot address all of the learning needs of your students. In addition, it does not provide enough material and resources to support remediation and extension of learning. THE FRAMEWORKS SHOULD NOT BE USED AS A TEXTBOOK where every lesson is taught in the specific order in which it is presented, nor should the frameworks tasks be given to students to do independently for homework.

- While collaborating for the upcoming unit, you and your team will pull in resources and tasks from multiple sources. The Math Wikispace provided by the DOE is an excellent place to begin.
    - Ga Math Wikispaces K-5
    - Ga Math Wikispaces 6-8
    - Ga Math Wikispaces 9-12
    - Illustrative Mathematics
    - Inside Mathematics
    - 3-Act Tasks found on the *CCGPS Effective Instructional Practices Guide* (pg. 3) provided by the DOE.

Step 7: Discuss vocabulary identified in the unit

- Identify key vocabulary terms that will be introduced throughout the unit. It is important to recognize that these terms should be introduced within the context of the lesson and not taught in isolation. Using the correct vocabulary within a context is extremely important as this ties back to the Standards for Mathematical Practice, particularly SMPs 6 and 8.

A unit plan template is included on page 13 and a completed sample unit plan is on page 14.
The completed unit plan is for 4th Grade Unit 3- Fraction Equivalents

<table>
<tr><td>Unit:
Timeframe:</td><td colspan="2" rowspan="2">Misconceptions:</td></tr>
<tr><td rowspan="2">Standards & Critical Areas:</td></tr>
<tr><td>Vocabulary:</td><td>Formative Assessment:</td></tr>
<tr><td>GA Frameworks Tasks:</td><td>Supplementary Tasks:</td><td>Resources:</td></tr>
</table>

<table>
<tr><td rowspan="2">Unit: 4th Grade Unit 3- Fraction Equivalents
Time Frame: 5-6 weeks</td><td colspan="2" rowspan="2">Misconceptions:
NF.1 & NF.2 - Students think that when generating equivalent fractions they need to multiply or divide either the numerator or denominator, such as, changing $^1/_2$ to sixths. They would multiply the denominator by 3 to get $^1/_6$, instead of multiplying the numerator by 3 also. Their focus is only on the multiple of the denominator, not the whole fraction.
Teacher note:
• Simplifying fractions is NOT a standard so it is not necessary to teach it.
• Use number lines that do not start at zero to deepen students understanding of number and the power of number lines.</td></tr>
<tr><td rowspan="2">Critical Areas: #2 Students develop understanding of fraction equivalence and operations with fractions.

Standards: MCC4.NF.1 & 2- equivalent fractions and comparing fractions

Fraction Progression –Illustrative Mathematics</td></tr>
<tr><td>Vocabulary: fraction denominator equivalent sets improper fraction increment mixed number numerator proper fraction term unit fraction whole number</td><td>Formative Assessment:
• Picking Fractions (included in GFW)
• Given the Part Find the Whole & Given the Whole Find the Part (Van de Walle Figure 12.10 and 12.11)</td></tr>
<tr><td>GA Frameworks Tasks:
• Fraction Kits PAGE 10
• Red Rectangles PAGE 14
• Pattern Block Puzzles PAGE 22
• Benchmark Fractions PAGE 29
• More or Less PAGE 33
• Close to 0, ½, or 1 PAGE 40
• Their Fair Share PAGE 44
• *Eating Cookies PAGE 48
• Equivalent Fractions PAGE 56
• Making Fractions PAGE 64
• Write About Fractions PAGE 68</td><td>Supplementary Tasks:
Illustrative Mathematics Tasks:
• Running Laps
• Fractions and Rectangles

Tasks from Van de Walle:
• How Far Did She Go? (Activity 12.5)
• Apples and Bananas (Activity 12.14)</td><td>Resources:
Fraction Progression –Illustrative Mathematics

Van de Walle Teaching Student Centered Mathematics 3-5 (chapter 12)

Tasks from Van de Walle for quick checks along the way and to assist in forming small groups and differentiation:
• Playground Fractions (12.1)
• Close Fractions (12.11)</td></tr>
</table>

Quantitative Reasoning

As students engage in the development of key mathematical concepts they will continuously reason quantitatively. With that being said, there are some ways that you can specifically focus on the quantitative reasoning of your students:

- Number Talks
- Fawn Nguyen Math Talks for middle and high school: How We Do Math Talks
- NZ Maths Numeracy Project

Number Talks *(taken from GaDOE) Further information on Number Talks can be found on the K-5 Wikispace - "How to Teach Math".*

A Number Talk is a 10 to 15 minute whole group mental math activity where students find the answer to a math problem in their heads, then share aloud the strategies they used to find that answer. This strategy helps to develop quality student discourse in a whole class setting as students are encouraged to explain their thinking, justify their reasoning, and make sense of each other's strategies.

During a Number Talk, the teacher steps away from his/her role of authority, and into the role of facilitator by asking students questions, recording student responses on the board, and encouraging students to make meaning out of the mathematics through verbal exchange.

A Number Talk can be used to address gaps in student skills or understanding, to confront anticipated misconceptions, to surface multiple strategies, and/or as a formative assessment when introducing new concepts. Number Talks also build flexibility, accuracy and efficiency with numbers for all students.

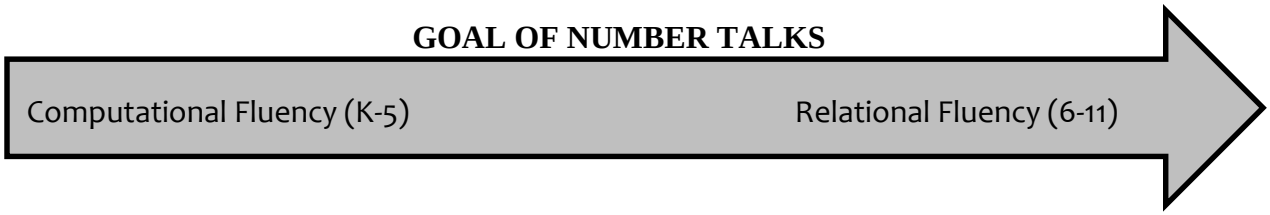
In **lower elementary**, students might experience a Number Talk where they have to look at a pattern of dots for 3 seconds, and share strategies for how they knew the total number of dots.

- In **upper elementary**, students may be asked to multiply 25 × 8 and may use different decomposition strategies or their knowledge of money to calculate.

- In **middle school**, students may be asked to mentally find 35% of 160.

- In **high school**, they may share multiple strategies for solving 1252/3 • 2. Number Talks may be used to make sense of grade-level content, but can also build from concepts from previous classes by starting with a dot talk or a simple arithmetic problem at any grade, based upon student needs.

GOAL OF NUMBER TALKS



When and why is this useful?

- To help students move from a reliance on memorization to truly understanding numbers and their relationships to each other.
- To help students recognize structure, and use that structure to understand more complex mathematics.
- As a regular routine where the problems in a series of Number Talks build on each other.
- To launch a task by activating students' prior knowledge.
- To provide students the opportunity to practice explaining their thinking and asking each other questions.
- To develop a stronger sense of mathematical identity and self-confidence in students since mistakes are treated as learning opportunities and everyone's opinion contributes to group knowledge.

What can students learn from this experience?

- Flexibility, accuracy and efficiency with mathematical thinking
- Ways to make sense of the mathematics and talk about the strategies used to solve a problem.
- Ease with composing and decomposing numbers
- Conceptual understanding of the relationships between numbers
- Computation strategies
- Mathematical reasoning skills
- Precision in explanations of mathematical thinking
- Multiple strategies and multiple representations for finding an answer
- Learning from the ideas of peers
- Confidence and motivation, contributing to a positive mathematical identity
- Empowerment through validation of each person's mathematical thinking process
- The value of both successes and errors in deepening understanding

More information about Number Talks can be found on the *CCGPS Effective Instructional Practices Guide* (pg. 11) provided by the DOE.

Number Talks At-A-Glance

What do Number Talks look like?

- Students are mentally solving problems
- Students are given thinking time
- Thumbs up show when they are ready
- Teacher is recording students' thinking

Communication

- Having to talk out loud about a problem helps students clarify their own thinking
- Allow students to listen to other's strategies and value other's thinking
- Gives the teacher the opportunity to hear student's thinking

Mental Math

- When you are solving a problem mentally you must rely on what you know and understand about the numbers instead of memorized procedures
- You must be efficient when computing mentally because you can hold a lot of quantities in your head

Thumbs Up

- This is just a signal to let you know that you have given your students enough time to think about the problem
- If will give you a picture of who is able to compute mentally and who is struggling
- It isn't as distracting as a waving hand

Teacher as Recorder

- Allows you to record students' thinking in the correct notation
- Provides a visual to look at and refer back to
- Allows you to keep a record of the problems posed and which students offered specific strategies

Purposeful Problems

- Start with small numbers so the students can learn to focus on the strategies instead of getting lost in the numbers
- Use a number string (a string of problems that are related to and scaffold each other)

Starting Number Talks in your Classroom

- Start with specific problems in mind (Number Talks planning sheet on next page)
- Be prepared to offer a strategy from a previous student
- It is ok to put a student's strategy on the backburner
- Limit your number talks to about 15 minutes
- Ask a question, don't tell

Number Talks Unit Planning Guide

<table>
<tr><td colspan="3">Unit Focus:</td></tr>
<tr><td>Prior Knowledge required:

Vocabulary:</td><td colspan="2" rowspan="2">Possible Strategies and Solutions</td></tr>
<tr><td>Questions:</td></tr>
<tr><td colspan="2">Additional Notes:</td><td>Assessment Note:</td></tr>
</table>

Number Talks Unit Planning Guide

<table>
<tr><td colspan="3"><u>Unit Focus:</u> The focus of Number Talks in this unit will be for students to develop computational fluency through the use of student-invented multiplication strategies.</td></tr>
<tr><td rowspan="2"><u>Prior Knowledge required:</u>
• Conceptual understanding of multiplication
• Understanding of the term “multiples” and how it can be applied to solve

<u>Vocabulary:</u>
Multiple, factor, product, benchmark</td><td colspan="2"><u>Possible Strategies and Solutions</u></td></tr>
<tr><td>Making Landmark or Friendly Numbers
Question: 9 x 29
Sample Solutions:

9 x 30 = 270 “that’s one group of 9 too much, so. . .”
270 – 9 = 261
Or
9 x 25 = 225 “because 8 25’s is 200, so 1 more 25 is 225”
9 x 2 = 18
9 x 2 = 18 and 18 + 18 = 36. 225 + 36 = 261</td><td>Partial Products
Question: 4 x 115
Sample Solution:
4 x 115 = 4 x 100 + 4 x 10 + 4 x 5

4 x 100 = 400
4 x 10 = 40
4 x 5 = 20
400 + 40 + 20 = 460</td></tr>
<tr><td><u>Questions:</u>
-Who would like to share how they got their answer?
-I heard you say, did I hear correctly?
-Did anyone use a different method?
-Can someone explain _____’s strategy in their own words?
-Which strategy is the most efficient?</td><td>Breaking Factors into Smaller Factors
Question: 8 x 25
Sample Solution:
8 = 2 x 4
25 x 4 = 100
100 x 2 = 200,
so 8 x 25 = 200</td><td>It is important to understand that these are only some of the strategies that students may develop during Number Talks. These strategies should NOT be explicitly taught to students but discovered by them</td></tr>
<tr><td colspan="2">Additional Notes: I must remember that the primary goal of Number Talks is to develop efficient thinkers. Some students may want to create a large quantity of partial products (which is fine at first), but I want to continue to push them to be efficient.</td><td>Assessment Note: Remember to keep track of the students that only show one thumb and call on them first.</td></tr>
</table>

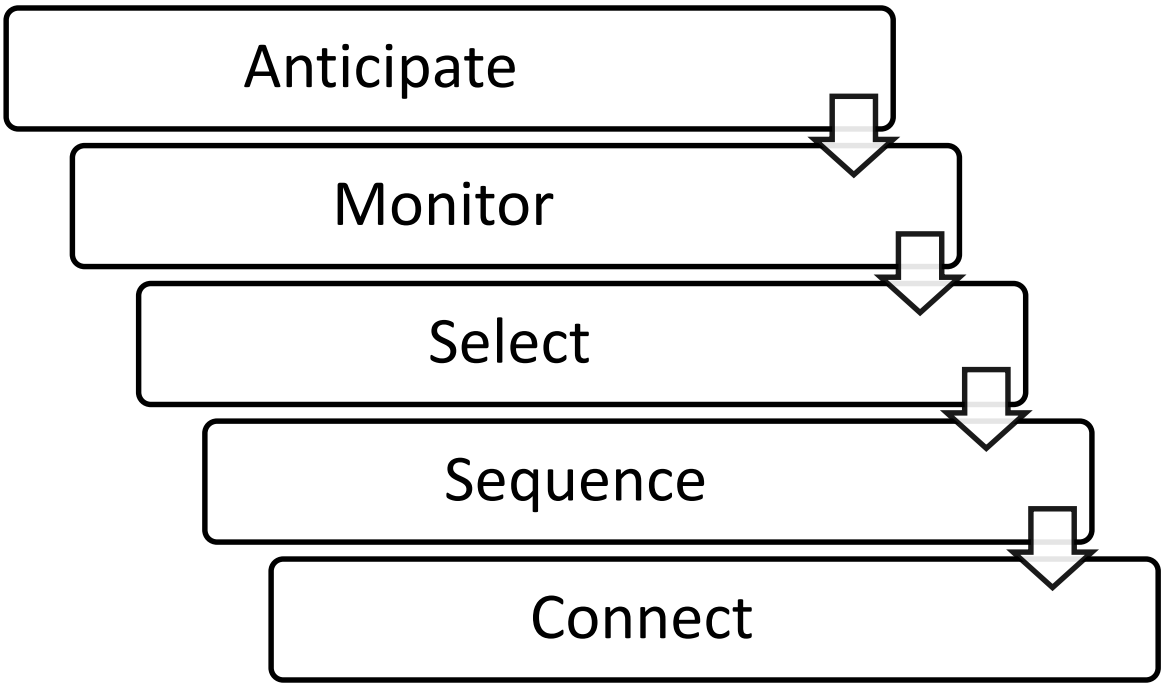
Communication

"Classroom discourse refers to the interactions between all the participants that occur throughout a lesson – in a whole-class, in small groups, between pairs of students, and with the teacher. The purpose of discourse is not for students to state their answer and get validation for the teacher but to engage all learners and keep the cognitive demand high."

Van de Walle, et al.

Promoting Student Discourse

Getting students engaged and communicating in problem-based lessons can be extremely difficult. Peg Smith from the University of Pittsburgh says that in order to get students talking in math class, we must be purposeful. Using the following sequence of steps will drastically improve your ability to foster conversation and discourse amongst your students.



- **Anticipate:** Identify possible solutions and misconceptions that students may use or encounter when working through the problem before doing the task with students. This will make you more prepared to ask the most effective questions.
- **Monitor:** As students are working through the task, the teacher is circulating the room, observing solutions, and engaging students through questions.
- **Selecting:** Purposefully choose student solutions to feature during the discussion.
- **Sequencing:** Purposefully choose the order in which to share the selected student responses.
- **Connect:** Make explicit mathematical connections between and *to* the student responses during the discussion.

Journaling *(taken from GaDOE Unit Overviews)*
"Students should be writing and talking about math topics every day. Putting thoughts into words helps to clarify and solidify thinking. By sharing their mathematical understandings in written and oral form with their classmates, teachers, and parents, students develop confidence in themselves as mathematical learners; this practice also enables teachers to better monitor student progress." NJ DOE

"Language, whether used to express ideas or to receive them, is a very powerful tool and should be used to foster the learning of mathematics. Communicating about mathematical ideas is a way for students to articulate, clarify, organize, and consolidate their thinking. Students, like adults, exchange thoughts and ideas in many ways—orally; with gestures; and with pictures, objects, and symbols. By listening carefully to others, students can become aware of alternative perspectives and strategies. By writing and talking with others, they learn to use more-precise mathematical language and, gradually, conventional symbols to express their mathematical ideas. Communication makes mathematical thinking observable and therefore facilitates further development of that thought. It encourages students to reflect on their own knowledge and their own ways of solving problems. Throughout the early years, students should have daily opportunities to talk and write about mathematics." NCTM

When beginning math journals, the teacher should model the process initially, showing students how to find the front of the journal, the top and bottom of the composition book, how to open to the next page in sequence (special bookmarks or ribbons), and how to date the page. Discuss the usefulness of the book, and the way in which it will help students retrieve their math thinking whenever they need it.

When beginning a task, you can ask, "What do we need to find out?" and then, "How do we figure it out?" Then figure it out, usually by drawing representations, and eventually adding words, numbers, and symbols. During the closing of a task, have students show their journals with a document camera or overhead when they share their thinking. This is an excellent opportunity to discuss different ways to organize thinking and clarity of explanations.

Use a composition notebook (the notebooks with graph paper are terrific for math) for recording or drawing answers to problems. The journal entries can be from Frameworks tasks, but should also include all mathematical thinking. Journal entries should be simple to begin with and become more detailed as the children's problem-solving, reasoning, and communication skills improve. Children should always be allowed to discuss their representations with classmates if they desire feedback. The children's journal entries demonstrate their thinking processes. Each entry could first be shared with a "buddy" to encourage discussion and explanation; then one or two children could share their entries with the entire class. Don't forget to praise children for their thinking skills and their journal entries! These journals are perfect for assessment and for parent conferencing. The student’s thinking is made visible!

Lesson Planning

There are several key components that should be evident in every math lesson. Routines drastically increase student engagement and accountability in addition to decreasing classroom management issues. The following components should be included in every lesson:

- Math Maintenance Activities
- Three-Phase Lesson Format/Math Instructional Framework
- Procedural Fluency
- Journaling/Communicating

Math Maintenance Activities *(taken from GaDOE Unit Overviews)*
In addition to instruction centered on the current unit of study, the math instructional block should include time devoted to reviewing mathematics that have already been taught, previewing upcoming mathematics, and developing mental math and estimation skills. There is a saying that if you don't use it, you'll lose it. If students don't have opportunities to continuously apply and refine the math skills they've learned previously, then they may forget how to apply what they've learned. Unlike vocabulary words for literacy, math vocabulary words are not used much outside math class, so it becomes more important to use those words in discussions regularly. Math maintenance activities incorporate review and preview of math concepts and vocabulary and help students make connections across domains. It's recommended that 15 to 30 minutes of the math instructional block be used for these math maintenance activities each day. It's not necessary nor is it recommended that teachers do every activity every day. Teachers should strive for a balance of math maintenance activities so that over the course of a week, students are exposed to a variety of these activities. Math maintenance time may occur before or after instruction related to the current math unit, or it can occur at a different time during the day.

The goals of this maintenance time should include:

- Deepening number sense, including subitizing, flexible grouping of quantities, counting forward and backward using whole numbers, fractions, decimals and skip counting starting at random numbers or fractional amounts
- Developing mental math skills by practicing flexible and efficient numerical thinking through the use of operations and the properties of operations
- Practicing estimation skills with quantities and measurements such as length, mass, and liquid volume, depending on grade level
- Practicing previously-taught skills so that students deepen and refine their understanding
- Reviewing previously-taught concepts that students struggled with as indicated on their assessments, including gaps in math concepts taught in previous grade levels
- Using a variety of math vocabulary terms especially those that are used infrequently
- Practicing basic facts using strategies learned in previous grade levels or in previous units to develop or maintain fluency
- Previewing prerequisite skills for upcoming math units of study
- Participating in mathematical discussions with others that require students to construct viable arguments and critique the reasoning of others

To accomplish these goals, math maintenance activities can take many different forms. Some activities include:

- Number Corner or Calendar Time
- Number Talks
- Estimation Activities/Estimation 180

In addition, math discussions, math journals and math games are appropriate not only for the current unit of study, but also for maintaining math skills that were previously taught.

Three Phase Lesson Format/Math Instructional Framework *(Van de Walle)*

The *Before Phase* of a Lesson (opening)
1) Get students mentally prepared to work on the problem and think about the previous knowledge they have that will be most helpful.
2) Be sure students understand the problem so they are ready to engage in solving it. You will not need to clarify or explain to individuals later in the lesson.
3) Clarify your expectations to students before they begin working on the problem. This includes both how they will be working (individually or in pairs or small groups) and what product you expect in addition to an answer.
The *During Phase* of a Lesson (work session)
1) Let go! Give students a chance to work without too much guidance. Allow and encourage students to embrace the struggle – it is an important part of doing mathematics.
2) Listen actively. Take this time to find out how different students are thinking, what ideas they are using, and how they are approaching the problem. This is a time for observation and assessment – not teaching.
3) Provide appropriate hints. Base any hints on students' ideas and ways of thinking. Be careful not to imply that you have the correct method of solving the problem.
4) Provide worthwhile extensions. Have something prepared for students who finish quickly.
The *After Phase* of a Lesson (closing)
1) Promote a mathematical community of learners. Includes all learners. Engage the class in productive discussion, helping students work together as a community of learners.
2) Listen actively without evaluation. Take this second major opportunity to find out how students are thinking – how they are approaching the problem. Evaluating methods and solutions is the duty of your students.
3) Summarize main ideas and identify future problems to explore. You can lay the groundwork for future activities as a natural part of this phase.

It is important to note that the length of task can vary from 20 minutes or longer than a single class period but the manner in which the task is introduced and taught remains consistent.

Fact and Procedural Fluency

Fluency is achieved AFTER strategies have been developed. In order to increase students' procedural fluency, they must be given an opportunity to practice implementing their student-invented strategy both mentally and in written form. The important thing to note here is that these activities are purposefully selected by the teacher to ensure that students are working towards mastery of specific content.

Procedural fluency can take place in many forms. Here a couple of examples:

- **Games:** games provide students an excellent opportunity to employ the strategies they have built through problem-solving. These games can be seen as spiral review or revisiting games that were previously introduced to the class.
- **Technology-Based Apps**
    - http://youcubed.org/teachers/2014/math-apps-and-games-we-like/
- **Problem-based tasks –** the same types of tasks that students use to develop strategies can be used as a means for them to practice their strategies and build fluency. These tasks can provide the purposeful practice needed for students to make the necessary connections to move from strategizing to automaticity.

Week of: Oct 13	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
Math Maintenance					
Number Talks **Focus:**					
Concept Development:					
Practice					
Journaling Communication					

Week of: Oct 13	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
Math Maintenance *5-10 minutes*	How tall is the giant wheel? *Students explain reasoning in journal before sharing*	How tall is Ms. Nguyen *Students explain reasoning in journal before sharing*	How Tall is Mr. Kraft? *Students explain reasoning in journal before sharing*	What's the height of the tractor tire? *Students explain reasoning in journal before sharing*	How many drops come in a bag? *Students explain reasoning in journal before sharing*
Number Talks *10-15 minutes* **Focus:** Partial Products	3 x 15 10 x 15 13 x 10 13 x 5 13 x 15	4 x 25 5 x 25 10 x 25 20 x 25 25 x 25	5 x 30 10 x 30 3 x 15 10 x 15 15 x 33	35 x 10 35 x 2 35 x 20 35 x 24	35 x 10 35 x 2 35 x 20 35 x 24
Concept Development: *45-60 minutes* Extend understanding of fraction equivalence and ordering.	Pattern Block Puzzles GFW Page 22	Benchmark Fractions GFW Page 29	Running Laps (Illustrative mathematics)	Their Fair Share GFW Page 44	FAL: Picking Fractions (included in GFW)
Fluency Focus *10-15 minutes* Factors and Multiples	The Factor Game Unit 2 Page 35 GFW	Multiplication Four Across	The Factor Trail Game Unit 2 Page 48 GFW	Multiplication Four Across	**Student Choice:** Factor Trail Multiplication Four Across The Factor Game
Journaling Communication *Students should journal throughout the lesson and/or use the following prompts:*	Using an area, length and set model, how many different ways can you model equivalent fractions for ½.	How are the numerator and denominator alike? How are they different?	Write 2 fractions with the same numerator. Which one is greater? Explain your thinking.	Choose and label 3 fractions on an open number line. Explain how you know your location on the number line is correct.	If you had two written fractions, how could you tell which one is less?

NZ MATHS Numeracy Project

Introduction

Teachers often struggle to identify where their students are in terms of the progressions for learning mathematics. The Ministry of Education for New Zealand has developed a comprehensive numeracy assessment complete with a diagnostic, formative, and summative student interview. This assessment can be used to form small groups and targeted intervention and extension activities/tasks for students. The NZ Maths Numeracy Project is applicable to students from Kindergarten through 9th grade. Many teachers, schools, and districts across Georgia are now looking to the Numeracy Project to better meet the needs of their students in the MTSS process.

The Number Framework

At the core of the Numeracy Projects is The Number Framework. The framework has been established to help teachers, parents, and students to understand the requirements of the Number knowledge and Number strategies sections of Common Core progression of building mathematical understanding and competence.

In the two main sections to the framework, the distinction is made between strategy and knowledge. The Strategy section describes the mental processes students use to estimate answers and solve operational problems with numbers. The Knowledge section describes the essential areas of mathematical knowledge for which students need automaticity. It is important that students make progress in both sections of the framework, as they develop in tandem.

The strategy section of the framework consists of a sequence of global stages. Progress through the stages indicates growth in knowledge and in the range of strategies that students have available.

The application of number knowledge and mental strategies is often described as 'number sense'. Strongly developed number sense leads to algebraic thinking.

Why the Numeracy Project?

The interviews are administered 3 times per year and serves as a diagnostic, formative, and summative assessment. The assessment is applicable to any K-8 students and to students struggling in high school mathematics. There are 2 separate assessments:

Global Strategy Stage (GloSS)

This assesses the strategies known and mastered by your student. From this assessment, a strategy stage is determined for each of three domains: addition/subtraction, multiplication/division, and ratio/proportions. An overall global stage is also identified. These strategy stages inform you how to assist your students develop more efficient strategies.

- 1st Benchmark Assessment
- 2nd Benchmark Assessment
- 3rd Benchmark Assessment

As a student is interviewed, the teacher records the responses to later identify where the student is in terms of their strategy stage (Number Framework on previous page). The record is placed in the student's permanent record at the end of the year, for reference and use by the teachers to come.

Individual Knowledge Assessment of Number (IKAN)

This assesses the knowledge stage of a student within four domains: place value, fractions, number order sequence, and basic facts. This is information a student should know and be able to recall without the application of a strategy. The IKAN assesses a student's ability to automatize numbers across multiple concepts (number order and sequence, basic facts, fractions, and place value). The assessments can be found here. As the Flash Player rolls through the questions, students record their answers on the student recording sheet.

The information gathered from these 2 assessments will help inform any teaching decisions moving forward such as small group placement, differentiation, and interventions.

Strategy Stage Descriptions

LEVEL 1	Stage 0 Emergent	Students at the Emergent stage are unable to consistently count a given number of objects because they lack knowledge of counting sequences and/or the ability to match things in one-to-one correspondence.
	Stage 1 One to One Counting	The One to One Counting stage is characterized by students who can count and form a set of objects up to ten but cannot solve simple problems that involve joining and separating sets, like 4 + 3.
	Stage 2 Counting From One on Materials	Students at the Counting From One on Materials stage rely on counting physical materials, like their fingers. They count all the objects in both sets to find an answer to a joining or separating of sets problem.
	Stage 3 Counting From One by Imaging	The Counting from One by Imaging stage is characterized by students counting all of the objects. Students at this stage are able to image visual patterns of the objects in their mind and count them.
	Stage 4 Advanced Counting	Students at the Advanced Counting stage understand that the end number in a counting sequence measures the whole set and can relate the addition or subtraction of objects to the forward and backward number sequences by ones, tens, etc.
LEVEL 2	Stage 5 Early Additive	At the Early Additive stage, students have begun to recognize that numbers are abstract units that can be treated simultaneously as wholes or can be partitioned and recombined. This is called part-whole thinking.
LEVEL 3	Stage 6 Advanced Additive	Students at the Advanced Additive stage are learning to choose appropriately from a repertoire of part-whole strategies. They see numbers as whole units in themselves but also understand that "nested" within these units is a range of possibilities for subdivision and recombining.
LEVEL 4-5	Stage 7 Advanced Multiplicative	Students at the Advanced Multiplicative stage are learning to choose appropriately from a range of part-whole strategies to solve and estimate the answers to problems involving multiplication and division. These strategies require one or more of the numbers involved in a multiplication or division to be partitioned, manipulated, then recombined.
	Stage 8 Advanced Proportional	Students at the Advanced Proportional stage are learning to select from a repertoire of part-whole strategies to solve and estimate the answers to problems involving fractions, proportions, and ratios. These strategies are based on finding common factors and include strategies for the multiplication of decimals and the calculation of percentages.

Resources Consulted in the Creation of this Document

GaDOE Resources

- GeorgiaStandards.Org
- Ga Math Wikispaces K-5
- Ga Math Wikispaces 6-8
- Ga Math Wikispaces 9-12

District Resources

- Oakland Unified School District

Supplementary Resources

Van de Walle, J. A., Karp, K., Lovin, L. A., & Bay-Williams, J. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades 3-5* (2nd ed.). Upper Saddle River, NJ: Pearson.

Further resources for teacher use:

- CCGPS Mathematics Frameworks and Comprehensive Course Overviews - https://www.georgiastandards.org/Common-Core/Pages/Math-9-12.aspx
- Mathematics Professional Learning Sessions (Summer Academy Presentations, Multi-Grade Webinars, Unit-by-Unit Webinar Series, Spring 2012 PL sessions of GPB, 2011 Standards for Mathematical Practices Webinars) - https://www.georgiastandards.org/Common-Core/Pages/Math-PL-Sessions.aspx
- Georgia Mathematics Teacher Forums - http://ccgpsmathematics9-10.wikispaces.com
- Mathematics Formative Assessment Lesson Videos - https://www.georgiastandards.org/Common-Core/Pages/Mathematics-Formative-Assessment-Lessons-Videos.aspx

GADOE Recommended Resources:

- SEDL videos - http://secc.sedl.org/common_core_videos/
- Georgia Virtual Learning - http://www.gavirtuallearning.org/Resources.aspx
- Dan Meyer – http://blog.mrmeyer.com/
- Robert Kaplinsky - http://robertkaplinsky.com/
- Estimation 180 - http://www.estimation180.com
- Graham Fletcher – http://gfletchy.com/
- Mike Wiernicki - http://mikewiernicki.com/
- Jenise Sexton - http://jenisesexton.wordpress.com/
- Graphing Stories - http://graphingstories.com
- Joe Schwartz - http://exit10a.blogspot.com
- Visual Patterns – http://www.visualpatterns/org
- Expeditionary Learning: Center for Student Work - http://elschools.org/student-work
- Illustrative Mathematics - http://www.illustrativemathematics.org/

- Mathematics Vision Project - http://www.mathematicsvisionproject.org/index.html
- LearnZillion - http://learnzillion.com
- ShareMyLesson - http://www.sharemylesson.com
- MAP - http://www.map.mathshell.org.uk/materials/index.php
- Inside Mathematics- http://www.insidemathematics.org/
- Edutopia – http://www.edutopia.org
- Teaching Channel - http://www.teachingchannel.org
- Achieve - http://www.achieve.org

The link below is a presentation (focusing on resources) delivered at the Special Education Conference in Athens and the Title Conference at Calloway Gardens Spring 2014.

http://prezi.com/m9abhhd0tmky/?utm_campaign=share&utm_medium=copy&rc=ex0share

This framework was created by Graham Fletcher for CSRA-RESA.

About

Graham has worked in education for over ten years as a classroom teacher, math coach, and as a district math specialist. He is a graduate of the University of Georgia where he earned his Ed.S in Math Education. Graham has conducted many workshops and professional development sessions at the local, district and state level as he continues to be an advocate for best practices in math class. He is the author of many tasks which are now included in the Georgia Instructional Frameworks and continues to be a change agent for K-12 mathematics.